

Monday 24 June 2013 – Afternoon

A2 GCE MATHEMATICS (MEI)

4798/01 Further Pure Mathematics with Technology (FPT)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4798/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator
- Computer with appropriate software

Duration: Up to 2 hours



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 This question concerns curves with parametric equations

$$x = a \cos t + \cos at, \quad y = a \sin t - \sin at$$

where a is a positive integer and $0 \leq t < 2\pi$.

- (i) Sketch the curves for the cases $a = 2$, $a = 3$ and $a = 4$ and describe two of the common features of these three curves. [7]
- (ii) For the case $a = 2$, find the values of t for the points where the curve intersects the axes and hence find the coordinates of the points of intersection with the axes. [5]
- (iii) For the case $a = 2$, confirm the feature of the curve at the point where $t = \frac{2\pi}{3}$ by investigating the gradient as $t \rightarrow \frac{2\pi}{3}$. [5]
- (iv) Sketch the curve

$$x = k \cos^3 t, \quad y = k \sin^3 t$$

where k is positive and $0 \leq t < 2\pi$.

You are given that, for the case $a = 3$, the curve

$$x = a \cos t + \cos at, \quad y = a \sin t - \sin at$$

can be written in the form

$$x = k \cos^3 t, \quad y = k \sin^3 t$$

for a particular positive value of k and $0 \leq t < 2\pi$. Find this value of k and obtain a cartesian equation for the curve in this case. [6]

2 This question concerns the function $f(z) = \sin z$ for $z \in \mathbb{C}$, with derivative $f'(z) = \cos z$.

- (i) Find, with real and imaginary parts given to 3 decimal places, the values of z_1 , z_2 and z_3 , where $z_1 = f(3 + 2i)$, $z_2 = f(3.1 + 2i)$ and $z_3 = f'(3 + 2i)$.

Plot these points on an Argand diagram.

Express $\frac{z_2 - z_1}{0.1}$ and z_3 in the form $re^{i\theta}$ and explain why they are approximately equal. [8]

- (ii) Construct a spreadsheet to demonstrate that

$$\lim_{h \rightarrow 0} \left(\frac{f(z+h) - f(z)}{h} \right) = \cos z \text{ for } z = 3 + 2i \text{ and } h \in \mathbb{R}.$$

State which values of h you have used and the expression(s) you have evaluated. Quoting sufficient values from your spreadsheet, explain how the result is demonstrated.

Find, correct to 1 significant figure, the largest value of h , $h \in \mathbb{R}$, such that

$$\left| \frac{f((3 + 2i) + h) - f(3 + 2i)}{h} - \cos(3 + 2i) \right| < 0.01. \quad [6]$$

- (iii) Find, correct to 1 significant figure, the largest value of h , $h \in \mathbb{R}$, such that

$$\left| \frac{f((3 + 2i) + hi) - f(3 + 2i)}{hi} - \cos(3 + 2i) \right| < 0.01. \quad [2]$$

- (iv) Use your software to find the roots of the equation $\cos z = 0$, where $-2\pi < \operatorname{Re}(z) < 2\pi$. Plot these roots on an Argand diagram.

Use the real and imaginary parts of $\cos z$ to show algebraically that all the roots of the equation $\cos z = 0$ are real. [8]

- 3 This question concerns arithmetic modulo 17. The unknowns x and y are integers such that $0 \leq x < 17$, $0 \leq y < 17$.

- (i) Create a program to find all the solutions, x and y , to the congruence

$$ax + by \equiv c \pmod{17}$$

where a , b and c are positive integers. You should write out your program in full.

Find the number of solutions to the congruence

$$x + 5y \equiv 13 \pmod{17}.$$

State the solution for which the sum $x + y$ is largest. [8]

- (ii) Edit your program to find the solutions, x and y , to the simultaneous congruences

$$ax + by \equiv c \pmod{17}$$

$$dx + ey \equiv f \pmod{17}$$

where a , b , c , d , e and f are positive integers. Indicate clearly all the changes to your program.

Use the edited program to solve the simultaneous congruences

$$3x + 5y \equiv 7 \pmod{17}$$

$$2x + 7y \equiv 1 \pmod{17}$$

and state the solution.

Check the solution by calculating the values of $3x + 5y$ and $2x + 7y$. [6]

- (iii) Explain how you would investigate the number of solutions, x and y , to the simultaneous congruences

$$kx + 5y \equiv 7 \pmod{17}$$

$$2x + 7y \equiv 1 \pmod{17}$$

for different integer values of k where $0 \leq k < 17$.

State the value of k for which the simultaneous congruences do not have a solution. Explain why the congruences do not have a solution for this value of k . [6]

- (iv) Find the number of solutions, x and y , to the simultaneous congruences

$$7x + y \equiv 6 \pmod{17}$$

$$x + 5y \equiv 13 \pmod{17}.$$

Explain your result. [5]



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A2 GCE MATHEMATICS (MEI)

4798/01 Further Pure Mathematics with Technology (FPT)

PRINTED ANSWER BOOK

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Candidate forename		Candidate surname	
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Centre number						Candidate number				
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1 (i)

(answer space continued on next page)

1 (i) (continued)	
1 (ii)	

1 (iii)	

1 (iv)	

2 (i)	

2 (ii)	

2 (iii)	

2 (iv)	

3 (i)	

3 (ii)	

3 (iii)	

3 (iv)	



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Mathematics (MEI)

Advanced GCE

Unit **4798**: Further Pure Mathematics with Technology (FPT)

Mark Scheme for June 2013

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c. The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g. Rules for replaced work

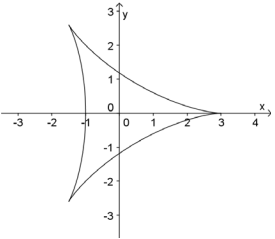
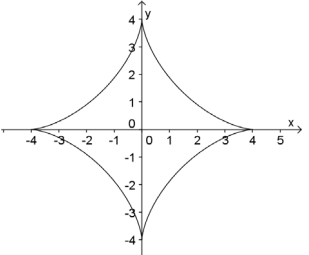
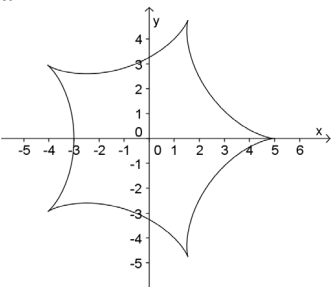
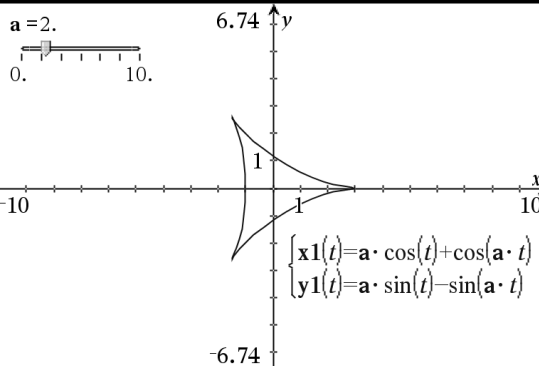
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

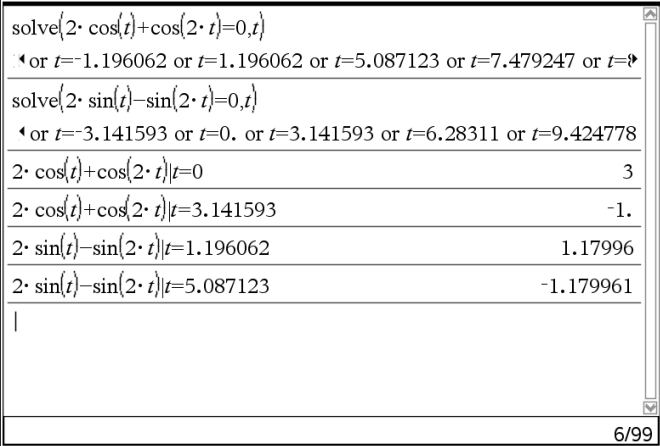
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

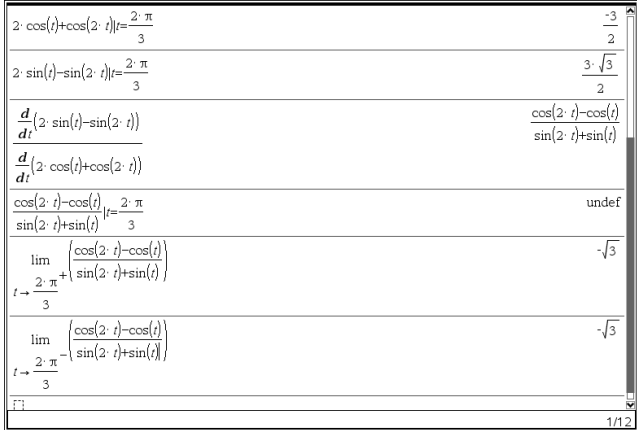
NB Follow these maths-specific instructions rather than those in the assessor handbook.

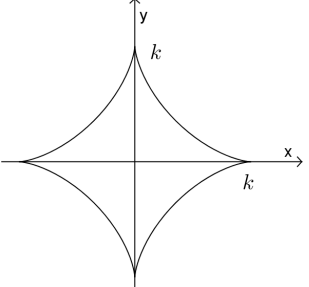
- h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.


Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

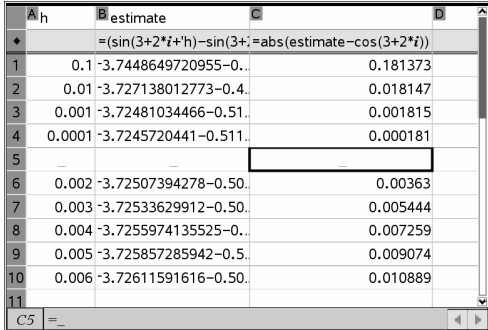
Question	Answer	Marks	Guidance
<p>1 (i)</p>	<p>$a=2$</p>  <p>$a=3$</p>  <p>$a=4$</p>  <p>Correct number of cusps in all 3 cases Intersection with +ve axis marked correctly.</p> <p>Any two distinct correct comments from:</p> <ul style="list-style-type: none"> • cusps, • curves being bounded / closed, • reflectional or rotational symmetry. 	<p>G1</p> <p>G1</p> <p>G1</p> <p>G1</p> <p>G1</p> <p>E1, E1</p> <p>[7]</p>	<p>Correct shape</p> <p>Correct shape</p> <p>Correct shape</p> <div data-bbox="1348 928 1899 1308" style="border: 1px solid black; padding: 5px;"> <p>$a=2.$</p>  <p>$\begin{cases} x1(t)=a \cdot \cos(t)+\cos(a \cdot t) \\ y1(t)=a \cdot \sin(t)-\sin(a \cdot t) \end{cases}$</p> </div>

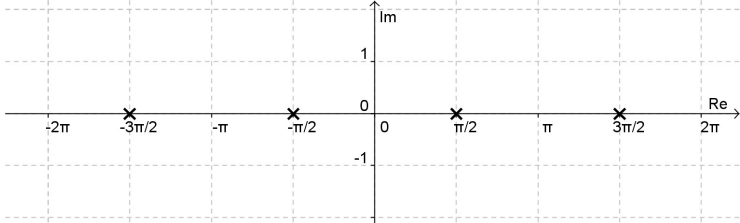
Question	Answer	Marks	Guidance
1 (ii)	$x = 0 \Rightarrow 2 \cos(t) + \cos(2t) = 0$ $\Rightarrow t = 1.1961$ or $t = 5.0871$ $y = 0 \Rightarrow 2 \sin(t) - \sin(2t) = 0$ $\Rightarrow t = 0$ or $t = 3.141593$ $(3, 0), (-1, 0), (0, 1.180), (0, -1.180)$	M1 A1 M1 A1 A1 A1 [5]	Must give evidence of solving an equation. π or 3.141593 acceptable. All 4 correct.  <p>The screenshot shows a calculator interface with the following text:</p> <pre> solve(2*cos(t)+cos(2*t)=0,t) *or t=-1.196062 or t=1.196062 or t=5.087123 or t=7.479247 or t= solve(2*sin(t)-sin(2*t)=0,t) *or t=-3.141593 or t=0. or t=3.141593 or t=6.28311 or t=9.424778 2*cos(t)+cos(2*t) t=0 3 2*cos(t)+cos(2*t) t=3.141593 -1. 2*sin(t)-sin(2*t) t=1.196062 1.17996 2*sin(t)-sin(2*t) t=5.087123 -1.179961 6/99 </pre>

Question	Answer	Marks	Guidance
1 (iii)	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $= \frac{\cos(2t) - \cos(t)}{\sin(2t) + \sin(t)}$ <p>There are two branches either side of $t = \frac{2\pi}{3}$. Considering the gradient on each branch:</p> $\lim_{t \rightarrow \frac{2\pi}{3}^+} \left(\frac{dy}{dx} \right) = -\sqrt{3}$ $\lim_{t \rightarrow \frac{2\pi}{3}^-} \left(\frac{dy}{dx} \right) = -\sqrt{3}$ <p>The curve is defined at $t = \frac{2\pi}{3}$ so there is a cusp.</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>[5]</p>	<p>Limit from one direction only scores max M1A1 M0 A0 B1</p> <p>Can be implied by subsequent working.</p> <p>Accept statement that the curve is defined. For reference: $\left(\frac{-3}{2}, \frac{3\sqrt{3}}{2} \right)$</p>  <p>The handwritten work shows the following steps:</p> <ul style="list-style-type: none"> Expression for $2 \cdot \cos(t) + \cos(2 \cdot t)$ at $t = \frac{2 \cdot \pi}{3}$ resulting in $\frac{-3}{2}$. Expression for $2 \cdot \sin(t) - \sin(2 \cdot t)$ at $t = \frac{2 \cdot \pi}{3}$ resulting in $\frac{3 \cdot \sqrt{3}}{2}$. Derivative $\frac{d}{dt} (2 \cdot \sin(t) - \sin(2 \cdot t))$ resulting in $\frac{\cos(2 \cdot t) - \cos(t)}{\sin(2 \cdot t) + \sin(t)}$. Derivative $\frac{d}{dt} (2 \cdot \cos(t) + \cos(2 \cdot t))$ resulting in $\frac{\cos(2 \cdot t) - \cos(t)}{\sin(2 \cdot t) + \sin(t)}$. Limit calculation: $\lim_{t \rightarrow \frac{2 \cdot \pi}{3}^+} \frac{\cos(2 \cdot t) - \cos(t)}{\sin(2 \cdot t) + \sin(t)} = -\sqrt{3}$. Limit calculation: $\lim_{t \rightarrow \frac{2 \cdot \pi}{3}^-} \frac{\cos(2 \cdot t) - \cos(t)}{\sin(2 \cdot t) + \sin(t)} = -\sqrt{3}$.

Question	Answer	Marks	Guidance
1 (iv)	 <p>$k=4$</p> <p>$x = 4 \cos^3 t, y = 4 \sin^3 t$</p> <p>$\left(\frac{x}{4}\right)^{1/3} = \cos t, \left(\frac{y}{4}\right)^{1/3} = \sin t$</p> <p>$\left(\frac{x}{4}\right)^{2/3} = \cos^2 t, \left(\frac{y}{4}\right)^{2/3} = \sin^2 t$</p> <p>$\left(\frac{x}{4}\right)^{2/3} + \left(\frac{y}{4}\right)^{2/3} = 1$</p>	<p>G1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[6]</p>	<p>Suitable rearrangement to allow elimination of t.</p> <p>Attempt to eliminate t.</p> <p>Correct equation.</p> <p>Equation that corresponds to the entire curve (multi-valued for x and y).</p>

Question	Answer	Marks	Guidance										
<p>2 (i)</p>	<p> $\sin(3 + 2i) = 0.531 - 3.591i$ $\sin(3.1 + 2i) = 0.156 - 3.624i$ $f'(3 + 2i) = \cos(3 + 2i)$ $= -3.725 - 0.512i$ </p>  <p> $\frac{z_2 - z_1}{0.1} = 3.760e^{-3.053i}$ $z_3 = 3.760e^{-3.005i}$ Appropriate comment relating to derivative. </p>	<p> B1 B1 M1 A1 G1 M1 A1 E1 [8] </p>	<table border="1"> <tr> <td>$\sin(3.+2 \cdot i)$</td> <td>$0.530921 - 3.59056 \cdot i$</td> </tr> <tr> <td>$\sin(3.1+2 \cdot i)$</td> <td>$0.156435 - 3.62372 \cdot i$</td> </tr> <tr> <td>$\cos(3.+2 \cdot i)$</td> <td>$-3.72455 - 0.511823 \cdot i$</td> </tr> <tr> <td>$(\sin(3.1+2 \cdot i) - \sin(3.+2 \cdot i)) \blacktriangleright$ Polar</td> <td>$e^{-3.05328 \cdot i} \cdot 0.375952$</td> </tr> <tr> <td>$(\cos(3.+2 \cdot i)) \blacktriangleright$ Polar</td> <td>$e^{-3.00503 \cdot i} \cdot 3.75955$</td> </tr> </table> <p style="text-align: right;">5/99</p>	$\sin(3.+2 \cdot i)$	$0.530921 - 3.59056 \cdot i$	$\sin(3.1+2 \cdot i)$	$0.156435 - 3.62372 \cdot i$	$\cos(3.+2 \cdot i)$	$-3.72455 - 0.511823 \cdot i$	$(\sin(3.1+2 \cdot i) - \sin(3.+2 \cdot i)) \blacktriangleright$ Polar	$e^{-3.05328 \cdot i} \cdot 0.375952$	$(\cos(3.+2 \cdot i)) \blacktriangleright$ Polar	$e^{-3.00503 \cdot i} \cdot 3.75955$
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$(\cos(3.+2 \cdot i)) \blacktriangleright$ Polar	$e^{-3.00503 \cdot i} \cdot 3.75955$												

Question	Answer	Marks	Guidance
<p>2 (ii)</p>	<p>Column for decreasing values of h.</p> <p>Column calculating $\frac{\sin(3+2i+h) - \sin(3+2i)}{h}$</p> <p>Comparing with the value of $\cos(3+2i)$ found in (i).</p> <p>Statement that the real and imaginary parts are tending to the value of $\cos(3+2i)$ as $h \rightarrow 0$.</p> $\left \frac{f(3+2i+0.006) - f(3+2i)}{0.006} - \cos(3+2i) \right > 0.01$ $\left \frac{f(3+2i+0.005) - f(3+2i)}{0.005} - \cos(3+2i) \right < 0.01$ <p>Largest value of h is 0.005. Accept 0.006.</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>Or equivalent argument using</p> $\left \frac{\sin(3+2i+h) - \sin(3+2i)}{h} - \cos(3+2i) \right $ <p>Accept use of polar form.</p> 
<p>2 (iii)</p>	$\left \frac{f(3+2i+0.006i) - f(3+2i)}{0.006i} - \cos(3+2i) \right > 0.01$ $\left \frac{f(3+2i+0.005i) - f(3+2i)}{0.005i} - \cos(3+2i) \right < 0.01$ <p>Largest value of h is 0.005. Accept 0.006.</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	

Question	Answer	Marks	Guidance
<p>2 (iv)</p>	$z = \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$  <p> $f'(z) = \cos(x + iy)$ $= \cos(x) \cosh(y) - i \sin(x) \sinh(y)$ </p> <p> $\operatorname{Re}(f'(z)) = \cos(x) \cosh(y)$ $\operatorname{Im}(f'(z)) = -\sin(x) \sinh(y)$ </p> <p>For $f'(z) = 0$ the real and imaginary parts must be 0.</p> <p>$\cosh(y) > 0$ for all y, therefore $\operatorname{Re}(\cos(x + iy)) = 0 \Rightarrow \cos(x) = 0$</p> <p> $\cos(x) = 0 \Rightarrow x = \frac{(2n-1)\pi}{2}, n \in \mathbf{Z}$ $\Rightarrow \sin(x) \neq 0$ </p> <p>Therefore $\operatorname{Im}(\cos(x + iy)) = 0 \Rightarrow \sinh(y) = 0$ Hence $y = 0 \Rightarrow z$ is real.</p>	<p>M1 A1</p> <p>B1 B1</p> <p>E1 E1 E1 E1</p> <p>[8]</p>	<p>Appropriate method using csolve or equivalent. Can be implied by 4 real solutions.</p> <p>All 4 correctly marked.</p> <p>Seen or implied.</p>

Question		Answer	Marks	Guidance
3	(i)	<p>Example program:</p> <pre>Define program1(a,b,c)= Prgm Local x,y For x,0,16 For y,0,16 If remain(a*x+b*y,17)=remain(c,17) Then Disp x,y EndIf EndFor EndFor EndPrgm</pre> <p>17 solutions $x = 16, y = 13$ gives the maximum sum.</p>	<p>M5</p> <p>A1 A2</p> <p>[8]</p>	<p>If the answers are incorrect allocate method marks as follows: M1 Appropriate structure program M1 Loop for x or equivalent M1 Loop for y or equivalent M1 values of 0-16 M1 Check (If) statement</p> <p>More efficient programs may be possible.</p> <p>Score A1A0 for correct x,y solution (with evidence of a program and modular arithmetic) that does not give the maximum sum.</p>
3	(ii)	<p>Change the Input variables: Define program1(a,b,c,d,e,f,p)=</p> <p>Changing the If statement:</p> <pre>If remain(a*x+b*y,p)=remain(c,p) and remain(d*x+e*y,p)=remain(f,p) Then x = 4, y = 16 3x + 5y = 3 × 4 + 5 × 16 = 92 = 5 × 17 + 7 2x + 7y = 2 × 4 + 7 × 16 = 120 = 7 × 17 + 1</pre>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>Changing the input variables (seen or implied)</p> <p>Changing the If statement</p> <p>Correct If statement given</p>

Question	Answer	Marks	Guidance
3 (iii)	<p>Systematic search through the values of k using the program. or Editing the program to loop through the values of k.</p> <p>$k = 16$ Multiplying the second congruence by 8 in $16x + 5y \equiv 7 \pmod{17}$ $2x + 7y \equiv 1 \pmod{17}$</p> <p>gives $16x + 5y \equiv 7 \pmod{17}$ $16x + 56y \equiv 8 \pmod{17}$</p> <p>$16x + 56y \equiv 8 \pmod{17}$ is equivalent to $16x + 5y \equiv 8 \pmod{17}$.</p> <p>$16x + 5y$ cannot be simultaneously 7 and 8 (mod17).</p>	<p>M1</p> <p>A1 M1</p> <p>M1A1</p> <p>E1</p> <p>[6]</p>	<p>Allow equivalent multiplications for elimination.</p>
3 (iv)	<p>17 solutions.</p> <p>17 solutions, or same solutions as in part (i), suggests that both congruences are equivalent to the single congruence $x + 5y \equiv 13 \pmod{17}$</p> <p>Multiplying the first congruence by 5 in $7x + y \equiv 6 \pmod{17}$ $x + 5y \equiv 13 \pmod{17}$</p> <p>gives $35x + 5y \equiv 30 \pmod{17}$ $x + 5y \equiv 13 \pmod{17}$ $35x + 5y \equiv 30 \pmod{17}$ is equivalent to $x + 5y \equiv 13 \pmod{17}$.</p>	<p>B1</p> <p>E1</p> <p>M1</p> <p>M1A1</p> <p>[5]</p>	<p>Can be implied by subsequent work.</p>

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Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

OCR Report to Centres

June 2013

4798 Further Pure Mathematics with Technology

General Comments

There were some excellent scripts showing a strong understanding of the content of the unit.

There were parts, or occasionally whole questions, left blank on some scripts. Candidates should always present any information they have from using the technology, such as programs, results from using CAS or graphing, as they might be able to gain partial credit for this.

Candidates' explanations were difficult to follow in some cases. This unit is designed so that candidates are able to explain the results of using software and they are advised to practise this. This unit contains a greater requirement to write explanations than other units in the scheme.

Comments on Individual Questions

- 1(i) This part was generally well done. Sketches of graphs should always include some evidence of scale, such as important points on the axes.
- 1(ii) Appropriate use of CAS to solve the equations was seen on almost all papers.
- 1(iii) Clearly evaluating the limit of the derivative from both directions and also giving evidence the curve is defined at the point was necessary to obtain full marks.
- 1(iv) Many candidates obtained the correct value of k but struggled with obtaining a Cartesian equation of the curve. An explicit equation in the form $y=f(x)$ was seen in a number of cases; however, an equation that is clearly multi-valued for both x and y was required for full marks.
- 2(i) This part was generally well done but there were some basic errors made on what was meant to be a straightforward first part to this question.
- 2(ii) Using a spreadsheet to demonstrate how a function behaves as it approaches a limit is explicitly stated in the specification but a number of candidates were not able to present a convincing explanation of how they would do this. The last part of these questions could have been interpreted as “the largest number that has 1 significant figure” or “the largest number, rounded to 1 significant figure”: either of these was acceptable.
- 2(iii)
- 2(iv) Where this was attempted, there were some clear explanations though some candidates were not able to give a convincing argument about why sin and cos cannot both be 0.
- 3(i) Where this was attempted, the programs given were generally correct, with the most common error being loops that went from 0 or 1 to 17, instead of 16.
- This question was left blank on a number of scripts. Candidates should be encouraged to write down their attempts at programs even if they don't think they are obtaining the correct numbers.
- 3(ii) Where this was attempted, it was generally well done.
- 3(iii) There is an expectation that candidates are familiar with modular arithmetic and able to manipulate congruences. Some candidates did not link the result in part (iv) to the result in i) which made this part more difficult.
- 3(iv)

Unit level raw mark and UMS grade boundaries June 2013 series
AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)		Max Mark	a	b	c	d	e	u
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	62	56	51	46	41	0
	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	54	48	43	38	33	0
	UMS	100	80	70	60	50	40	0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	46	40	33	0
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	66	59	53	47	41	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
	UMS	100	80	70	60	50	40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	61	54	48	42	36	0
	UMS	100	80	70	60	50	40	0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	60	52	44	36	28	0
	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	62	56	51	46	40	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	57	49	41	33	25	0
	UMS	100	80	70	60	50	40	0
4762/01 (M2) MEI Mechanics 2	Raw	72	50	43	36	29	22	0
	UMS	100	80	70	60	50	40	0
4763/01 (M3) MEI Mechanics 3	Raw	72	64	56	48	41	34	0
	UMS	100	80	70	60	50	40	0
4764/01 (M4) MEI Mechanics 4	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4766/01 (S1) MEI Statistics 1	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw	72	58	52	46	41	36	0
	UMS	100	80	70	60	50	40	0
4768/01 (S3) MEI Statistics 3	Raw	72	61	55	49	44	39	0
	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	58	52	46	40	35	0
	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	58	52	46	41	36	0
	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
	UMS	100	80	70	60	50	40	0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	44	38	31	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4798/01 (FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0
GCE Statistics (MEI)		Max Mark	a	b	c	d	e	u
G241/01 (Z1) Statistics 1	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
G242/01 (Z2) Statistics 2	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
G243/01 (Z3) Statistics 3	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0