

## **Tuesday 24 June 2014 – Morning**

### **A2 GCE MATHEMATICS (MEI)**

**4798/01** Further Pure Mathematics with Technology (FPT)

#### **QUESTION PAPER**

Candidates answer on the Printed Answer Book.

#### **OCR** supplied materials:

- Printed Answer Book 4798/01
- MEI Examination Formulae and Tables (MF2)

#### Other materials required:

- Scientific or graphical calculator
- Computer with appropriate software



#### **INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer all the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

#### **INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

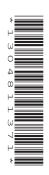
- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive no marks unless you show sufficient detail
  of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

#### **COMPUTING RESOURCES**

 Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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1 This question concerns curves with equation

$$y = \frac{x^3 + ax^2 + 1}{x^n}$$

for various values of a and n.

- (i) For the case n = 1, sketch the curves when a = 2, a = 3 and a = 4. Describe two common features of these three curves.
- (ii) For the case n = 1 and a = 3, find the number of stationary points and identify their nature, justifying your answers. [5]
- (iii) For the case a = 3, sketch the curves when n = 2 and n = 3. [4]
- (iv) Given that a > 0, find the equations of the asymptotes for each of the cases n = 2 and n = 3. For any non-vertical asymptotes, state whether they are approached from above or below, justifying your answers.
- 2 (i) The function f is defined by  $f(z) = z^3 + (2-2i)z^2 + (7-12i)z + 6-10i$ . Solve the equation f(z) = 0 and plot the roots as points on an Argand diagram.

Show that these points lie on a straight line.

[6]

[8]

(ii) Find the roots of f'(z) = 0. Plot these roots on the Argand diagram drawn in part (i).

Show that the roots of f'(z) = 0 lie on the same straight line as the roots of f(z) = 0.

(iii) The function g is defined by  $g(z) = z^3 - (k+1)az^2 + ka^2z$  where  $a \in \mathbb{C}$ ,  $k \in \mathbb{R}$ .

Show that the roots of g(z) = 0 lie on a straight line.

Show that the roots of g'(z) = 0 lie on this same line.

(iv) Now consider a function h which is a cubic with real coefficients. Identify the two distinct conditions under which the roots of h(z) = 0 lie on a straight line in the Argand diagram. Give, in expanded form, an example of such a cubic for each case.

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- 3 This question concerns Pythagorean triples: positive integers a, b and c such that  $a^2 + b^2 = c^2$ . The integer n is defined by c = b + n.
  - (i) Create a program that will find all such triples for a given value of *n*, where both *a* and *b* are less than or equal to a maximum value, *m*. You should write out your program in full.

For the case n = 1, find all the triples with  $1 \le a \le 100$  and  $1 \le b \le 100$ .

For the case n = 3, find all the triples with  $1 \le a \le 200$  and  $1 \le b \le 200$ .

- (ii) For the case n = 1, prove that there is a triple for every odd value of a where a > 1. [4]
- (iii) For the case n = p, where p is prime, show that a must be a multiple of p. [3]
- (iv) For the case n = b, determine whether there are any triples. [4]
- (v) Edit your program from part (i) so that it will only find values of a and b where b is not a multiple of n. Indicate clearly all the changes to your program.

Use the edited program to find all such triples for the case n = 2 with  $1 \le a \le 100$  and  $1 \le b \le 100$ .

#### **END OF QUESTION PAPER**

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4798/01 Further Pure Mathematics with Technology (FPT)

#### PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

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- Question Paper 4798/01 (inserted)
- MEI Examination Formulae and Tables (MF2)

#### Other materials required:

- Scientific or graphical calculator
- Computer with appropriate software

**Duration:** up to 2 hours



| Candidate forename |  | Candidate surname |  |                  |  |  |  |
|--------------------|--|-------------------|--|------------------|--|--|--|
|                    |  |                   |  |                  |  |  |  |
| Centre number      |  |                   |  | Candidate number |  |  |  |

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  working to indicate that a correct method is being used.
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## **GCE**

## **Mathematics (MEI)**

Unit 4789/01: Further Pure Mathematics with Technology

**GCE** 

Mark Scheme for June 2014

#### 1. Annotations and abbreviations

| Annotation in scoris | Meaning   |
|----------------------|---|
| BP                   | Blank Page – this annotation <b>must</b> be used on all blank pages within an answer booklet (structured or |
|                      | unstructured) and on each page of an additional object where there is no candidate response.                |
| ✓and ×               |   |
| BOD                  | Benefit of doubt  |
| FT                   | Follow through  |
| ISW                  | Ignore subsequent working   |
| M0, M1               | Method mark awarded 0, 1  |
| A0, A1               | Accuracy mark awarded 0, 1  |
| B0, B1               | Independent mark awarded 0, 1   |
| SC                   | Special case  |
| ۸                    | Omission sign   |
| MR                   | Misread   |
| Highlighting         |   |
|                      |   |
| Other abbreviations  | Meaning   |
| in mark scheme       |   |
| E1                   | Mark for explaining   |
| U1                   | Mark for correct units  |
| G1                   | Mark for a correct feature on a graph   |
| M1 dep*              | Method mark dependent on a previous mark, indicated by *  |
| cao                  | Correct answer only   |
| oe                   | Or equivalent   |
| rot                  | Rounded or truncated  |
| soi                  | Seen or implied   |
| www                  | Without wrong working   |
|                      |   |
|                      |   |

#### 2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

#### М

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

#### Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

#### В

Mark for a correct result or statement independent of Method marks.

#### Ε

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.
  - Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

| Question | Answer   | Marks    | Guidance   |
|----------|--|----------|--|
| 1 (i)    | a=2  | G1       | Correct shape  |
|          | a = 3  |          |  |
|          | 5-1<br>4-1<br>3-2-1<br>1-1<br>-7-6-5-4-3-2-1 0 1 2 3   | G1       | Correct shape  |
|          | a = 4  |          |  |
|          | 5  | G1       | Correct shape  |
|          | -7 -6 -6 -4 -3 -2 -1 0 1 2 5 4 6   |          | $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ |
|          | Correct number of stationary points in all 3 cases   | G1       | 1 1 10   |
|          | Any two distinct comments, e.g.<br>Asymptote at $x = 0$ or The curve has two branches<br>Minimum in the first quadrant<br>Single point of intersection with the $x$ -axis when $x$ is negative.<br>$x^2$ is dominant for large $x$ . | E1<br>E1 | -6.74  |
|          |  | [6]      |  |

| Question | Answer   | Marks | Guidance  |
|----------|--|-------|---|
| 1 (ii)   | $\frac{dy}{dx} = \frac{2x^3 + 3x^2 - 1}{x^2}$  | M1    |   |
|          | $\frac{1}{dx} - \frac{1}{x^2}$   |       |   |
|          | $\frac{dy}{dx} = 0 \Rightarrow x = -1 \text{ or } x = \frac{1}{2}$ : 2 stationary points       | A1    |   |
|          | By comparing these values to the graph:  | M1    |   |
|          | The stationary point $x = \frac{1}{2}$ is a minimum.   | E1    |   |
|          | The stationary point at $x = -1$ is a point of inflection                                      | E1    |   |
|          |  | Or    |   |
|          | $\frac{d^2 y}{dx^2} = \frac{2(x^3 + 1)}{x^3}$  | M1    | Evaluation of second derivative.  |
|          | At $x = \frac{1}{2} \frac{d^2 y}{dx^2} = 18$ i.e. a minimum                                    | A1    |   |
|          | At $x = -1$ $\frac{d^2y}{dx^2} = 0$ , examining values either side shows that it is a point of | E1    | Any appropriate comment about about points/curve/gradient explaining why inflection |
|          | inflection.  |       |   |
|          |  | [5]   |   |

| Question | Answer   | Marks    | Guidance   |  |  |  |  |
|----------|--|----------|--|--|--|--|--|
| 1 (iii)  | n=2  | G1       | Correct general shape.   |  |  |  |  |
|          | 5  | G1       | Evidence of oblique asymptote.   |  |  |  |  |
|          | 3<br>1<br>1<br>1<br>2<br>2<br>4<br>3<br>2<br>3<br>3<br>3<br>3<br>3<br>3<br>3<br>3<br>3<br>3<br>3<br>3<br>3 |          |  |  |  |  |  |
|          | n=3  | G1<br>G1 | Correct general shape. Clearly showing interception with negative <i>x</i> -axis and horizontal asymptote. |  |  |  |  |
|          | 4 5 4 3 2 1 0 1 2 3 4 7 1 1 1 1 2 3 4 7 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1                                |          |  |  |  |  |  |
|          |  | [4]      |  |  |  |  |  |

| Question | Answer   | Marks | Guidance   |
|----------|--|-------|--|
| 1 (iv)   | n = 2<br>Vertical asymptote at $x = 0$ .   | B1    |  |
|          | $\frac{x^3 + ax^2 + 1}{x^2} = x + a + \frac{1}{x^2}$   | M1    | Condone a numerical argument SC M0 B1 B1   |
|          | Oblique asymptote: $y = x + a$   | A1    |  |
|          | Approached from above as $\frac{1}{x^2}$ is positive for all $x$ .                                 | A1    | Justification required. Accept numerical argument or reference to graph with suitable explanation. |
|          | n=3  |       |  |
|          | Vertical asymptote at $x = 0$ .  | B1    |  |
|          | $\frac{x^3 + ax^2 + 1}{x^3} = 1 + \frac{a}{x} + \frac{1}{x^3}$                                     | M1    | Condone a numerical argument SC M0 B1 B1   |
|          | Horizontal asymptote: $y=1$  | A1    |  |
|          | Approached from above as $x \to +\infty$ $\frac{a}{x} + \frac{1}{x^3}$ is positive for positive x. | A1    | Both required, with justification. Accept numerical argument.                                      |
|          | Approached from below as $x \to -\infty$ $\frac{a}{x} + \frac{1}{x^3}$ is negative for negative x. |       |  |
|          |  | [8]   |  |

| Question | Answer   | Marks    | Guidance   |
|----------|--|----------|--|
| 2 (i)    | z = 1 + 4i, -2 - 2i, -1  | B2<br>B2 | B1 for 1 or 2 correct only. B2 for points marked correctly. B1 for two correct.  |
|          | Z <sub>3</sub> 0 Re  |          |  |
|          | $z_1 - z_3 = 2 + 4i$<br>$z_2 - z_3 = -1 - 2i$<br>$z_1 - z_3 = -2(z_2 - z_3)$ | M1<br>A1 | Evaluation of differences.<br>Shown to be in same direction.<br>Award full marks: M1A1 for showing each of the $z = x + iy$ is on the line $y = 2x + 2$<br>Any other acceptable method should be awarded full marks. |
|          |  | [6]      |  |

| Question | Answer  | Marks | Guidance   |
|----------|---|-------|--|
| 2 (ii)   | $f'(z) = 3z^2 + (4-4i)z + 7 - 12i$  | M1    |  |
|          | $f'(z) = 0 \Rightarrow z = \frac{-\sqrt{7} - 2}{3} + \frac{2 - 2\sqrt{7}}{3}i \text{ or } \frac{\sqrt{7} - 2}{3} + \frac{2 + 2\sqrt{7}}{3}i$  | A1    |  |
|          |   | A1    | Points marked approximately correctly.   |
|          | 3<br>•W2  | M1    | Finding the difference between each root of $f'(z) = 0$ and a root of $f(z) = 0$ . |
|          | Z <sub>3</sub> 0 Re   | M1    | Showing this difference is a multiple of 1+2i or equivalent.                       |
|          | $\frac{-\sqrt{7}-2}{3} + \frac{2-2\sqrt{7}}{3}i - (-1) = \frac{-\sqrt{7}+1}{3} + \frac{2-2\sqrt{7}}{3}i$ $\frac{-\sqrt{7}+1}{3} + \frac{2-2\sqrt{7}}{3}i = \frac{\sqrt{7}-1}{3} \text{ which is real.}$ $\frac{\sqrt{7}-2}{3} + \frac{2+2\sqrt{7}}{3}i - (-1) = \frac{\sqrt{7}+1}{3} + \frac{2+2\sqrt{7}}{3}i$ $\frac{\sqrt{7}+1}{3} + \frac{2+2\sqrt{7}}{3}i = \frac{-\sqrt{7}-1}{3} \text{ which is real.}$ | A1    | Both shown correctly.  Award full marks: M1M1A1 for showing each of the            |
|          |   |       | Z = x + iy is on the line $y = 2x + 2$   |
|          |   | [6]   |  |

| Q | uestion | Answer  | Marks    | Guidance                                   |
|---|---------|---|----------|--|
| 2 | (iii)   | $z^{3} - (k+1)az^{2} + ka^{2}z = z(z-a)(z-ka)$  | M1       | Attempt at factorising or solving.         |
|   |         | Roots are $z = 0, z = a, z = ka$  | A1       |  |
|   |         | As k is real $ka - 0 = ka$ is in the same direction as $a - 0 = a$ .  | E1       | Must reference <i>k</i> real.              |
|   |         | $g'(z) = 3z^2 - 2a(k+1)z + a^2k$  | M1       |  |
|   |         | g'(z) = 0 $\Rightarrow$ z = $-\frac{(\sqrt{k^2 - k + 1} - k - 1)}{3}a$ and z = $\frac{(\sqrt{k^2 - k + 1} + k + 1)}{3}a$  | M1<br>A1 |  |
|   |         | The discriminant of $k^2 - k + 1$ is $-3$ , so $k^2 - k + 1$ is positive for all real $k$ and hence $\sqrt{k^2 - k + 1}$ is real. Therefore each of these is in the same direction as $a$ . | M1<br>E1 | Showing that $\sqrt{k^2 - k + 1}$ is real. |
|   |         |   | [8]      |  |
| 2 | (iv)    | When either all 3 roots are real.   | E1       |  |
|   |         | Any cubic with three real roots.  | B1       |  |
|   |         | Or when $z_1$ and $z_2$ are complex conjugates and $z_3 = \text{Re}(z_1) = \text{Re}(z_2)$ .  | E1       |  |
|   |         | Attempt at multiplying out, e.g. $(z-(1+2i))(z-(1-2i))(z-1)$  | M1       | soi  |
|   |         | Any cubic with roots in this orientation.   | B1       |  |
|   |         |   | [5]      |  |

| Question | Answer                                |     | Guidance  |  |  |  |
|----------|---------------------------------------|-----|---|--|--|--|
| 3 (i)    | Example program:                      | M5  | If the answers are incorrect allocate method marks as |  |  |  |
|          |                                       |     | follows:  |  |  |  |
|          | Define program1(n,m)=                 |     | M1 Appropriate structure program                      |  |  |  |
|          | Prgm                                  |     | M1 Loop for a or equivalent                           |  |  |  |
|          | Local a,b                             |     | M1 Loop for b or equivalent                           |  |  |  |
|          | For a,1,m                             |     | M1 maximum values m or 100 & 200.                     |  |  |  |
|          | For b,1,m                             |     | M1 Check (If) statement                               |  |  |  |
|          | If $a^{(2)}+b^{(2)}=(b+n)^{(2)}$ Then |     |   |  |  |  |
|          | Disp a,b,b+n                          |     | More efficient programs may be possible.              |  |  |  |
|          | EndIf                                 |     |   |  |  |  |
|          | EndFor                                |     |   |  |  |  |
|          | EndFor                                |     |   |  |  |  |
|          | EndPrgm                               |     |   |  |  |  |
|          | n=1                                   | A2  | All 6 correct. A1 for at least 4 correct.             |  |  |  |
|          | 3,4,5                                 |     |   |  |  |  |
|          | 5,12,13                               |     |   |  |  |  |
|          | 7,24,25                               |     |   |  |  |  |
|          | 9,40,41                               |     |   |  |  |  |
|          | 11,60,61                              |     |   |  |  |  |
|          | 13,84,85                              |     |   |  |  |  |
|          | n=3                                   | A2  | All 5 correct. A1 for at least 3 correct.             |  |  |  |
|          | 9,12,15                               |     |   |  |  |  |
|          | 15,36,39                              |     |   |  |  |  |
|          | 21,72,75                              |     |   |  |  |  |
|          | 27,120,123                            |     |   |  |  |  |
|          | 33,180,183                            |     |   |  |  |  |
|          |                                       | [9] |   |  |  |  |

| Ç | uestion      | Answer  | Marks    | Guidance   |
|---|--------------|---|----------|--|
| 3 | (ii)         | $a^2 + b^2 = (b+1)^2$   | M1       |  |
|   |              | $a^2 = b^2 + 2b + 1 - b^2$  |          |  |
|   |              | $a^2 = 2b + 1$  | A1       |  |
|   |              | If $a$ is odd then $a^2$ is odd.                                  | E1       | Full marks for obtaining b explicitly from a=2n+1.   |
|   |              | Therefore $b = \frac{a^2 - 1}{2}$ will also be an integer $> 0$ . | E1       |  |
|   |              |   | [4]      |  |
| 3 | (iii)        | $a^2 + b^2 = (b+p)^2$   | M1       |  |
|   |              | $a^2 = b^2 + 2bp + p^2 - b^2$                                     |          |  |
|   |              | $a^2 = 2bp + p^2$   |          |  |
|   |              | $a^2 = p(2b+p)$   | A1       |  |
|   |              | As $a^2$ is a multiple of p then a must be a multiple of p.       | E1       |  |
|   |              |   | [3]      |  |
| 3 | (iv)         | $a^2 + b^2 = (b+b)^2$   | M1       |  |
|   |              | $a^2 = 4b^2 - b^2$  |          |  |
|   |              | $a^2 = 3b^2$  |          |  |
|   |              | $a = \sqrt{3}b$   | A1       |  |
|   |              | So $a$ and $b$ cannot both be integers.                           | E1       |  |
|   |              | Therefore there are no triples where $n = b$ .                    | E1       |  |
|   |              |   | [4]      |  |
| 3 | ( <b>v</b> ) | Change If statement   | M1       | B1 all correct. B1 no values with b a multiple of 2. |
|   |              | 4,3,5<br>8,15,17  | M1<br>B2 |  |
|   |              | 12,35,37  | D2       |  |
|   |              | 16,63,65  |          |  |
|   |              | 20,99,101   |          |  |
|   |              |   | [4]      |  |



# Unit level raw mark and UMS grade boundaries June 2014 series AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

| GCE Mat         | thematics (MEI)  |            | Max Mark       | а              | b           | С              | d              | e              | - 11          |
|-----------------|--|------------|----------------|----------------|-------------|----------------|----------------|----------------|---------------|
| 4751/01         | (C1) MEI Introduction to Advanced Mathematics  | Raw        | 72             | 61             | 56          | 51             | 46             | 42             | <b>u</b><br>0 |
| 4731/01         | (OT) WET Introduction to Advanced Mathematics  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
| 4752/01         | (C2) MEI Concepts for Advanced Mathematics   | Raw        | 72             | 57             | 51          | 45             | 39             | 33             | 0             |
|                 |  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
|                 | (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper   | Raw        | 72             | 58             | 52          | 47             | 42             | 36             | 0             |
|                 | (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework  | Raw        | 18             | 15             | 13          | 11             | 9              | 8              | 0             |
| 4753/82<br>4753 | (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark (C3) MEI Methods for Advanced Mathematics with Coursework | Raw<br>UMS | 18<br>100      | 15<br>80       | 13<br>70    | 11<br>60       | 9<br>50        | 8<br>40        | 0             |
| 4754/01         | (C4) MEI Applications of Advanced Mathematics  (C4) MEI Applications of Advanced Mathematics   | Raw        | 90             | 68             | 61          | 54             | 47             | 41             | 0             |
| +70-701         | (O4) WEIT phoduons of Navanoed Wathernatios  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
| 4755/01         | (FP1) MEI Further Concepts for Advanced Mathematics  | Raw<br>UMS | 72<br>100      | 63<br>80       | 57<br>70    | 51<br>60       | 45<br>50       | 40<br>40       | 0<br>0        |
| 1756/01         | (FP2) MEI Further Methods for Advanced Mathematics   | Raw        | 72             | 60             | 54          | 48             | 42             | 36             | 0             |
| +7 30/01        | (1 F 2) WELL Until et Wethous for Advanced Wathernatics  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
| 4757/01         | (FP3) MEI Further Applications of Advanced Mathematics   | Raw        | 72             | 57             | 51          | 45             | 39             | 34             | 0             |
|                 |  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
|                 | (DE) MEI Differential Equations with Coursework: Written Paper   | Raw        | 72             | 63             | 56          | 50             | 44             | 37             | 0             |
|                 | (DE) MEI Differential Equations with Coursework: Coursework  | Raw        | 18             | 15             | 13          | 11             | 9              | 8              | 0             |
| 4758/82<br>4758 | (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark (DE) MEI Differential Equations with Coursework                     | Raw<br>UMS | 18<br>100      | 15<br>80       | 13<br>70    | 11<br>60       | 9<br>50        | 8<br>40        | 0             |
|                 | (M1) MEI Mechanics 1   | Raw        | 72             | 57             | 49          | 41             | 34             | 27             | 0             |
| 4701/01         | (MT) MET MOCHAINGS T   | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
| 4762/01         | (M2) MEI Mechanics 2   | Raw        | 72             | 57             | 49          | 41             | 34             | 27             | 0             |
|                 |  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
| 4763/01         | (M3) MEI Mechanics 3   | Raw        | 72             | 55             | 48          | 42             | 36             | 30             | 0             |
| 4=0.4/0.4       |  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
| 4/64/01         | (M4) MEI Mechanics 4   | Raw<br>UMS | 72<br>100      | 48<br>80       | 41<br>70    | 34<br>60       | 28<br>50       | 22<br>40       | 0<br>0        |
| 1766/01         | (S1) MEI Statistics 1  | Raw        | 72             | 61             | 53          | 46             | 39             | 32             | 0             |
| 1700701         |  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
| 1767/01         | (S2) MEI Statistics 2  | Raw        | 72             | 60             | 53          | 46             | 40             | 34             | 0             |
|                 |  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
| 4768/01         | (S3) MEI Statistics 3  | Raw        | 72             | 61             | 54          | 47             | 41             | 35             | 0             |
| 4760/04         | (S4) MEI Statistics 4  | UMS        | 100<br>72      | 80<br>56       | 70<br>49    | 60<br>42       | 50<br>35       | 40<br>28       | 0             |
| +769/01         | (S4) MEI Statistics 4  | Raw<br>UMS | 100            | 80             | 70          | 42<br>60       | 50             | 40             | 0<br>0        |
| 4771/01         | (D1) MEI Decision Mathematics 1  | Raw        | 72             | 51             | 46          | 41             | 36             | 31             | 0             |
|                 |  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
| 4772/01         | (D2) MEI Decision Mathematics 2  | Raw        | 72             | 46             | 41          | 36             | 31             | 26             | 0             |
|                 |  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
| 4773/01         | (DC) MEI Decision Mathematics Computation  | Raw        | 72             | 46             | 40          | 34             | 29             | 24             | 0             |
| 4776/01         | (NM) MEI Numerical Methods with Coursework: Written Paper  | UMS<br>Raw | 100<br>72      | 80<br>54       | 70<br>48    | 60<br>43       | 50<br>38       | 40<br>32       | 0             |
|                 | (NM) MEI Numerical Methods with Coursework: Coursework   | Raw        | 18             | 14             | 12          | 43<br>10       | 8              | 32<br>7        | 0             |
|                 | (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark  | Raw        | 18             | 14             | 12          | 10             | 8              | 7              | 0             |
| 4776            | (NM) MEI Numerical Methods with Coursework   | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
| 4777/01         | (NC) MEI Numerical Computation   | Raw        | 72             | 55             | 47          | 39             | 32             | 25             | 0             |
|                 |  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
| 4798/01         | (FPT) Further Pure Mathematics with Technology   | Raw<br>UMS | 72<br>100      | 57<br>80       | 49<br>70    | 41<br>60       | 33<br>50       | 26<br>40       | 0<br>0        |
| GCE Sta         | tistics (MEI)  |            |                |                |             |                |                |                |               |
| G2/11/01        | (Z1) Statistics 1  | Raw        | Max Mark<br>72 | <b>a</b><br>61 | <b>b</b> 53 | <b>c</b><br>46 | <b>d</b><br>39 | <b>e</b><br>32 | <b>u</b><br>0 |
| J24 I/U I       | (21) Statistics 1  | UMS        | 100            | 80             | 53<br>70    | 46<br>60       | 39<br>50       | 32<br>40       | 0             |
| G242/01         | (Z2) Statistics 2  | Raw        | 72             | 55             | 48          | 41             | 34             | 27             | 0             |
|                 |  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
| G243/01         | (Z3) Statistics 3  | Raw        | 72             | 56             | 48          | 41             | 34             | 27             | 0             |
|                 |  | UMS        | 100            | 80             | 70          | 60             | 50             | 40             | 0             |
|                 |  |            |                |                |             |                |                |                |               |