

Friday 17 May 2013 – Morning

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

- 1 (i) Show by sketching two curves on the same axes that the equation

$$x^2 = \cos x,$$

where x is in radians, has exactly one positive root. Give a rough initial estimate of the root. [3]

- (ii) By re-arranging the equation, find an iterative formula for x_{r+1} in terms of x_r . Use this iterative formula to find the root correct to 2 decimal places. [5]

- 2 This question concerns binomial coefficients of the form $\binom{2n}{n}$, where $\binom{2n}{n} = \frac{(2n)!}{(n!)^2}$.

An approximate formula for $\binom{2n}{n}$ is $\frac{4^n}{\sqrt{n\pi}}$.

- (i) Calculate the absolute and relative errors in the approximate formula for $n = 5$ and $n = 10$. Comment briefly on how the absolute errors and relative errors appear to change with n . [5]

- (ii) It can be shown that the relative errors in part (i) are approximately equal to $\frac{1}{kn}$ for some integer k . Use the values calculated in part (i) to determine k . [2]

- 3 The function $f(x)$ has the values shown in the table.

x	0.1	0.2	0.3	0.4
$f(x)$	1.641	1.990	1.840	1.192

- (i) Show by means of a difference table that $f(x)$ can be closely approximated by a quadratic function. [3]

- (ii) Use Newton's forward difference interpolation formula to obtain an estimate of $f(0.15)$. [4]

- 4 (i) Show, graphically or otherwise, that the equation

$$2^x + 3^x = 4 \quad (*)$$

has exactly one root.

Show that the root lies in the interval $[0.7, 0.8]$. [4]

- (ii) Use the method of false position to find the root of (*) correct to 2 decimal places. [4]

- 5 The values of the function $g(x)$ in the table are correct to 4 decimal places.

x	-0.2	-0.15	-0.1	-0.05	0	0.05	0.1	0.15	0.2
$g(x)$	1.1292	1.1540	1.1766	1.1974	1.2163	1.2335	1.2489	1.2625	1.2745

- (i) Use the central difference formula with suitable values of h to obtain a sequence of three estimates of $g'(0)$. [4]
- (ii) Hence give a value for $g'(0)$ to an appropriate degree of accuracy, explaining your reasoning. [2]

Section B (36 marks)

- 6 In this question, $I = \int_0^{0.5} \sqrt{1 + \tan x} dx$, where x is in radians. Estimates of I should be given correct to 6 decimal places.

- (i) Obtain the trapezium rule and mid-point rule estimates of I with $h = 0.5$.

Use these two values to obtain a Simpson's rule estimate of I . [3]

- (ii) Find, as efficiently as possible, two further trapezium rule estimates, two further mid-point rule estimates, and two further Simpson's rule estimates.

Give the value of I to the accuracy that is justified. [7]

- (iii) Find the differences and the ratio of differences for the trapezium rule estimates and also for the mid-point rule estimates.

What do the ratios of differences indicate?

State, with a reason, whether either of the mid-point and trapezium rules gives more accurate estimates than the other. [8]

[Question 7 is printed overleaf]

7 The series $S_n = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}}$ is summed, for various values of n , using a spreadsheet. The spreadsheet gives the answers $S_{100} = 18.5896$ and $S_{200} = 26.8593$. For the purposes of this question, these values may be regarded as exact.

- (i) The same calculations are now carried out with each term in the series rounded to 4 decimal places. The answers obtained are 18.5897 and 26.8589 respectively.

Explain how it arises that one sum is too large and the other is too small. [2]

- (ii) Now suppose that the same calculations were carried out with each term in the series chopped to 4 decimal places. Estimate the answers that would be obtained, explaining your reasoning. [4]

- (iii) Show, by using the mid-point rule on the integral $\int_{k-0.5}^{k+0.5} \frac{1}{\sqrt{x}} dx$, that

$$\frac{1}{\sqrt{k}} \approx 2(\sqrt{k+0.5} - \sqrt{k-0.5}). \quad [4]$$

- (iv) It follows from the result in part (iii) that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} \approx 2(\sqrt{n+0.5} - \sqrt{0.5}).$$

Use this result to find approximations for S_{100} and S_{200} . Find the errors in these approximations. What do you notice about the values of these errors? [5]

- (v) Making a suitable assumption about the error, obtain as accurate an estimate of S_{1000} as you can. [3]

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4776/01 Numerical Methods

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Candidate forename		Candidate surname	
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Centre number						Candidate number				
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Section A (36 marks)

1 (i)	
1 (ii)	

3 (i)	
3 (ii)	

5 (i)	
5 (ii)	

Section B (36 marks)

6 (i)	

6 (ii)	

6 (iii)	

7 (i)	
7 (ii)	
7 (iii)	
(answer space continued on next page)	

7 (iii)	(continued)
7 (iv)	

7 (v)	



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Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4776**: Numerical Methods

Mark Scheme for June 2013

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance																									
1	(i)	Convincing sketches of x^2 and $\cos x$. Single intersection. Estimate of root in $[0.5, 1]$	G2 B1 [3]	G1 for each graph Accept $\pi/4$. Accept an interval in $[0.5, 1]$																									
1	(ii)	Iteration $x_{r+1} = (\cos x_r)^{0.5}$ <table style="margin-left: auto; margin-right: auto;"> <tr> <td>r</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>x_r</td> <td>0.8</td> <td>0.83469</td> <td>0.819395</td> <td>0.826235</td> <td>0.823195</td> <td>0.82455</td> </tr> </table> 0.82 correct to 2 dp	r	0	1	2	3	4	5	x_r	0.8	0.83469	0.819395	0.826235	0.823195	0.82455	M1 A1 M1 A1 A1 [5]	For any valid rearrangement For writing it as an iteration (soi) Max 1 for a diverging iteration A1 requires agreement to 2 dp Dependent on previous A1											
r	0	1	2	3	4	5																							
x_r	0.8	0.83469	0.819395	0.826235	0.823195	0.82455																							
2	(i)	<table style="margin-left: auto; margin-right: auto;"> <tr> <td>n</td> <td>exact</td> <td>approx</td> <td>error</td> <td>rel error</td> </tr> <tr> <td>5</td> <td>252</td> <td>258.3688</td> <td>6.36877</td> <td>0.025273</td> </tr> <tr> <td>10</td> <td>184756</td> <td>187079</td> <td>2322.973</td> <td>0.012573</td> </tr> </table> errors increase but relative errors decrease with n	n	exact	approx	error	rel error	5	252	258.3688	6.36877	0.025273	10	184756	187079	2322.973	0.012573	M1 B1 B1 B1 B1 [5]	Requires method for abs and rel error Approximations Errors Relative errors										
n	exact	approx	error	rel error																									
5	252	258.3688	6.36877	0.025273																									
10	184756	187079	2322.973	0.012573																									
2	(ii)	$10k = \frac{1}{0.01257} = 79.5548$ $k = 8$ to nearest integer OR $5k = \frac{1}{0.0257} = 39.5726$ $k = 8$ to nearest integer	M1 A1 M1 A1 [2]	Must be an integer Must be an integer																									
3	(i)	<table style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>$f(x)$</td> <td>Δ</td> <td>Δ^2</td> <td></td> </tr> <tr> <td>0.1</td> <td>1.641</td> <td></td> <td></td> <td></td> </tr> <tr> <td>0.2</td> <td>1.990</td> <td>0.349</td> <td></td> <td></td> </tr> <tr> <td>0.3</td> <td>1.840</td> <td>-0.150</td> <td>-0.499</td> <td>these almost equal</td> </tr> <tr> <td>0.4</td> <td>1.192</td> <td>-0.648</td> <td>-0.498</td> <td>(so approx qdratic)</td> </tr> </table>	x	$f(x)$	Δ	Δ^2		0.1	1.641				0.2	1.990	0.349			0.3	1.840	-0.150	-0.499	these almost equal	0.4	1.192	-0.648	-0.498	(so approx qdratic)	M1 A1 E1 [3]	
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Question	Answer	Marks	Guidance																				
3 (ii)	$f(1.5) = 1.641 + \frac{0.349(0.15 - 0.1)}{0.1} - \frac{0.499(0.15 - 0.1)(0.15 - 0.2)}{2(0.1)^2}$ <p>= 1.878 to 3dp</p>	M1 A1 A1 A1 [4]	For recognizable attempt at correct formula 2 out 3 for formula with x and no 0.15 Either second or third term correct All three terms correct. Accept cubic. Accept any awrt 1.878																				
4 (i)	Sketch or convincing argument to an increasing function hence a single root <table style="margin-left: 40px;"> <tr> <td>x</td> <td>0.7</td> <td>3.782174</td> <td>< 4</td> </tr> <tr> <td>function</td> <td>0.8</td> <td>4.149326</td> <td>> 4</td> </tr> </table> (Hence root)	x	0.7	3.782174	< 4	function	0.8	4.149326	> 4	G2/E2 M1 A1 [4]	If comparing with zero: -0.2178, 0.1493 Max 1 if function sign but not values												
x	0.7	3.782174	< 4																				
function	0.8	4.149326	> 4																				
4 (ii)	<table style="margin-left: 40px;"> <tr> <td>a</td> <td>$f(a)$</td> <td>b</td> <td>$f(b)$</td> <td>x</td> <td>$f(x)$</td> </tr> <tr> <td>0.7</td> <td>-0.21783</td> <td>0.8</td> <td>0.149326</td> <td>0.759329</td> <td>-0.00431</td> </tr> <tr> <td>0.759329</td> <td>-0.00431</td> <td>0.8</td> <td>0.149326</td> <td>0.760469</td> <td></td> </tr> </table> 0.76 to 2dp	a	$f(a)$	b	$f(b)$	x	$f(x)$	0.7	-0.21783	0.8	0.149326	0.759329	-0.00431	0.759329	-0.00431	0.8	0.149326	0.760469		M1 A1 M1 A1 [4]	Allow a maximum of 3 out 4 for a solution which goes wrong but self corrects For correct interval and calculating x Must follow from false position		
a	$f(a)$	b	$f(b)$	x	$f(x)$																		
0.7	-0.21783	0.8	0.149326	0.759329	-0.00431																		
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h	$g(-h)$	$g(h)$	$g'(0)$																				
0.2	1.1292	1.2745	0.36325																				
0.1	1.1766	1.2489	0.3615																				
0.05	1.1974	1.2335	0.361																				
5 (ii)	0.36 because last figure still changing and so unreliable Or 0.361 if some argument about convergence or extrapolation is used	A1 E1 [2]	Any sensible comment or attempt to analyse errors																				
6 (i)	<table style="margin-left: 40px;"> <tr> <td>x</td> <td>$f(x)$</td> <td>T</td> <td>M</td> <td>S</td> </tr> <tr> <td>0</td> <td>1</td> <td></td> <td></td> <td></td> </tr> <tr> <td>0.5</td> <td>1.243504</td> <td></td> <td></td> <td></td> </tr> <tr> <td>0.25</td> <td>1.12042</td> <td>0.560876</td> <td>0.560210</td> <td>0.560432</td> </tr> </table>	x	$f(x)$	T	M	S	0	1				0.5	1.243504				0.25	1.12042	0.560876	0.560210	0.560432	M1 M1 M1 [3]	M Award these marks for a correct answer S or a correct method with wrong answer T Do not penalise no. of sf
x	$f(x)$	T	M	S																			
0	1																						
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0.25	1.12042	0.560876	0.560210	0.560432																			

Question		Answer	Marks	Guidance																																			
6	(ii)	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center;">x</td> <td style="text-align: center;">$f(x)$</td> <td style="text-align: center;">T</td> <td style="text-align: center;">M</td> <td style="text-align: center;">S</td> </tr> <tr> <td style="text-align: center;">0.125</td> <td style="text-align: center;">1.060969</td> <td></td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">0.375</td> <td style="text-align: center;">1.18052</td> <td style="text-align: center;">0.560543</td> <td style="text-align: center;">0.560372</td> <td style="text-align: center;">0.560429</td> </tr> <tr> <td style="text-align: center;">0.0625</td> <td style="text-align: center;">1.030816</td> <td></td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">0.1875</td> <td style="text-align: center;">1.090747</td> <td></td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">0.3125</td> <td style="text-align: center;">1.150255</td> <td></td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">0.4375</td> <td style="text-align: center;">1.211499</td> <td style="text-align: center;">0.560458</td> <td style="text-align: center;">0.560415</td> <td style="text-align: center;">0.560429</td> </tr> </table> <p>0.560429 is justified (for information only: 0.5604289 is justified if more sf used)</p>	x	$f(x)$	T	M	S	0.125	1.060969				0.375	1.18052	0.560543	0.560372	0.560429	0.0625	1.030816				0.1875	1.090747				0.3125	1.150255				0.4375	1.211499	0.560458	0.560415	0.560429	<p>A6</p> <p>A1</p> <p>[7]</p>	<p>Values Lose 1 for each error Lose 1 overall if no. of sf is not 6 FT sensible but incorrect M and/or T to S</p>
x	$f(x)$	T	M	S																																			
0.125	1.060969																																						
0.375	1.18052	0.560543	0.560372	0.560429																																			
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6	(iii)	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center;">T</td> <td style="text-align: center;">diffs</td> <td style="text-align: center;">ratio</td> <td style="text-align: center;">M</td> <td style="text-align: center;">diffs</td> <td style="text-align: center;">ratio</td> </tr> <tr> <td style="text-align: center;">0.560876</td> <td></td> <td></td> <td style="text-align: center;">0.560210</td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">0.560543</td> <td style="text-align: center;">-0.000333</td> <td></td> <td style="text-align: center;">0.560372</td> <td style="text-align: center;">0.000162</td> <td></td> </tr> <tr> <td style="text-align: center;">0.560458</td> <td style="text-align: center;">-0.000085</td> <td style="text-align: center;">0.256788</td> <td style="text-align: center;">0.560415</td> <td style="text-align: center;">0.000043</td> <td style="text-align: center;">0.262091</td> </tr> </table> <p>Ratios about 0.25 in each case; indicates both have 2nd order convergence But M is more accurate than T; smaller differences so nearer the correct answer</p>	T	diffs	ratio	M	diffs	ratio	0.560876			0.560210			0.560543	-0.000333		0.560372	0.000162		0.560458	-0.000085	0.256788	0.560415	0.000043	0.262091	<p>M1 A1</p> <p>M1 A1</p> <p>E1 E1 E1 E1</p> <p>[8]</p>	<p>T Allow small errors that still give ratios ...</p> <p>M ... of approximately 0.25</p> <p>Allow correct explanations using 0.25 if the ratios come out wrong Allow correct statements about M and T even if not supported by the numbers</p>											
T	diffs	ratio	M	diffs	ratio																																		
0.560876			0.560210																																				
0.560543	-0.000333		0.560372	0.000162																																			
0.560458	-0.000085	0.256788	0.560415	0.000043	0.262091																																		
7	(i)	In the first 100 terms the positive rounding errors exceed the negative rounding errors The opposite occurs in the first 200 terms.	<p>E1</p> <p>E1</p> <p>[2]</p>	<p>Allow E1 for an incomplete explanation that shows some understanding</p>																																			
7	(ii)	Chopping will reduce the sum by an average of 0.00005 per term ie by 0.005 and 0.01 in S_{100} and S_{200} Hence estimate as 18.5846 (18.585) and 26.8493 (26.85)	<p>E1</p> <p>M1A1 A1</p> <p>[4]</p>	<p>M1 for 0.00005, A1 rest</p>																																			

Question		Answer	Marks	Guidance																
7	(iii)	$\int_{k-0.5}^{k+0.5} \frac{1}{\sqrt{x}} dx = \left[2\sqrt{x} \right]_{k-0.5}^{k+0.5}$ = RHS Midpoint rule Gives LHS	M1 A1 M1 A1 [4]	Answer given Must be convincing Answer given																
7	(iv)	<table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td>approx</td> <td>exact</td> <td>error</td> </tr> <tr> <td>S_{100}</td> <td>18.63572</td> <td>18.5896</td> <td>0.046124</td> </tr> <tr> <td>S_{200}</td> <td>26.90539</td> <td>26.8593</td> <td>0.046091</td> </tr> </table> Errors almost exactly equal		approx	exact	error	S_{100}	18.63572	18.5896	0.046124	S_{200}	26.90539	26.8593	0.046091	B1 B1 M1 A1 E1 [5]	Approximations Errors				
	approx	exact	error																	
S_{100}	18.63572	18.5896	0.046124																	
S_{200}	26.90539	26.8593	0.046091																	
7	(v)	<table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td></td> <td>assumed</td> <td></td> </tr> <tr> <td></td> <td>approx</td> <td>error</td> <td>estimate</td> </tr> <tr> <td>S_{1000}</td> <td>61.84715</td> <td>0.046</td> <td>61.80115</td> </tr> <tr> <td></td> <td></td> <td></td> <td>(61.801 or 61.80)</td> </tr> </table> (For information, correct sum is 61.80101 to 5dp)			assumed			approx	error	estimate	S_{1000}	61.84715	0.046	61.80115				(61.801 or 61.80)	B1 M1 A1 [3]	Approx Correction using 0.046 (or similar) Penalize more dp
		assumed																		
	approx	error	estimate																	
S_{1000}	61.84715	0.046	61.80115																	
			(61.801 or 61.80)																	

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Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

OCR Report to Centres

June 2013

4776 Numerical Methods (Written Examination)

General Comments

The purely computational parts of this question paper were found straightforward by most candidates, though closer attention needs to be paid to the accuracy with which answers are given. The interpretation of results was less good this session than in recent years.

The standard of presentation of work, and in particular the systematic setting out of numerical algorithms, continues to be better than it was a few years ago. However some candidates are still resort to scattering calculations haphazardly on the page, making it difficult for examiners to detect and reward any correct work.

Comments on Individual Questions

- 1) **Solution of an equation, fixed point iteration**
This proved to be a very straightforward question for the majority of candidates. In part (i), some of the graphs were very inaccurate, and it was common to have no indication of scale on one or both of the axes. In part (ii), almost everyone made the obvious rearrangement using the square root.
- 2) **Absolute and relative errors**
This question, too, was straightforward for most candidates. The calculations in part (i) were generally done well and the comments were accurate. In part (ii), a surprising number of candidates did not see (or did not use) the information given that k is an integer. Indeed, many of those who did give k as an integer appeared to be doing so as a way of avoiding being too precise rather than because of the given information.
- 3) **Newton's forward difference interpolation formula**
The difference table in part (i) was generally done well, though inevitably some candidates made sign errors. A few omitted to comment on the near equality of the second differences. Part (ii) was a little more testing, but a majority gained full marks.
- 4) **Solution of an equation, false position method**
This question proved rather challenging for many candidates. The sketches of the graph in part (i) were often of poor quality and unconvincing: an increasing function of vague shape was as much as many candidates could come up with. However, showing that the root lies in the given interval was easy. The method of false position, required in part (ii), was sometimes confused with the secant method. It was good to see more candidates laying out their work in tabular form.
- 5) **Numerical differentiation, central difference method**
Most candidates knew which formula to use but did not always do so on a sensible set of three values of h . Given the need to estimate the likely accuracy in part (ii), the best values to use are 0.2, 0.1, 0.05 so that the rate of convergence can be judged as h is halved. In part (ii), credit was given for either 0.36 or 0.361 with appropriate justification.

6) **Numerical integration**

Part (i) was almost invariably done well. In part (ii), however, errors did build up. Candidates were told to give answers to 6 decimal places, but this instruction was frequently ignored. Consideration of the Simpson's rule values in part (ii) shows that the integral is known accurately to 6 decimal places. Part (iii) required a clear understanding of the difference between the order of a method and its accuracy. The mid-point and trapezium rules are both second order (as indicated by a ratio of differences approximating to 0.25) but the mid-point rule is more accurate (as indicated by the differences being smaller in magnitude). Many candidates carried out the calculations correctly but were unable to give full and clear comments on what the calculations show.

7) **Errors and approximations; computer representation of numbers**

This was the most challenging question on the paper – perhaps surprisingly so, as questions exploring similar ground have been set before. In part (i), candidates were required to deduce that in S_{100} there must be more rounding up than rounding down, with the opposite happening in S_{200} . The explanations offered frequently missed the point. In part (ii), very few candidates could calculate correctly the likely effect of chopping. In part (iii), many were able to come up with the left hand side of the equation by using the mid-point rule as directed, but very few realised that the right hand side simply came from doing the integration. In part (iv), most candidates picked up on the given formula and carried out the calculations correctly. Applying what they had done to part (v) was straightforward, though many candidates gave the final answer to an unjustified number of significant figures.

Unit level raw mark and UMS grade boundaries June 2013 series
AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)		Max Mark	a	b	c	d	e	u
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	62	56	51	46	41	0
	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	54	48	43	38	33	0
	UMS	100	80	70	60	50	40	0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	46	40	33	0
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	66	59	53	47	41	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
	UMS	100	80	70	60	50	40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	61	54	48	42	36	0
	UMS	100	80	70	60	50	40	0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	60	52	44	36	28	0
	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	62	56	51	46	40	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	57	49	41	33	25	0
	UMS	100	80	70	60	50	40	0
4762/01 (M2) MEI Mechanics 2	Raw	72	50	43	36	29	22	0
	UMS	100	80	70	60	50	40	0
4763/01 (M3) MEI Mechanics 3	Raw	72	64	56	48	41	34	0
	UMS	100	80	70	60	50	40	0
4764/01 (M4) MEI Mechanics 4	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4766/01 (S1) MEI Statistics 1	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw	72	58	52	46	41	36	0
	UMS	100	80	70	60	50	40	0
4768/01 (S3) MEI Statistics 3	Raw	72	61	55	49	44	39	0
	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	58	52	46	40	35	0
	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	58	52	46	41	36	0
	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
	UMS	100	80	70	60	50	40	0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	44	38	31	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4798/01 (FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0
GCE Statistics (MEI)		Max Mark	a	b	c	d	e	u
G241/01 (Z1) Statistics 1	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
G242/01 (Z2) Statistics 2	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
G243/01 (Z3) Statistics 3	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0