



**Thursday 22 May 2014 – Morning**

**A2 GCE MATHEMATICS (MEI)**

**4763/01** Mechanics 3

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

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- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 (a) The speed  $v$  of sound in a solid material is given by  $v = \sqrt{\frac{E}{\rho}}$ , where  $E$  is Young's modulus for the material and  $\rho$  is its density.

(i) Find the dimensions of Young's modulus. [3]

The density of steel is  $7800 \text{ kg m}^{-3}$  and the speed of sound in steel is  $6100 \text{ m s}^{-1}$ .

(ii) Find Young's modulus for steel, stating the units in which your answer is measured. [2]

A tuning fork has cylindrical prongs of radius  $r$  and length  $l$ . The frequency  $f$  at which the tuning fork vibrates is given by  $f = kc^\alpha E^\beta \rho^\gamma$ , where  $c = \frac{l^2}{r}$  and  $k$  is a dimensionless constant.

(iii) Find  $\alpha$ ,  $\beta$  and  $\gamma$ . [4]

- (b) A particle P is performing simple harmonic motion along a straight line, and the centre of the oscillations is O. The points X and Y on the line are on the same side of O, at distances 3.9 m and 6.0 m from O respectively. The speed of P is  $1.04 \text{ m s}^{-1}$  when it passes through X and  $0.5 \text{ m s}^{-1}$  when it passes through Y.

(i) Find the amplitude and the period of the oscillations. [5]

(ii) Find the time taken for P to travel directly from X to Y. [4]

- 2 (a) The fixed point A is vertically above the fixed point B. A light inextensible string of length 5.4 m has one end attached to A and the other end attached to B. The string passes through a small smooth ring R of mass 0.24 kg, and R is moving at constant angular speed in a horizontal circle. The circle has radius 1.6 m, and  $AR = 3.4$  m,  $RB = 2.0$  m, as shown in Fig. 2.

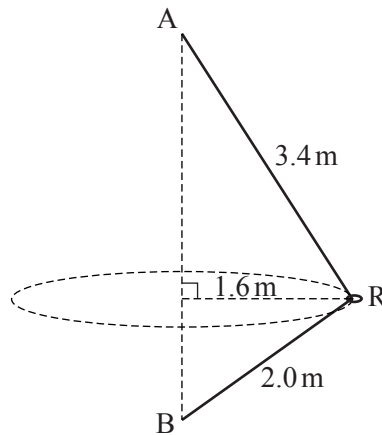


Fig. 2

- (i) Find the tension in the string. [3]
- (ii) Find the angular speed of R. [3]
- (b) A particle P of mass 0.3 kg is joined to a fixed point O by a light inextensible string of length 1.8 m. The particle P moves without resistance in part of a vertical circle with centre O and radius 1.8 m. When OP makes an angle of  $25^\circ$  with the downward vertical, the tension in the string is 15 N.
- (i) Find the speed of P when OP makes an angle of  $25^\circ$  with the downward vertical. [3]
- (ii) Find the tension in the string when OP makes an angle of  $60^\circ$  with the upward vertical. [5]
- (iii) Find the speed of P at the instant when the string becomes slack. [5]

- 3 The fixed points A and B lie on a line of greatest slope of a smooth inclined plane, with B higher than A. The horizontal distance from A to B is 2.4 m and the vertical distance is 0.7 m. The fixed point C is 2.5 m vertically above B. A light elastic string of natural length 2.2 m has one end attached to C and the other end attached to a small block of mass 9 kg which is in contact with the plane. The block is in equilibrium when it is at A, as shown in Fig. 3.

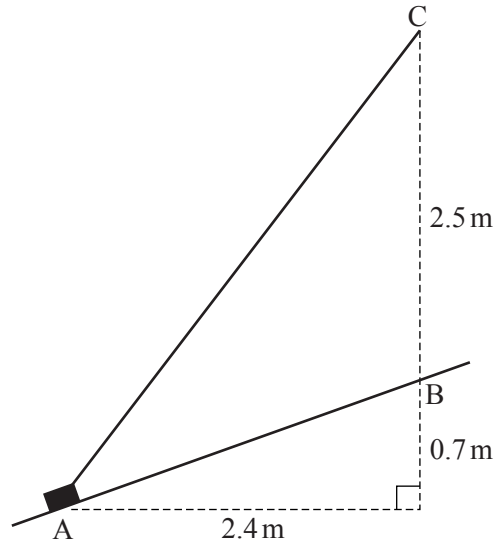


Fig. 3

- (i) Show that the modulus of elasticity of the string is 37.73 N. [5]

The block starts at A and is at rest. A constant force of 18 N, acting in the direction AB, is then applied to the block so that it slides along the line AB.

- (ii) Find the magnitude and direction of the acceleration of the block
- (A) when it leaves the point A,
- (B) when it reaches the point B. [6]
- (iii) Find the speed of the block when it reaches the point B. [6]

- 4 The region  $R$  is bounded by the  $x$ -axis, the  $y$ -axis, the curve  $y = e^{-x}$  and the line  $x = k$ , where  $k$  is a positive constant.

- (i) The region  $R$  is rotated through  $2\pi$  radians about the  $x$ -axis to form a uniform solid of revolution. Find the  $x$ -coordinate of the centre of mass of this solid, and show that it can be written in the form

$$\frac{1}{2} - \frac{k}{e^{2k} - 1}. \quad [7]$$

- (ii) The solid in part (i) is placed with its larger circular face in contact with a rough plane inclined at  $60^\circ$  to the horizontal, as shown in Fig. 4, and you are given that no slipping occurs.

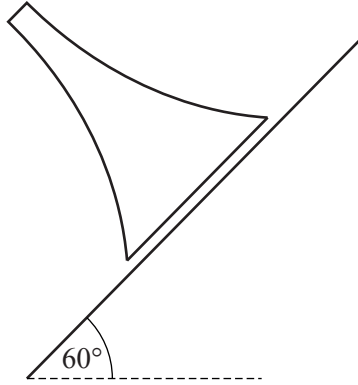


Fig. 4

Show that, whatever the value of  $k$ , the solid will not topple. [4]

- (iii) A uniform lamina occupies the region  $R$ . Find, in terms of  $k$ , the coordinates of the centre of mass of this lamina. [7]

**END OF QUESTION PAPER**



**Thursday 22 May 2014 – Morning**

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**4763/01 Mechanics 3**

**PRINTED ANSWER BOOK**

Candidates answer on this Printed Answer Book.

**OCR supplied materials:**

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- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



Candidate forename		Candidate surname	
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Centre number						Candidate number				
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<b>1 (a) (i)</b>	

<b>1 (a) (ii)</b>	

<b>1 (a) (iii)</b>	



<b>1(b)(i)</b>	

<b>1 (b) (ii)</b>	

<b>2(a)(i)</b>	

<b>2(a)(ii)</b>	

<b>2(b)(i)</b>	

<b>2(b)(ii)</b>	

<b>2 (b) (iii)</b>	



<b>3(ii)(A)</b>	

<b>3(ii)(B)</b>	



<b>3 (iii)</b>	



<b>4(ii)</b>	

<b>4 (iii)</b>	

(answer space continued on next page)

<b>4 (iii) (continued)</b>	



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**GCE**

**Mathematics (MEI)**

Unit **4763**: Mechanics 3 (M3)

Advanced GCE

**Mark Scheme for June 2014**

## 1. Annotations and abbreviations

Annotation in scoris	Meaning
<b>BP</b>	Blank Page – this annotation <b>must</b> be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

**2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand**

- a. Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c. The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.



**E**

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep \*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (eg lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

**When a value is given in the paper**

Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

**When a value is not given in the paper**

Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

There is no penalty for using a wrong value for  $g$ . E marks will be lost except when results agree to the accuracy required in the question.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

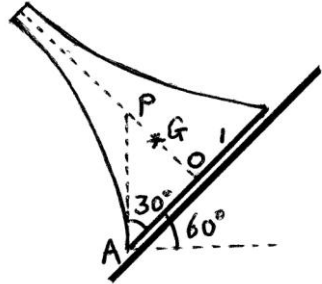
j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	Guidance
1	(a)	(i)	$[\rho] = \text{ML}^{-3}$ $[E] = [\rho v^2] = (\text{ML}^{-3})(\text{LT}^{-1})^2$ Dimensions of Young's modulus are $\text{ML}^{-1}\text{T}^{-2}$	B1 M1 A1 <b>[3]</b>	Obtaining dimensions of $E$
1	(a)	(ii)	$E = \rho v^2 = 7800 \times 6100^2 = 2.90 \times 10^{11}$ (3 sf) Units are $\text{kg m}^{-1} \text{s}^{-2}$	B1 B1 <b>[2]</b>	OR $\text{Nm}^{-2}$ OR Pa FT provided all powers are non-zero No FT if derived units involved
1	(a)	(iii)	$\text{T}^{-1} = \text{L}^\alpha (\text{ML}^{-1}\text{T}^{-2})^\beta (\text{ML}^{-3})^\gamma$ $\beta = \frac{1}{2}$ $\gamma = -\frac{1}{2}$ $\alpha - \beta - 3\gamma = 0$ $\alpha = -1$	B1 B1 M1 A1 <b>[4]</b>	CAO FT $\gamma = -\beta$ Equation from powers of L CAO Provided non-zero
1	(b)	(i)	$1.04^2 = \omega^2(A^2 - 3.9^2)$ $0.5^2 = \omega^2(A^2 - 6.0^2)$ $\frac{A^2 - 15.21}{A^2 - 36} = 4.3264$ Amplitude ( $A$ ) is 6.5 m Period ( $\frac{2\pi}{\omega}$ ) is $10\pi = 31.4$ s (3 sf)	M1 A1 M1 A1 A1 <b>[5]</b>	Using $v^2 = \omega^2(A^2 - x^2)$ Both equations correct Eliminating $\omega$ or $A$ $A = 6.5, \omega = 0.2$
1	(b)	(ii)	$x = 6.5 \sin 0.2t$ When $x = 3.9, t = 3.2175$ When $x = 6.0, t = 5.8800$ Time from X to Y is 2.66 s (3 sf)	B1 M1 M1 A1 <b>[4]</b>	FT For $6.5 \sin 0.2t$ or $6.5 \cos 0.2t$ Using $x = 3.9$ or $x = 6.0$ to find a time Fully correct strategy for required time CAO OR ( $v =$ ) $1.3 \sin 0.2t$ etc OR using $v = 1.04$ or $v = 0.5$ 4.6365 – 1.9740 if cos is used

Question			Answer	Marks	Guidance
2	(a)	(i)	$T \cos \alpha = T \cos \beta + 0.24 \times 9.8$ $\frac{15}{17}T = \frac{3}{5}T + 2.352$ Tension is 8.33 N	M1 A1 A1 [3]	Resolving vertically (three terms) Accept $\cos 28.1^\circ$ etc $\alpha = \hat{A} = 28.1^\circ, \beta = \hat{B} = 53.1^\circ$
2	(a)	(ii)	$T \sin \alpha + T \sin \beta = m(r\omega^2)$ $\frac{8}{17}T + \frac{4}{5}T = (0.24)(1.6\omega^2)$ Angular speed is $5.25 \text{ rad s}^{-1}$	M1 A1 A1 [3]	Eqn with resolved tension and $r\omega^2$ <i>One tension sufficient for M1</i> Allow $\frac{v^2}{r}$ for M1 FT is $1.819\sqrt{T}$
2	(b)	(i)	$15 - (0.3)(9.8) \cos 25^\circ = (0.3) \frac{v_1^2}{1.8}$ Speed is $8.60 \text{ ms}^{-1}$ (3 sf)	M1 A1 A1 [3]	Equation with tension, resolved weight and $v_1^2 / r$ <i>Accept use of mass instead of weight throughout for M marks</i>
2	(b)	(ii)	$\frac{1}{2}m(v_1^2 - v_2^2) = mg(1.8 \cos 25^\circ + 1.8 \cos 60^\circ)$ $v_2^2 = 24.40$ $T + (0.3)(9.8) \cos 60^\circ = (0.3) \frac{v_2^2}{1.8}$ Tension is 2.60 N (3 sf)	M1 A1 M1 A1 A1 [5]	Equation with initial KE, final KE and attempt at PE Equation with tension, resolved weight (using $60^\circ$ ) and $v_2^2 / r$ FT is $\frac{v_1^2}{6} - 9.739$ SC For $60^\circ$ with <i>downward</i> vertical give A1 for 11.4 N (after M1A0M1A0), i.e. 3/5

Question			Answer	Marks	Guidance	
2	(b)	(iii)	$(m)g \cos \theta = (m) \frac{v_3^2}{1.8}$ $\frac{1}{2}m(v_2^2 - v_3^2) = mg(1.8)(\cos \theta - \cos 60^\circ)$ $24.40 - v_3^2 = 2v_3^2 - 9.8 \times 1.8$ Speed is $3.74 \text{ ms}^{-1}$ (3 sf)	M1 A1 M1 A1 A1 <b>[5]</b>	Equation with resolved weight in general position, and $v_3^2 / r$ Equation with KE and attempt at PE in general position OR CAO	May also include $T$ $\theta$ is angle between OP and upward vertical OR $\frac{1}{2}m(v_1^2 - v_3^2) = mg(1.8)(\cos \theta + \cos 25^\circ)$ $\cos \theta = 0.794$ , $\theta = 0.653 \text{ rad} = 37.4^\circ$
3	(i)		$T \cos \beta = mg \sin \alpha$ $0.8T = (9)(9.8)(0.28)$	M1 A1	Resolving parallel to slope Accept $\cos 36.9^\circ$ etc	$\alpha$ is angle of slope, $\beta = \hat{CAB}$ $\alpha = 16.26^\circ$ , $\beta = 53.13 - \alpha = 36.87^\circ$
		OR	$T \sin \gamma + R \cos \alpha = mg$ $T \cos \gamma = R \sin \alpha$ $0.8T + 0.96R = 9 \times 9.8$ $0.6T = 0.28R$		M1 Resolving vertically <i>and</i> horizontally A1 Both equations correct	$\gamma$ is between string and horizontal $\gamma = 53.13^\circ$ ( $R$ is normal reaction)
			$T = 30.87$ $T = \frac{\lambda(4.0 - 2.2)}{2.2}$ Modulus of elasticity is $37.73 \text{ N}$	A1 B1 E1 <b>[5]</b>	Accept anything rounding to 31 Correct equation linking $T$ and $\lambda$ <i>Working must lead to 37.73 to 4 sf</i>	<i>Dep on M1A1 (May be implied)</i>
3	(ii)	(A)	Resultant force is $18 \text{ N}$ (up the slope) Acceleration is $2 \text{ ms}^{-2}$ in direction AB	M1 A1 <b>[2]</b>	Or $18 + T \cos \beta - mg \sin \alpha$ <i>Accept positive direction indicated clearly on diagram</i>	<i>Just '2' implies M1A0</i>

Question			Answer	Marks	Guidance
3	(ii)	(B)	At B, tension is $\frac{37.73 \times (2.5 - 2.2)}{2.2}$ (= 5.145) $18 + T_B \sin \alpha - mg \sin \alpha = ma$ $18 + 5.145 \times 0.28 - 9 \times 9.8 \times 0.28 = 9a$ Acceleration is $0.584 \text{ ms}^{-2}$ in direction BA (3 sf)	B1  M1 A1 A1 <b>[4]</b>	Equation of motion FT for wrong tension CAO  At least two forces required for M1
3	(iii)		WD by force is $18 \times 2.5$ (= 45) EE at A is $\frac{37.73 \times 1.8^2}{2 \times 2.2}$ (= 27.783) EE at B is $\frac{37.73 \times 0.3^2}{2 \times 2.2}$ (= 0.77175)  Change in PE is $9 \times 9.8 \times 0.7$ (= 61.74)  $45 + 27.783 = 0.77175 + 61.74 + \frac{1}{2}(9)v^2$ Speed is $1.51 \text{ ms}^{-1}$ (3 sf)	B1  B1  B1 M1 A1 A1 <b>[6]</b>	For either of these  Equation involving KE and at least two of WD, EE, PE FT from any B0 above, but all 5 terms must be non-zero and all signs correct CAO  <i>Dependent on previous 5 marks</i>

Question		Answer	Marks	Guidance	
4	(i)	<p>Volume is <math>\int_0^k \pi(e^{-x})^2 dx</math></p> $= \pi \left[ -\frac{1}{2}e^{-2x} \right]_0^k \quad \left\{ = \frac{1}{2}\pi(1 - e^{-2k}) \right\}$ <p><math>\int \pi xy^2 dx</math></p> $= \int_0^k \pi x e^{-2x} dx = \pi \left[ -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} \right]_0^k$ $= \frac{1}{4}\pi(1 - 2ke^{-2k} - e^{-2k})$ $\bar{x} = \frac{1 - 2ke^{-2k} - e^{-2k}}{2(1 - e^{-2k})}$ $= \frac{1 - e^{-2k}}{2(1 - e^{-2k})} - \frac{2ke^{-2k}}{2(1 - e^{-2k})} = \frac{1}{2} - \frac{k}{e^{2k} - 1}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1A1</p> <p>A1</p> <p>E1</p> <p>[7]</p>	<p><math>\pi</math> may be omitted throughout</p> <p>For <math>-\frac{1}{2}e^{-2x}</math></p> <p>For <math>-\frac{1}{2}xe^{-2x}</math> and <math>-\frac{1}{4}e^{-2x}</math></p> <p>Any correct form</p>	
4	(ii)	<p><math>OG &lt; \frac{1}{2}</math> for all values of <math>k</math></p> <p><math>OP = (1) \tan 30^\circ = \frac{1}{\sqrt{3}} (= 0.577)</math></p> <p><math>OG &lt; OP</math> (or <math>\hat{OAG} &lt; 30^\circ</math>) so G is to the right of AP and solid will not topple</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>E1</p> <p>[4]</p>	<p>OR <math>\frac{k}{e^{2k} - 1} &gt; 0</math> o.e. stated or implied</p> <p>Allow <math>\bar{x} \rightarrow \frac{1}{2}</math> as <math>k \rightarrow \infty</math> for B1</p> <p>Trigonometry in OAP or OAG</p> <p>Or <math>\hat{OAG} &lt; \tan^{-1} \frac{1}{2} (= 26.6^\circ)</math></p> <p>Fully correct explanation</p>	

4	Question	Answer	Marks	Guidance
	(iii)	Area is $\int_0^k e^{-x} dx = [-e^{-x}]_0^k (=1-e^{-k})$ $\int xy dx$ $= \int_0^k xe^{-x} dx = [-xe^{-x} - e^{-x}]_0^k$ $\bar{x} = \frac{1-ke^{-k} - e^{-k}}{1-e^{-k}}$ $\int \frac{1}{2}y^2 dx$ $= \int_0^k \frac{1}{2}e^{-2x} dx = [-\frac{1}{4}e^{-2x}]_0^k$ $\bar{y} = \frac{1-e^{-2k}}{4(1-e^{-k})}$	B1 M1 A1 A1 M1 A1 A1 [7]	Any correct form For $\int \dots y^2 dx$ Any correct form Any correct form e.g. $1 - \frac{k}{e^k - 1}$ e.g. $\frac{1}{4}(1 + e^{-k})$



## 4763 Mechanics 3

### General Comments:

Most of the candidates demonstrated a sound understanding of the topics involved. The extent to which they could apply these mechanical principles to the particular problems set varied considerably and resulted in a wide range of marks for the paper. The questions on dimensional analysis (Q.1(a)), simple harmonic motion (Q.1(b)) and circular motion (Q.2) were generally well answered. The situations in Q.3 (based on elasticity) presented challenges to very many candidates, because care was required in setting up the equations of motion and the work-energy principle. The centres of mass question (Q.4) presented challenges in the integration and the application to toppling. Most candidates obtained the bulk of their marks from Q.1 and Q.2, and the average mark for the paper was somewhat lower than it has been in the recent past.

### Comments on Individual Questions:

#### Question No. 1 (Dimensional Analysis and Simple Harmonic Motion)

Most candidates answered this question well. In part (a)(i) the dimensions of density were almost always stated correctly and used to find the dimensions of Young's modulus. The only common error was to use  $E = \rho v$  instead of  $E = \rho v^2$ .

In part (a)(ii) most candidates calculated the value correctly. The relationship between the dimensions found in part (a)(i) and the SI units of Young's modulus was well understood.

In part (a)(iii) most candidates knew how to use dimensional analysis to find the indices in the equation, and very many completed this accurately.

In part (b)(i) most candidates attempted to apply the equation  $v^2 = \omega^2(A^2 - x^2)$  at the points  $X$  and  $Y$ , and a good number followed this through to find the amplitude and the period. A common error was to obtain  $\omega = 0.04$  instead of  $\omega^2 = 0.04$ ; another was, after finding  $\omega$ , to omit finding the period.

In part (b)(ii) most candidates used a suitable displacement-time (or, rarely, velocity-time) equation, with their  $A$  and  $\omega$ , to find the time taken.

#### Question No. 2 (Circular Motion)

In this question most candidates were able to demonstrate their competence at solving problems involving circular motion and conservation of energy.

In part (a)(i) it was necessary to realise that the tensions in both parts of the string were equal, and both parts had vertical components; candidates then usually completed the calculation successfully.

In part (a)(ii) most candidates wrote down a correct equation of motion. Many found the speed of the ring correctly but did not convert this into the required angular speed.

In part (b)(i) candidates needed to form an equation of motion in the direction of the string, and most candidates did this correctly.

In part (b)(ii) a successful solution required a radial equation of motion and an equation involving kinetic and potential energy. Most candidates formed the radial equation correctly, with others making sign errors or mistakes in resolving the weight. Many candidates made errors in the energy equation, notably in the potential energy. Those who considered potential energy at the start and finish (rather than finding the change in potential energy directly) quite often used two different reference points.

In part (b) (iii) it was again necessary to form a radial equation of motion (with zero tension in the string) and an energy equation. Candidates performed in a similar way to part (b) (ii).

Question No. 3 (Elasticity and Energy)

Most candidates demonstrated their understanding of elastic strings and elastic energy. This question assessed their skills in applying these principles, and produced a wide range of performance.

In part (i) the simplest approach was to resolve parallel to the plane, and candidates who did this were often successful. To earn full marks it was necessary to justify the given value to an accuracy of at least 4 significant figures, so finding the angles to one decimal place, as many candidates did, resulted in the loss of one mark here. A fair number of candidates preferred to resolve horizontally and vertically; this approach was sometimes successful, provided that the normal reaction was included in the equations.

In part (ii) (A) the resultant force is simply the extra applied force of 18 N up the plane. Many candidates started again and repeated much of the work already done in part (i).

In part (ii) (B) most candidates obtained the new tension in the string correctly, and attempted to form an equation of motion parallel to the plane. Common errors in this equation included resolving incorrectly and omitting one of the three forces (usually the 18 N). The question asked for magnitude (which must be positive) and direction, so an answer ‘-0.584 up the plane’ did not earn the final mark. Some candidates persisted in forming vertical and horizontal equations of motion, and it was very difficult (but not impossible) to earn any marks using this approach.

In part (iii) most candidates formed an equation based on energy. The most common error was omission of the work done by the 18 N force. There were also many sign errors, and the change in elastic energy was sometimes calculated incorrectly.

Question No. 4 (Centres of Mass)

Most candidates showed a good understanding of how to find the centre of mass of a solid of revolution in part (i) and a lamina in part (iii). This question also tested integration (notably integration by parts) and an application of the centre of mass in part (ii).

In part (i) there were often errors, such as incorrect signs, in the integration of  $xe^{-2x}$ . When a correct expression for the  $x$ -coordinate of the centre of mass had been obtained it was necessary to rearrange it into the given form, and this was not always done convincingly.

In part (ii) most candidates attempted to show that the centre of mass was vertically above a point in the base and applied some relevant trigonometry. Many had the radius of the larger circular face as  $e^{-k}$  (which is the radius of the smaller circular face) or  $k$  (which is the ‘height’ of the solid) instead of 1. To earn full marks it was necessary to say that the  $x$ -coordinate of the centre of mass was always *less than*  $\frac{1}{2}$  (for example, *tends to*  $\frac{1}{2}$  was not sufficient). Many candidates did not attempt this part at all.

Part (iii) was completed correctly by many candidates. The main difficulty was the integration by parts when finding the  $x$ -coordinate.

**Unit level raw mark and UMS grade boundaries June 2014 series**  
**AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award**

<b>GCE Mathematics (MEI)</b>		<b>Max Mark</b>	<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>	<b>u</b>
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	61	56	51	46	42	0
	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	57	51	45	39	33	0
	UMS	100	80	70	60	50	40	0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	47	42	36	0
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	68	61	54	47	41	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
	UMS	100	80	70	60	50	40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	60	54	48	42	36	0
	UMS	100	80	70	60	50	40	0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	57	51	45	39	34	0
	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4762/01 (M2) MEI Mechanics 2	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4763/01 (M3) MEI Mechanics 3	Raw	72	55	48	42	36	30	0
	UMS	100	80	70	60	50	40	0
4764/01 (M4) MEI Mechanics 4	Raw	72	48	41	34	28	22	0
	UMS	100	80	70	60	50	40	0
4766/01 (S1) MEI Statistics 1	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw	72	60	53	46	40	34	0
	UMS	100	80	70	60	50	40	0
4768/01 (S3) MEI Statistics 3	Raw	72	61	54	47	41	35	0
	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	51	46	41	36	31	0
	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	46	41	36	31	26	0
	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
	UMS	100	80	70	60	50	40	0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	54	48	43	38	32	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4798/01 (FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0
<b>GCE Statistics (MEI)</b>		<b>Max Mark</b>	<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>	<b>u</b>
G241/01 (Z1) Statistics 1	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
G242/01 (Z2) Statistics 2	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
G243/01 (Z3) Statistics 3	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0