OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)

Numerical Computation

4777

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)
- Graph paper

Other Materials Required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Monday 28 June 2010
Afternoon

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet routines to carry out various numerical analysis processes.
- You will not receive credit for using any numerical analysis functions which are provided within the spreadsheet. For example, many spreadsheets provide a solver routine; you will not receive credit for using this routine when asked to write your own procedure for solving an equation.
  You may use the following built-in mathematical functions: square root, sin, cos, tan, arcsin, arccos, arctan, ln, exp.
- For each question you attempt, you should submit print-outs showing the spreadsheet routine you have written and the output it generates. It will be necessary to print out the formulae in the cells as well as the values in the cells.
  You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 The table shows some values of x and y that have been obtained experimentally. The values are assumed to be correct to the numbers of significant figures shown.

x	0.09	0.93	1.91	4.10	4.91	6.04
y	1.076	0.897	0.498	–0.544	–0.740	–0.900

(i) Estimated values of y are required for various values of x. Explain briefly why Newton's divided difference formula might be used here in preference to other methods of interpolation. **[3]**

(ii) Use a spreadsheet to obtain a sketch of the data. **[2]**

(iii) Set up a spreadsheet, using divided differences, to produce a sequence of estimates, linear, quadratic, cubic and quartic, of y when $x = 3$.

Discuss briefly the likely accuracy of the value of y when $x = 3$. **[14]**

(iv) Modify the spreadsheet so that it will estimate y for user-specified values of x near to 3. Hence determine, to 2 decimal places, the value of x for which y is zero. **[5]**

2 **(i)** The trapezium rule, using n strips of equal width h, is used to find an estimate T_n of the integral

$$I = \int_a^b f(x)\mathrm{d}x.$$

You are given that the global error in T_n is of the form

$$A_2 h^2 + A_4 h^4 + A_6 h^6 + \ldots,$$

where the coefficients $A_2, A_4, A_6, \ldots$ are independent of n and h.

Show that $T_n{}^* = \frac{1}{3}(4\,T_{2n} - T_n)$ is an estimate of I with global error of order h^4.

Write down, without proof, an expression, $T_n{}^{**}$, in terms of $T_{2n}{}^*$ and $T_n{}^*$, that represents an estimate of I with global error of order h^6. **[6]**

(ii) Use a spreadsheet to obtain a graph of $y = \ln(1 + \sin x)$ for $0 \leqslant x \leqslant 4.5$. **[2]**

(iii) Set up a spreadsheet that uses Romberg's method to find, correct to 5 decimal places, the integral

$$\int_0^\pi \ln(1 + \sin x)\,\mathrm{d}x.$$ **[11]**

(iv) Modify your spreadsheet so that it finds the value of

$$\int_0^c \ln(1 + \sin x)\,\mathrm{d}x$$

for a user-specified value of c. Hence find, correct to 3 decimal places, the value of c for which the integral is zero. **[5]**

3 The differential equation

$$\frac{dy}{dx} = \sqrt{1 + xy}\text{ , with } y = 1 \text{ when } x = 1,$$

is to be solved numerically. When $x = 2$, the value of y is α.

(i) Use the modified Euler method with $h = 0.1, 0.05, 0.025, \ldots$ to obtain a sequence of estimates of α. Show that the convergence of this sequence is second order. Obtain the value of α correct to 4 decimal places. **[12]**

(ii) Now set up a predictor-corrector routine to find a sequence of estimates of α. Use the Euler method as predictor and the modified Euler method as corrector. Apply the corrector 3 times at each step. As before take $h = 0.1, 0.05, 0.025, \ldots$ until α is secure to 4 decimal places. **[8]**

(iii) Compare briefly the computational merits of the methods in parts **(i)** and **(ii)**. **[4]**

4 The system of linear equations with augmented matrix

$$\left(\begin{array}{cccc|c} 7+\alpha & 6 & 5 & 4 & 1+\beta \\ 6 & 5+\alpha & 4 & 3 & 1 \\ 5 & 4 & 3+\alpha & 2 & 1 \\ 4 & 3 & 2 & 1+\alpha & 1 \end{array}\right)$$

is to be investigated numerically for various values of α and β.

(i) For the case $\alpha = 0.1$ and $\beta = 0$, solve the equations using Gaussian elimination with partial pivoting. Find the magnitude of the determinant of the coefficient matrix. **[14]**

(ii) For the case $\alpha = 0.01$, solve the equations for

(A) $\beta = 0$,

(B) $\beta = 0.01$,

and find the magnitude of the determinant of the coefficient matrix.

Comment on your results. **[10]**

THERE ARE NO QUESTIONS ON THIS PAGE

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **4777**

Numerical Computation

Mark Scheme for June 2010

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

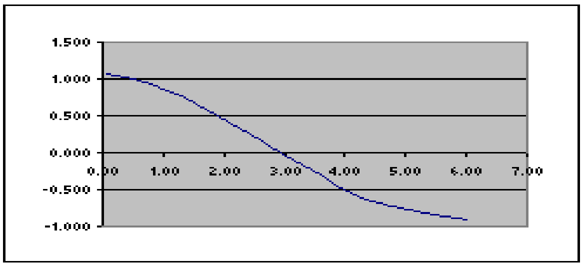
OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

1 (i) The data are not evenly spaced so (ordinary) differences will not work **[E1]**
Lagrange's method is not well suited to increasing the degree of the **[E1]**
approximating polynomial because it requires complete recalculation **[E1]**
[subtotal 3]

(ii)

x	f
0.09	1.076
0.93	0.897
1.91	0.498
4.10	-0.544
4.91	-0.740
6.04	-0.900



[G2]
[subtotal 2]

(iii)

x	f	1DD	2DD	3DD	4DD	5DD
1.91	0.498					
4.10	-0.544	-0.4758				
4.91	-0.740	-0.24198	0.077941			
0.93	0.897	-0.41131	0.053417	0.025025		
0.09	1.076	-0.2131	-0.04112	0.023576	0.000796	
6.04	-0.900	-0.3321	-0.02329	0.015782	-0.00402	-0.00117

re-order: **[M1A1]**
table: **[M1A1]**

f(3)				
=	0.498			
+	-0.51862	-0.021	linear	**[M1A1]**
+	-0.09345	-0.114	quadratic	**[M1A1]**
+	0.057309	-0.057	cubic	**[M1A1]**
+	0.003774	-0.053	quartic	**[M1A1]**

f(3) approximately zero, but difficult to say whether -0.05 or -0.06, -0.1 or 0.0. **[E1E1]**
[subtotal 14]

(iv)

x	f	1DD	2DD	3DD	4DD	5DD
1.91	0.498					
4.10	-0.544	-0.4758				
4.91	-0.740	-0.24198	0.077941			
0.93	0.897	-0.41131	0.053417	0.025025		
0.09	1.076	-0.2131	-0.04112	0.023576	0.000796	
6.04	-0.900	-0.3321	-0.02329	0.015782	-0.00402	-0.00117

user-specified x:	2.89	0.498		*adjust SS to allow*	
		-0.46628	0.032	*user-specified x:*	**[M1A1]**
		-0.09242	-0.061		
		0.056679	-0.004	*trial and error:*	**[M1A1]**
		0.003738	0.000	*answer:*	**[A1]**

[subtotal 5]
[TOTAL 24]

2 (i) $T_n - I = A_2h^2 + A_4h^4 + A_6h^6 + \ldots$

$T_{2n} - I = A_2(h/2)^2 + A_4(h/2)^4 + A_6(h/2)^6 + \ldots$ **[M1A1]**

$4(T_{2n} - I) - (T_n - I) = b_4h^4 + b_6h^6 + \ldots$ **[M1]**

$4T_{2n} - T_n - 3I = b_4h^4 + b_6h^6 + \ldots$ **[A1]**

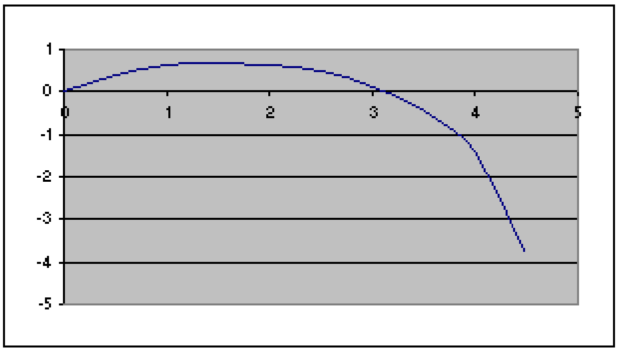
$(4T_{2n} - T_n)/3 - I = B_4h^4 + B_6h^6 + \ldots$ **[A1]**

($T_n^* = (4T_{2n} - T_n)/3$ has error of order h^4 as given)

$T_n^{**} = (16T_{2n}^* - T_n^*)/15$ has error of order h^6 **[B1]**

[subtotal 6]

(ii)



[G2]

[subtotal 2]

(iii)

x	f(x)	T	T*	T**	T***	(T****)		
0	0							
3.141593	2.22E-16	3.49E-16						
1.570796	0.693147	1.088793	1.451724					
0.785398	0.5348						f:	[A1]
2.356194	0.5348	1.384458	1.483014	1.485099				
0.392699	0.324026						*T:*	[M1A2]
1.178097	0.654344							
1.963495	0.654344						*T*:*	[M1A1]
2.748894	0.324026	1.460639	1.486033	1.486234	1.486252		*T**:*	[M1A1]
0.19635	0.178222						*T****	[M1A1]
0.589049	0.441842							
0.981748	0.605119						*answer:*	[A1]
1.374447	0.683493							
1.767146	0.683493							
2.159845	0.605119							
2.552544	0.441842							
2.945243	0.178222	1.479855	1.48626	1.486275	1.486276	1.486276		

[subtotal 11]

(iv) Spreadsheet as above, but seen to work for user-specified c in place of 3.141593 **[M2]**

Sequence of values representing trial and error towards solution:

c	4	4.5	4.4	4.45	4.44	4.442
I	0.977343	-0.20713	0.133659	-0.02687	0.006681	0.00003

[M1A1]

Answer 4.442 to 3 decimal places **[A1]**

[subtotal 5]
[TOTAL 24]

3 (i) Modified Euler method

h	x	y	k1	k2	new y		
0.1	1	1	0.141421	0.150185	1.145803		
	1.1	1.145803	0.150346	0.159856	1.300904	*setup:*	**[M2]**
	1.2	1.300904	0.160034	0.170271	1.466056		
	1.3	1.466056	0.170466	0.181415	1.641997	*first run:*	**[A2]**
	1.4	1.641997	0.181626	0.193273	1.829446		
	1.5	1.829446	0.193499	0.205833	2.029112		
	1.6	2.029112	0.206072	0.219085	2.24169		
	1.7	2.24169	0.219337	0.23302	2.467869		
	1.8	2.467869	0.233284	0.247633	2.708328		
	1.9	2.708328	0.247908	0.262916	2.963739		
	2	2.963739					

h	α	diffs	ratio of diffs		
0.1	2.963739				
0.05	2.964219	0.000480		*further runs:*	**[A1A1A1]**
0.025	2.964341	0.000122	0.254789		
0.0125	2.964372	0.000031	0.252418	*differences:*	**[M1]**
0.00625	2.964380	0.000008	0.251215	*ratios:*	**[M1A1]**

Correct to 4 dp, α = 2.9644 **[A1]**
Ratio of differences indicates 2nd order convergence **[E1]**
[subtotal 12]

(ii) Predictor corrector method

h	x	y	y pred	y corr1	y corr2	y corr3		
0.1	1	1	1.141421	1.145803	1.145884	1.145885		
	1.1	1.145885	1.296234	1.300989	1.301078	1.30108	*setup:*	**[M2]**
	1.2	1.30108	1.46112	1.466239	1.466336	1.466338		
	1.3	1.466338	1.636815	1.64229	1.642395	1.642397	*first run:*	**[A2]**
	1.4	1.642397	1.824039	1.829862	1.829975	1.829978		
	1.5	1.829978	2.023497	2.029664	2.029784	2.029786		
	1.6	2.029786	2.235885	2.242392	2.242518	2.24252		
	1.7	2.24252	2.461889	2.468732	2.468864	2.468866		
	1.8	2.468866	2.702189	2.709364	2.709501	2.709504		
	1.9	2.709504	2.957457	2.964961	2.965104	2.965107		
	2	2.965107						

h	α	diffs	ratio of diffs			
0.1	2.965107					
0.05	2.964564	-0.000543			*further runs:*	**[A1A1A1]**
0.025	2.964428	-0.000136	0.250154			
0.0125	2.964394	-0.000034	0.250039	*these -->*	*differences*	
0.00625	2.964385	-0.000008	0.25001	*may appear in (iii)*	*and ratios:*	**[M1]**

[subtotal 8]

(iii) The rate of convergence (see ratio of differences) is the same for both methods. **[E1]**
Magnitude of errors about the same for a given h **[E1]**
More programming required for predictor-corrector **[E1]**
Modified Euler (at least in this case) is preferable **[E1]**
[subtotal 4]
[TOTAL 24]

4 (i)

7.1	6	5	4	1	x1 =	0.320827	*Gauss elim:*
6	5.1	4	3	1			**[M2A2]**
5	4	3.1	2	1			*pivoting:*
4	3	2	1.1	1			**[M1A2]**
	0.029577	-0.22535	-0.38028	0.15493			
	-0.22535	-0.42113	-0.8169	0.295775			*back subn:*
	-0.38028	-0.8169	-1.15352	0.43662	x2 =	0.103317	**[M1A2]**
		-0.28889	-0.47	0.188889	x3 =	-0.11419	
		0.062963	-0.13333	0.037037			*solutions:*
			-0.23577	0.078205	x4 =	-0.3317	**[A2]**

product of pivots: -0.18390 magnitude of determinant: 0.18390 **[M1A1]**

[subtotal 14]

(ii) α = 0.01 β = 0.01

7.01	6	5	4	1.01	x1 =	0.599796
6	5.01	4	3	1		
5	4	3.01	2	1		
4	3	2	1.01	1		
	-0.12552	-0.2796	-0.42368	0.135521		
	-0.2796	-0.55633	-0.85307	0.279601		
	-0.42368	-0.85307	-1.27245	0.42368	x2 =	-0.2999
		-0.02687	-0.0467	0.01	x3 =	-0.1996
		0.006633	-0.01333	0		
			-0.02486	0.002469	x4 =	-0.09929

product of pivots: -0.00198 magnitude of determinant: 0.001984 **[M1A1]**

α = 0.01	(A) β = 0	(B)β = 0.1	
x1	0.302	0.600	*solutions:*
x2	0.100	-0.300	**[M1A1]**
x3	-0.101	-0.200	**[M1A1]**
x4	-0.303	-0.099	

Very large changes in the solution for small change in one coefficient. **[E1E1]**

The determinant is very small in relation to the magnitude of the coefficients. **[E1E1]**

[subtotal 10]

[TOTAL 24]

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

4777 Numerical Computation

General comments
A usual, the entry for this paper was small. However, candidates were mostly well prepared and there was some excellent work seen.

Comments on individual questions

1) *(Interpolation; divided differences)*
This was, perhaps surprisingly, found a little more difficult than the other questions. The approach required is one in which a sequence of estimates is examined in order to assess accuracy. Individual values do not provide any indication of accuracy.

2) *(Integration; Romberg's method)*
This was handled well by most candidates. The method does, of course, involve formulae of higher order than the T^* and T^{**} explicitly mentioned. The approach of iterating T^{**} to convergence without using higher order formulae is not as efficient.

3) *(First order differential equation; modified Euler and predictor-corrector methods)*
Again, this question was done well by most candidates who tackled it. The comparison asked for at the end of the question should have led to candidates saying that, in this case at least, modified Euler appears to deliver about the same accuracy as predictor-corrector with rather less programming effort.

4) *(System of linear equations; Gaussian elimination)*
Most, but not all, candidates clearly understood the Gaussian elimination method. Equally importantly, they could implement it on a spreadsheet. In the final part, candidates were expected to notice that a small change in one of the coefficients produces a large change in the solution and that the determinant is very small in relation to the size of the coefficients. The latter is a good indicator of ill-conditioning.