

**ADVANCED SUBSIDIARY GCE
MATHEMATICS (MEI)**

4752/01

Concepts for Advanced Mathematics (C2)

WEDNESDAY 9 JANUARY 2008

Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

Section A (36 marks)

1 Differentiate $10x^4 + 12$. [2]

2 A sequence begins

1 2 3 4 5 1 2 3 4 5 1 ...

and continues in this pattern.

(i) Find the 48th term of this sequence. [1]

(ii) Find the sum of the first 48 terms of this sequence. [2]

3 You are given that $\tan \theta = \frac{1}{2}$ and the angle θ is acute. Show, without using a calculator, that $\cos^2 \theta = \frac{4}{5}$. [3]

4

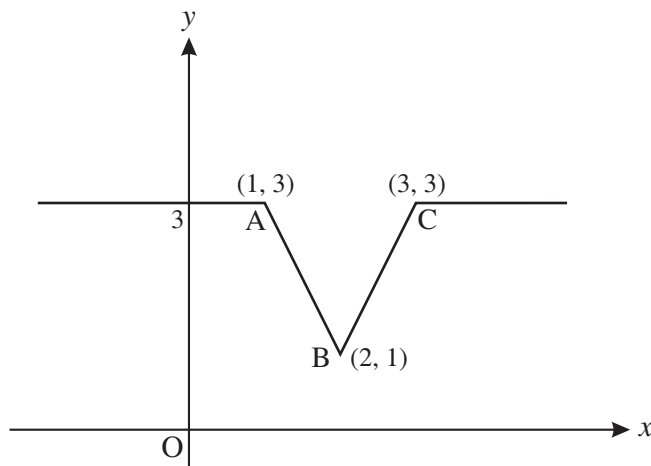


Fig. 4

Fig. 4 shows a sketch of the graph of $y = f(x)$. On separate diagrams, sketch the graphs of the following, showing clearly the coordinates of the points corresponding to A, B and C.

(i) $y = 2f(x)$ [2]

(ii) $y = f(x + 3)$ [2]

5 Find $\int (12x^5 + \sqrt[3]{x} + 7) dx$. [5]

6 (i) Sketch the graph of $y = \sin \theta$ for $0 \leq \theta \leq 2\pi$. [2]

(ii) Solve the equation $2 \sin \theta = -1$ for $0 \leq \theta \leq 2\pi$. Give your answers in the form $k\pi$. [3]

- 7 (i) Find $\sum_{k=2}^5 2^k$. [2]
- (ii) Find the value of n for which $2^n = \frac{1}{64}$. [1]
- (iii) Sketch the curve with equation $y = 2^x$. [2]
- 8 The second term of a geometric progression is 18 and the fourth term is 2. The common ratio is positive. Find the sum to infinity of this progression. [5]
- 9 You are given that $\log_{10} y = 3x + 2$.
- (i) Find the value of x when $y = 500$, giving your answer correct to 2 decimal places. [1]
- (ii) Find the value of y when $x = -1$. [1]
- (iii) Express $\log_{10}(y^4)$ in terms of x . [1]
- (iv) Find an expression for y in terms of x . [1]

Section B (36 marks)

10

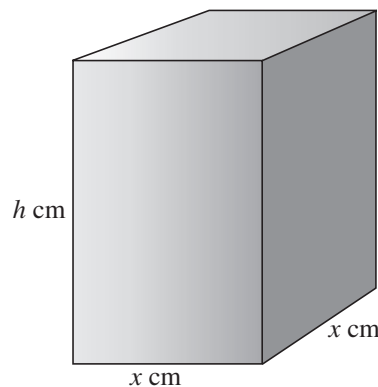


Fig. 10

Fig. 10 shows a solid cuboid with square base of side x cm and height h cm. Its volume is 120 cm^3 .

- (i) Find h in terms of x . Hence show that the surface area, $A \text{ cm}^2$, of the cuboid is given by $A = 2x^2 + \frac{480}{x}$. [3]
- (ii) Find $\frac{dA}{dx}$ and $\frac{d^2A}{dx^2}$. [4]
- (iii) Hence find the value of x which gives the minimum surface area. Find also the value of the surface area in this case. [5]

- 11 (i) The course for a yacht race is a triangle, as shown in Fig. 11.1. The yachts start at A, then travel to B, then to C and finally back to A.

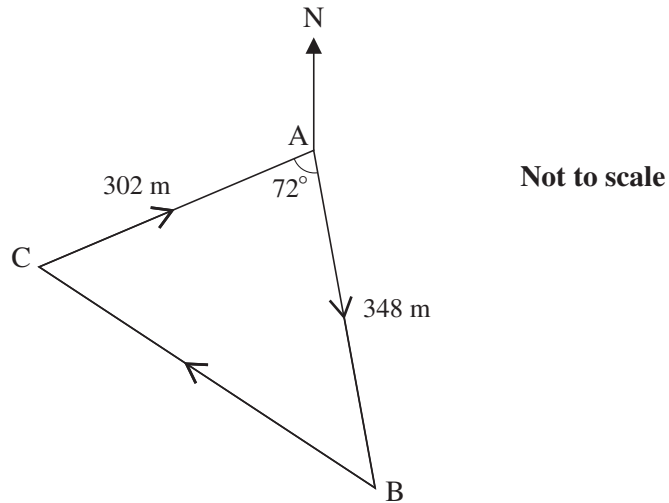


Fig. 11.1

- (A) Calculate the total length of the course for this race. [4]
- (B) Given that the bearing of the first stage, AB, is 175° , calculate the bearing of the second stage, BC. [4]
- (ii) Fig. 11.2 shows the course of another yacht race. The course follows the arc of a circle from P to Q, then a straight line back to P. The circle has radius 120 m and centre O; angle POQ = 136° .

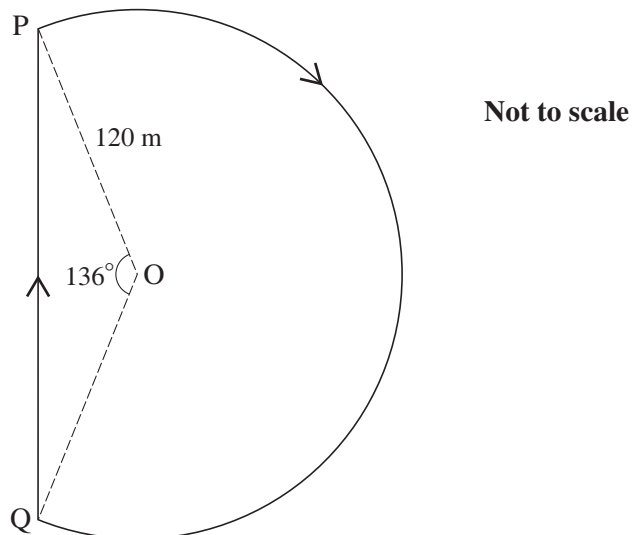


Fig. 11.2

Calculate the total length of the course for this race.

[4]

12 (i)

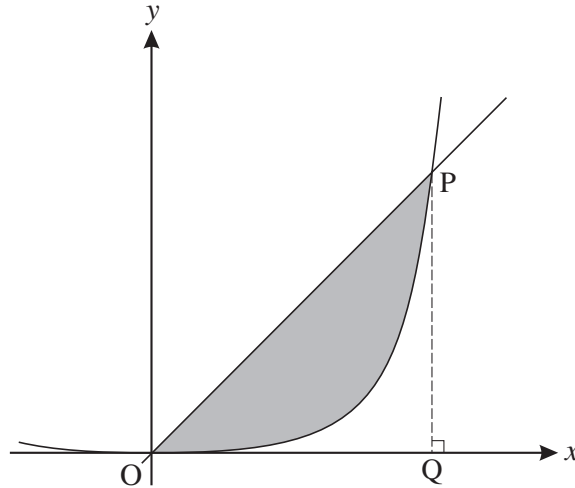


Fig. 12

Fig. 12 shows part of the curve $y = x^4$ and the line $y = 8x$, which intersect at the origin and the point P.

(A) Find the coordinates of P, and show that the area of triangle OPQ is 16 square units. [3]

(B) Find the area of the region bounded by the line and the curve. [3]

(ii) You are given that $f(x) = x^4$.

(A) Complete this identity for $f(x+h)$.

$$f(x+h) = (x+h)^4 = x^4 + 4x^3h + \dots \quad [2]$$

(B) Simplify $\frac{f(x+h) - f(x)}{h}$. [2]

(C) Find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. [1]

(D) State what this limit represents. [1]

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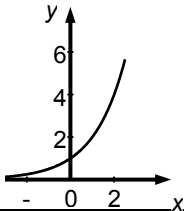
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4752 (C2) Concepts for Advanced Mathematics

Section A

1	$40x^3$	2	-1 if extra term	2
2	(i) 3 (ii) 141	1 2	M1 for $9 \times (1 + 2 + 3 + 4 + 5) + 1 + 2 + 3$	3
3	right angled triangle with 1 and 2 on correct sides Pythagoras used to obtain hyp = $\sqrt{5}$ $\cos \theta = \frac{a}{h} = \frac{2}{\sqrt{5}}$	M1 M1 A1	or M1 for $\sin \theta = \frac{1}{2} \cos \theta$ and M1 for substituting in $\sin^2 \theta + \cos^2 \theta = 1$ E1 for sufficient working	3
4	(i) line along $y = 6$ with V (1, 6), (2, 2), (3, 6) (ii) line along $y = 3$ with V (-2, 3), (-1, 1), (0, 3)	2 2	1 for two points correct 1 for two points correct	4
5	$2x^6 + \frac{3}{4}x^{\frac{4}{3}} + 7x + c$	5	1 for $2x^6$; 2 for $\frac{3}{4}x^{\frac{4}{3}}$ or 1 for other $kx^{\frac{4}{3}}$; 1 for $7x$; 1 for $+c$	5
6	(i) correct sine shape through O amplitude of 1 and period 2π shown (ii) $7\pi/6$ and $11\pi/6$	1 1 3	B2 for one of these; 1 for $-\pi/6$ found	5
7	(i) 60 (ii) -6 (iii) 	2 1 1 1	M1 for $2^2 + 2^3 + 2^4 + 2^5$ o.e. Correct in both quadrants Through (0, 1) shown dep.	5
8	$r = 1/3$ s.o.i. $a = 54$ or ft $18 \div$ their r $S = \frac{a}{1-r}$ used with $-1 < r < 1$ $S = 81$ c.a.o.	2 M1 M1 A1	1 mark for $ar = 18$ and $ar^3 = 2$ s.o.i.	5
9	(i) 0.23 c.a.o. (ii) 0.1 or $1/10$ (iii) $4(3x + 2)$ or $12x + 8$ (iv) $[y =] 10^{3x+2}$ o.e.	1 1 1 1	10^{-1} not sufficient	4

Section B

10	i	$h = 120/x^2$ $A = 2x^2 + 4xh$ o.e. completion to given answer	B1 M1 A1	at least one interim step shown	3
	ii	$A' = 4x - 480/x^2$ o.e. $A'' = 4 + 960/x^3$	2 2	1 for kx^2 o.e. included ft their A' only if kx^2 seen ; 1 if one error	4
	iii	use of $A' = 0$ $x = \sqrt[3]{120}$ or 4.9(3..) Test using A' or A'' to confirm minimum Substitution of their x in A $A = 145.9$ to 146	M1 A1 T1 M1 A1	Dependent on previous M1	5
11	iA	$BC^2 = 348^2 + 302^2 - 2 \times 348 \times 302 \times \cos 72^\circ$ $BC = 383.86\dots$ $1033.86\dots$ [m] or ft $650 +$ their BC	M2 A1 1	M1 for recognisable attempt at Cosine Rule to 3 sf or more accept to 3 sf or more	4
	iB	$\frac{\sin B}{302} = \frac{\sin 72}{\text{their } BC}$ $B = 48.4\dots$ $355 -$ their B o.e. answer in range 306 to 307	M1 A1 M1 A1	Cosine Rule acceptable or Sine Rule to find C or $247 +$ their C	4
	ii	Arc length PQ = $\frac{224}{360} \times 2\pi \times 120$ o.e. or 469.1... to 3 sf or more QP = 222.5... to 3 sf or more answer in range 690 to 692 [m]	M2 B1 A1	M1 for $\frac{136}{360} \times 2\pi \times 120$	4
12	iA	$x^4 = 8x$ (2, 16) c.a.o. PQ = 16 and completion to show $\frac{1}{2} \times 2 \times 16 = 16$	M1 A1 A1	NB answer 16 given	3
	iB	$x^5/5$ evaluating their integral at their co-ord of P and zero [or $32/5$ o.e.] 9.6 o.e.	M1 M1 A1	ft only if integral attempted, not for x^4 or differentiation c.a.o.	3
	iiA	$6x^2h^2 + 4xh^3 + h^4$	2	B1 for two terms correct.	2
	iiB	$4x^3 + 6x^2h + 4xh^2 + h^3$	2	B1 for three terms correct	2
	iiC	$4x^3$	1		1
	iiD	gradient of [tangent to] curve	1		1

4752: Concepts for Advanced Mathematics (C2)

General Comments

Most candidates found this paper accessible, but there was enough challenging material to stretch the best candidates. There was a full range of achievement, but there were fewer very poor scripts than usual, and fewer candidates scored full marks. Section A was generally better received than section B. Many candidates set their work out clearly. Nevertheless, many marks were lost by failing to show sufficient detail of the method or by failing to annotate diagrams adequately.

Comments on Individual Questions

Section A

1) An overwhelming majority of candidates scored full marks here. A minority lost a mark by including an extra term. A few made careless errors and gave the answer as $40x$ or $30x^3$.

2) Part (i) was very well done, with nearly everyone presenting the correct answer.

Part (ii) proved more challenging, although a significant minority scored both marks. Common errors were: " 9.6×15 ", " $9 \times 15 + 3$ " and " $8 \times 15 + 1 + 2 + 3$ ".

3) This question elicited a variety of approaches. Generally speaking only the better candidates scored well. The usual method was a right angled triangle with θ clearly marked and Pythagoras used to find the hypotenuse. Some candidates lost marks by failing to show enough detail in their working, although they may have fully understood what they were doing. There was some excellent work seen based on substituting $\sin\theta = \frac{1}{2}\cos\theta$ or $\sin\theta = \cos\theta\tan\theta$ in the identity $\sin^2\theta + \cos^2\theta = 1$. Some weaker candidates used their calculator to find $\tan^{-1}\frac{1}{2}$, and then find $\cos\theta$ and square the answer, in spite of the clear instruction in the question. This approach invariably scored zero.

4) Part (i) was often done well by stronger candidates, but many thought the transformation was an enlargement, or a translation of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

Part (ii) was very well done by most candidates, although some translated to the right by 3 units, or vertically upwards by 3 units, thus scoring no marks. In both parts, far too many candidates lost marks either by incorrect labelling of the points, or by ambiguous labelling.

Report on the Units taken in January 2008

- 5) Most candidates scored 4 or 5 marks on this question. A fully simplified answer was required, so answers such as $\frac{12}{6}x^6$ were penalised. A few candidates omitted “+ c” or gave the third term as $\frac{7^2}{2}$. However, the most common error was failing to deal with $\sqrt[3]{x}$. Some simply integrated \sqrt{x} – often correctly – but lost two marks. Others failed to simplify the fraction, and a few candidates gave the answer as

$$\frac{(\sqrt[3]{x})^2}{\frac{2}{3}}$$

- 6) Part (i) was accessible to most. Nearly all candidates were able to draw the correct sine shape over one period. However, many lost the second mark by omitting some indication of scale (in radians) or some indication of the amplitude.

There were some excellent answers to part (ii) However, many chose to work in degrees, in spite a clear request for radians, and a good number failed to give the answer in the required form. A surprising number of candidates supplied only one answer, and a few gave answers such as $2\pi - 30$, which scored zero.

- 7) Many good candidates scored full marks on this question. However, there were a number of common errors. In part (i) an extra term was often included – usually 64, but sometimes 2. Some candidates calculated 5×2^2 and others evaluated $2^2 + 3^2 + 4^2 + 5^2$. In part (ii) some candidates used logarithms, and ended up working with rounded decimals, giving their answer as -5.99, which scored zero. In part (iii) the most common error was the omission of the y-intercept, but a minority of candidates only sketched the curve in one quadrant. Some candidates wasted valuable time on an accurate plot on graph paper.

- 8) Most were able to score well on this question, with many candidates obtaining full marks. The decreasing geometrical progression caused some problems, and those who thought $r=3$ could make no progress with the sum to infinity. A small number of candidates seemed unfamiliar with the appropriate formula, and used their calculators to sum a large number of terms, usually giving an answer close to 81.

- 9) Most candidates obtained the correct answers to parts (i) and (ii). However, parts (iii) and (iv) defeated many. Common errors were $(3x + 2)^4$ for part (iii) and e^{3x+2} in part (iv)

Section B

- 10) (i) There were many very good answers to this question, but a significant number of candidates could not progress beyond stating an expression for h, and some even failed to do this. A number of candidates showed insufficient working to obtain all three marks.
- (ii) Many candidates scored full marks on this section. There were, however, some common errors. Most notable of these was $\frac{dy}{dx} = 4x + 480$, but some weaker candidates gave the answer as $\frac{dy}{dx} = 4x + x^{-480}$ and a few made slips with the signs in either the first or the second derivatives.

- (iii) Surprisingly few candidates took the hint and attempted to solve $\frac{dy}{dx} = 0$. Of those who did, solving the equation caused problems, with many obtaining an extra root $x = 0$. Very few candidates used the second derivative appropriately, and substituting the x -value in the formula for A often led to arithmetical slips. A common error was to attempt to solve $\frac{d^2y}{dx^2} = 0$ or $\frac{d^2y}{dx^2} > 0$.
- 11) (i) Most candidates scored full marks for this part. Candidates were clearly familiar with the Cosine Rule, but a disappointingly large number evaluated $(b^2 + c^2 - 2bc)\cos A$. Even some good candidates omitted to give the total length of the course.
- (ii) Most knew to use the Sine Rule to find angle B – or occasionally angle C – but a surprising number seemed unfamiliar with bearings, and this was sometimes left unanswered. Many simply calculated $360 - "B"$.
- (iii) Most candidates successfully found the length of the chord PQ , but far too many candidates simply calculated $r\theta$, with θ as 224 (or 136) and did not appreciate that this answer was simply ridiculous. Surprisingly few of the successful candidates adopted the expected approach – more often they converted the angle to radians and then used $s = r\theta$.
- 12) (i) Part (A) was generally done very well indeed, although some candidates gave an extra root of -2 .
- Part (B) was also very well done, although some candidates substituted an upper limit of 16 instead of 2 .
- (ii) (A) Those who used the Binomial expansion generally obtained both marks in this part. Those who expanded the brackets longhand often made slips.
- (B) Many went on to obtain both marks with this part, too. However, many candidates simply ignored their previous answer and manipulated $\frac{f(x+h) - f(x)}{h}$, often submitting the answer f . It seemed clear that many candidates were unfamiliar with differentiation from first principles.
- (C) Only a few candidates connected this part with its immediate predecessors. Consequently it was often not answered. However, some candidates gave the correct answer, but had been unable to cope with the algebra in parts A and B.
- (D) This elicited a wide variety of incorrect answers, some of which were clearly flippant. More often it was left blank, a correct response occurring only rarely.