OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4752

Concepts for Advanced Mathematics (C2)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Insert for Question 10 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 22 May 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- There is an **insert** for use in Question **10**.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Use an isosceles right-angled triangle to show that $\cos 45° = \dfrac{1}{\sqrt{2}}$. **[2]**

2 Find $\displaystyle\int_1^2 (12x^5 + 5)\,dx$. **[4]**

3 **(i)** Find $\displaystyle\sum_{k=3}^{8}(k^2 - 1)$. **[2]**

(ii) State whether the sequence with kth term $k^2 - 1$ is convergent or divergent, giving a reason for your answer. **[1]**

4 A sector of a circle of radius 18.0 cm has arc length 43.2 cm.

(i) Find in radians the angle of the sector. **[2]**

(ii) Find this angle in degrees, giving your answer to the nearest degree. **[2]**

5 **(i)** On the same axes, sketch the graphs of $y = \cos x$ and $y = \cos 2x$ for values of x from 0 to 2π. **[3]**

(ii) Describe the transformation which maps the graph of $y = \cos x$ onto the graph of $y = 3\cos x$. **[2]**

6 Use calculus to find the x-coordinates of the turning points of the curve $y = x^3 - 6x^2 - 15x$.

Hence find the set of values of x for which $x^3 - 6x^2 - 15x$ is an increasing function. **[5]**

7 Show that the equation $4\cos^2\theta = 4 - \sin\theta$ may be written in the form

$$4\sin^2\theta - \sin\theta = 0.$$

Hence solve the equation $4\cos^2\theta = 4 - \sin\theta$ for $0° \leqslant \theta \leqslant 180°$. **[5]**

8 The gradient of a curve is $3\sqrt{x} - 5$. The curve passes through the point (4, 6). Find the equation of the curve. **[5]**

9 Simplify

(i) $10 - 3\log_a a$, **[1]**

(ii) $\dfrac{\log_{10} a^5 + \log_{10}\sqrt{a}}{\log_{10} a}$. **[2]**

Section B (36 marks)

10 Answer part (i) of this question on the insert provided.

Ash trees grow quickly for the first years of their life, then more slowly. This table shows the height of a tree at various ages.

Age (t years)	4	7	10	15	20	40
Height (h m)	4	9	12	17	19	26

The height, h m, of an ash tree when it is t years old may be modelled by an equation of the form

$$h = a\log_{10} t + b.$$

(i) **On the insert**, complete the table and plot h against $\log_{10} t$, drawing by eye a line of best fit. **[3]**

(ii) Use your graph to find an equation for h in terms of $\log_{10} t$ for this model. **[3]**

(iii) Find the height of the tree at age 100 years, as predicted by this model. **[1]**

(iv) Find the age of the tree when it reaches a height of 29 m, according to this model. **[3]**

(v) Comment on the suitability of the model when the tree is very young. **[2]**

11 (i) In a 'Make Ten' quiz game, contestants get £10 for answering the first question correctly, then a further £20 for the second question, then a further £30 for the third, and so on, until they get a question wrong and are out of the game.

(A) Haroon answers six questions correctly. Show that he receives a total of £210. **[1]**

(B) State, in a simple form, a formula for the total amount received by a contestant who answers n questions correctly.

Hence find the value of n for a contestant who receives £10 350 from this game. **[4]**

(ii) In a 'Double Your Money' quiz game, contestants get £5 for answering the first question correctly, then a further £10 for the second question, then a further £20 for the third, and so on doubling the amount for each question until they get a question wrong and are out of the game.

(A) Gary received £75 from the game. How many questions did he get right? **[1]**

(B) Bethan answered 9 questions correctly. How much did she receive from the game? **[2]**

(C) State a formula for the total amount received by a contestant who answers n questions correctly.

Hence find the value of n for a contestant in this game who receives £2 621 435. **[4]**

[Question 12 is printed overleaf.]

12 **(i)** Calculate the gradient of the chord joining the points on the curve $y = x^2 - 7$ for which $x = 3$ and $x = 3.1$. **[2]**

(ii) Given that $f(x) = x^2 - 7$, find and simplify $\dfrac{f(3+h) - f(3)}{h}$. **[3]**

(iii) Use your result in part **(ii)** to find the gradient of $y = x^2 - 7$ at the point where $x = 3$, showing your reasoning. **[2]**

(iv) Find the equation of the tangent to the curve $y = x^2 - 7$ at the point where $x = 3$. **[2]**

(v) This tangent crosses the x-axis at the point P. The curve crosses the positive x-axis at the point Q. Find the distance PQ, giving your answer correct to 3 decimal places. **[3]**

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4752

Concepts for Advanced Mathematics (C2)

INSERT for Question 10

Friday 22 May 2009
Morning

Duration: 1 hour 30 minutes

Candidate Forename		Candidate Surname	
Centre Number		Candidate Number	

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- This insert should be used to answer Question **10** part **(i)**.
- Write your answers to Question **10** part **(i)** in the spaces provided in this insert, and **attach it to your Answer Booklet**.

INFORMATION FOR CANDIDATES

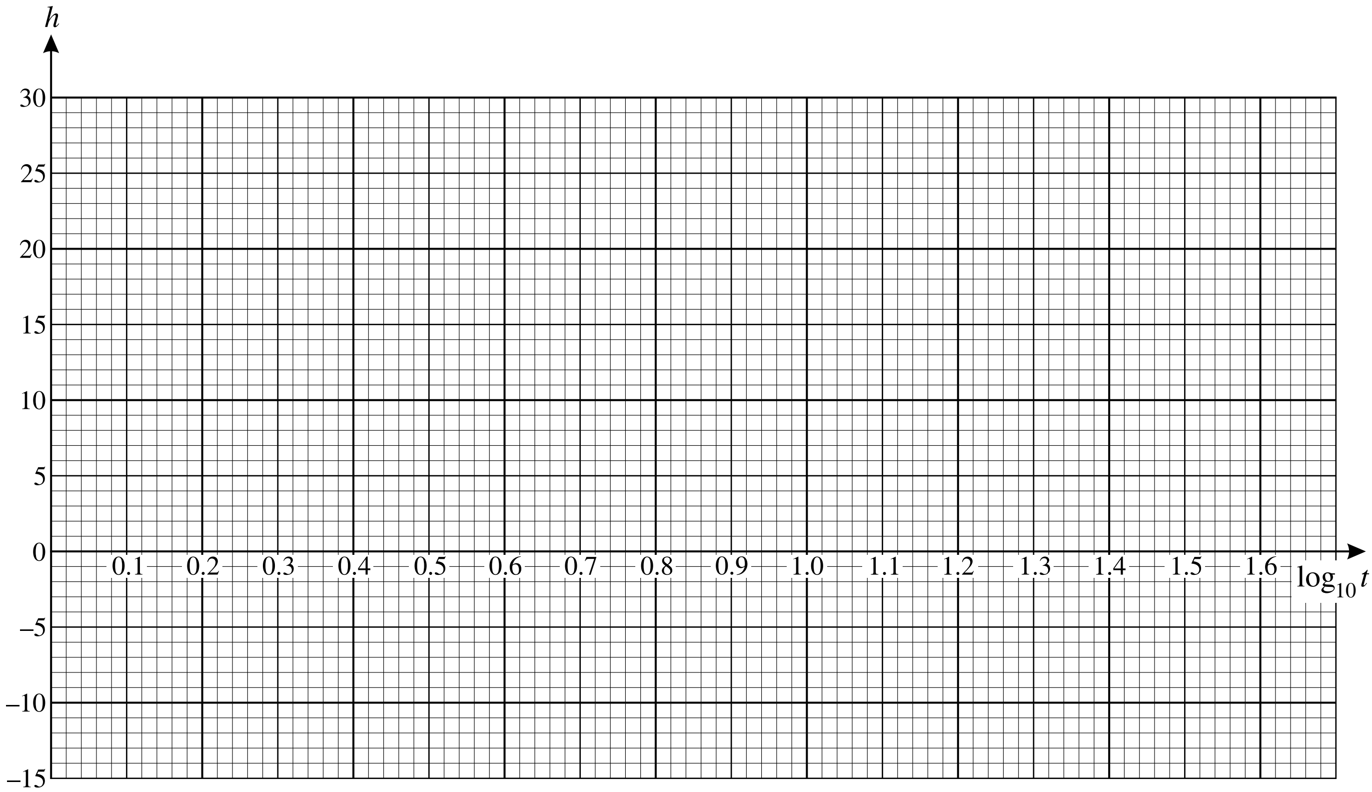
- This document consists of **2** pages. Any blank pages are indicated.

2R–8K22

Turn over

10 (i)

Age (t years)	4	7	10	15	20	40
$\log_{10} t$			1			
Height (h m)	4	9	12	17	19	26



4752 (C2) Concepts for Advanced Mathematics

Section A

1	using Pythagoras to show that hyp. of right angled isos. triangle with sides *a* and *a* is $\sqrt{2}a$ completion using definition of cosine	M1 A1	www *a* any letter or a number NB answer given	2
2	$2x^6 + 5x$ value at 2 − value at 1 131	M2 M1 A1	M1 if one error ft attempt at integration only	4
3	(i) 193 (ii) divergent + difference between terms increasing o.e.	2 1	M1 for 8 + 15 + …+ 63	3
4	(i) 2.4 (ii) 138	2 2	M1 for 43.2 ÷ 18 M1 for their (i) × $\frac{180}{\pi}$ or θ = $\frac{43.2 \times 360}{36\pi}$ o.e. or for other rot versions of 137.50…	4
5	(i)sketch of cos*x* ; one cycle, sketch of cos2*x*; two cycles, Both axes scaled correctly (ii) (1-way) stretch parallel to *y* axis sf 3	1 1 D1 1 D1		5
6	$y' = 3x^2 - 12x - 15$ use of $y' = 0$, s.o.i. ft *x* = 5, – 1 c.a.o. $x < -1$ or $x > 5$ f.t.	M1 M1 A1 A1 A1	for two terms correct	5
7	use of $\cos^2\theta = 1 - \sin^2\theta$ at least one correct interim step in obtaining $4\sin^2\theta - \sin\theta = 0$. θ = 0 and 180, 14.(47…) 165 - 166	M1 M1 B1 B1 B1	 NB answer given r.o.t to nearest degree or better -1 for extras in range	5

8	attempt to integrate $3\sqrt{x} - 5$ [*y*=] $2x^{\frac{3}{2}} - 5x + c$ subst of (4, 6) in their integrated eqn *c* = 10 or [*y*=] $2x^{\frac{3}{2}} - 5x + 10$	M1 A2 M1 A1	A1 for two terms correct	5
9	(i) 7 (ii) 5.5 o.e.	1 2	M1 for at least one of 5 $\log_{10}a$ or ½ $\log_{10}a$ or $\log_{10}a^{5.5}$ o.e.	3

Section B

10	**i**	0.6(0..), 0.8(45..), [1], 1.1(76..) 1.3(0..), 1.6(0..) points plotted correctly f.t. ruled line of best fit	T 1 P1 L1	Correct to 2 d.p. Allow 0.6, 1.3 and 1.6 tol. 1 mm	3
	ii	*b* = their intercept *a* = their gradient -11 ≤ b ≤ -8 *and* 21 ≤ a ≤ 23.5	M1 M1 A1		3
	iii	34 to 35 m	1		1
	iv	29 = "22"logt – "9" t = $10^{\text{"1.727.."}}$ 55 [years] approx	M1 M1 A1	accept 53 to 59	3
	v	For small t the model predicts a negative height (or h = 0 at approx 2.75) Hence model is unsuitable	1 D1		2

11	**iA**	10+20+30+40+50+60	B1	or $\frac{6}{2}(2\times10+5\times10)$ or $\frac{6}{2}(10+60)$	1
	iB	correct use of AP formula with a = 10 and d = 10 $n(5 + 5n)$ or $5n(n + 1)$ or $5(n^2 + n)$ or $(5n^2 + 5n)$ $10n^2 + 10n - 20700 = 0$ 45 c.a.o.	M1 A1 M1 A1	 Or better	 4
	iiA	4	1		1
	iiB	£2555	2	M1 for $5(1 + 2 + \ldots 2^8)$ or $5(2^9 - 1)$ o.e.	2
	iiC	correct use of GP formula with a =5, r = 2 $5(2^n - 1)$ o.e.= 2621435 2^n = 524288 www 19 c.a.o.	M1 DM1 M1 A1	 "S" need not be simplified	 4
12	**i**	6.1	2	M1 for $\frac{(3.1^2-7)-(3^2-7)}{3.1-3}$ o.e.	2
	ii	$\frac{((3+h)^2-7)-(3^2-7)}{h}$ numerator = $6h + h^2$ $6 + h$	M1 M1 A1	s.o.i.	 3
	iii	as h tends to 0, grad. tends to 6 o.e. f.t.from "6"+h	M1 A1		 2
	iv	$y - 2$ = "6" $(x - 3)$ o.e. $y = 6x - 16$	M1 A1	6 may be obtained from $\frac{dy}{dx}$	 2
	v	At P, x = 16/6 o.e. or ft At Q, $x = \sqrt{7}$ 0.021 cao	M1 M1 A1		 3

4752 Concepts for Advanced Mathematics (C2)

General Comments

The paper was generally well received, with most candidates able to make some headway with questions in both sections. However, very few obtained full marks or even close to full marks. A significant minority of candidates lost easy marks because they were unable to use terminology and definitions expected of Higher Level GCSE candidates correctly. Some candidates presented wildly inaccurate answers, seemingly without the sense that something must be wrong, and failure to show adequate working cost easy marks for others. It seemed that many candidates understood the concepts in this course, but failed to do themselves full justice in the examination because of poor algebra (e.g. factorising) and poor arithmetic (e.g. inability to deal with fractional indices).

Comments on Individual Questions

Section A

1) There were some good clear answers to this question, but many candidates failed to score both marks. Elementary mistakes with the right angled triangle, such as $1^2 + 1^2 = 2^2$, were common. For the second mark, $\cos\theta = \frac{\text{opp}}{\text{hyp}}$ was seen quite often.

2) This question was generally well done, with many candidates scoring full marks. Occasional errors were $6x^6$ for the first term, and $\frac{5x^2}{2}$ for the second term. A few made slips with the arithmetic, or evaluated 138 + 7 as a final step.

3) Part (i) was tackled successfully by most. A few candidates added two extra terms or began the sum at 1. Some evaluated $3^2 - 1$ six times and found the sum. Other mistakes were to simply compute each term, or slip up with the arithmetic – a final term of 65 in the sum was quite frequent.

   Part (ii) was seldom answered correctly. Approximately half the candidates gave "convergent" as the answer. Most of those who correctly stated "divergent" were unable to give the correct reason – comments such as "$r > 1$", "the terms are increasing exponentially" and "the terms are getting further from 0" were quite common.

4) Part (i) was well done. Some candidates spoiled their answer by adding a gratuitous π, and a few wrote "$2.4° = \frac{\pi}{75}$ radians".

   Most candidates successfully multiplied 2.4 by $\frac{180}{\pi}$, but a good number then failed to round their answer to the nearest degree, thus losing an easy mark.

5) Part (i) was generally well done. Most successfully drew the correct cosine waves, but a good number lost the second mark because they only sketched the curve for half the range. Some candidates sketched $y = 2\cos x$ or $y = \cos x + 2$. A few candidates sketched $y = \sin x$ and $y = \sin 2x$. Clear labelling on both axes was required. Many candidates unnecessarily used graph paper and attempted an accurate plot.

Many candidates failed to score in part (ii) because they did not know the correct terminology. Even those who did identify "stretch" as the appropriate word often lacked precision in their description, referring to the scale factor as an "increase" or even a vector.

6) The first three marks were obtained by the vast majority of candidates, but the final two were not usually earned. Most candidates examined the second derivative, identified the corresponding y-values and then ran out of steam. Some solved $\frac{d^2y}{dx^2} = 0$ and then used their value for an inequality. Those who did obtain the correct inequality often wrote it down clumsily: $-1 > x > 5$ was common.

7) Most candidates knew that the substitution $\cos^2\theta = 1 - \sin^2\theta$ was expected, and were able to show at least one correct step in obtaining the required result. A few incurred a penalty by failing to make clear what they were doing. Some weak candidates simply manipulated the original expression, "went round the houses" and ended up back at the start point. Most candidates went on to obtain 14.47° and 165.53°, but omitted either 0° or 180° - or in some instances both.

8) There were many excellent responses to this question, with full marks awarded on many occasions. Some slipped up by writing $3\sqrt{x}$ as $x^{1/3}$ before integrating, and some evaluated $2\times4^{1.5}$ as $(2\times4^{1/2})^3$ when finding "c". However, a significant minority of candidates scored no marks at all because they went straight to "$y = mx + c$" and inserted $m = 3\sqrt{x} - 5$.

9) Part (i) was very well done, although some candidates wrote $3\log_a a = \log_a a^3 = 1^3$ and $10 - 1 = 9$.

Part (ii) was sometimes done well by weaker candidates, but there were often mistakes on better scripts. Most scored M1 for a correct use of one of the log laws, but a surprisingly high number obtained the answer $4\frac{1}{2} \log a$ in various different ways.

Section B

10) (i) This was done very well indeed, with many candidates scoring full marks. Two decimal place accuracy was expected for $\log_{10}7$ and $\log_{10}15$. A few slipped up with plotting the points (usually (1,12) was plotted as (1,10)) and some drew a free hand line or failed to cover the domain given in the table.

(ii) There were many excellent answers; most knew how the gradient and the intercept related to *a* and *b*. Some lost accuracy by taking values from the table rather than the line of best fit to solve a pair of simultaneous equations. In catastrophic cases, such as "$b = +12.5$", it was seldom appreciated that the value had to be wrong. The most fruitful approach was to use the gradient and vertical intercept of the line of best fit for *a* and *b*. However, a few candidates used the intercept on the horizontal axis for *b*.

(iii) This was generally well done, although many failed to score because their values for *a* and *b* were outside tolerance. A few candidates attempted to find *t* at h =100.

(iv) There were many good answers to this part of the question, although a few slipped up in manipulating the equation. Some candidates obtained "$\log_{10}t$ = 1.727.." and then ran out of steam.

Those who attempted extrapolation of the graph were generally so far away from the allowed range of values that they failed to score.

(v) A good number of candidates scored both marks here, but many candidates failed to comment on the model. Instead a paragraph on the growth of trees was presented. However plausible this may have been, it earned no marks.

11) (i) Part (A) was very well done.

In part (B), most candidates were able to use $S_n = \frac{n}{2}(2\times10 + (n - 1)\times10)$, but few obtained a satisfactory simplified version. The correct quadratic was generally obtained, but many resorted to trial and improvement to find the solution. Some made slips with algebra, obtaining $n^2 + 2n - 2070 = 0$, or $n^2 - n - 2070 = 0$. A few candidates went straight to an arithmetic approach, which scored a maximum of two marks. A minority of candidates started out with $u_n = 10 + (n - 1)\times10$

(ii) Parts (A) and (B) were very well done, although a small number of candidates used the formula for the *n*th term instead of using the formula for the sum of the first *n* terms or summing directly.

In part (C) there were many excellent solutions. Some spoiled their work by incorrect simplification of the formula, or by incorrect manipulation of the equation. A few candidates failed to score because they used the A.P. formula, or because they went straight to trial and error. Failure to "state the formula" was heavily penalised.

12) (i) The correct answer of 6.1 was often obtained. Occasionally this was arrived at by evaluating $\frac{dy}{dx}$ at x =3 and 3.1 and finding the mean. Some slipped up by rounding $3.1^2 - 7$ to 2.6 when finding "m" using the usual formula.

(ii) A minority of candidates adopted the correct approach, but often slipped up on the algebra. Of those who correctly obtained "6 + h", a significant minority went on to write "so h = - 6", showing that they did not really know what they were doing. Many candidates did not understand the notation at all – for example, f(3 + h) = 3f + 3h was surprisingly common, as was $(x^2 - 7)(3 + h)$ etc.

(iii) Hardly anyone realised that consideration of $h \rightarrow 0$ was expected. Most candidates re-started with differentiation and didn't score.

(iv) Many candidates scored full marks, having obtained m = 6 from differentiation. A significant minority scored zero, however, because they used $m = -\frac{1}{6}$.

(v) Those candidates who attempted this question generally did very well. A few found the y-intercepts instead, and a surprising number failed to leave the answer to the specified degree of accuracy, thus losing an easy mark.