OCR
RECOGNISING ACHIEVEMENT

Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754 Applications of Advanced Mathematics (C4)

INSTRUCTIONS

4715680612

The examination is in two parts:

Paper A (1 hour 30 minutes)
Paper B (up to 1 hour)

Supervisors are requested to ensure that Paper B **is not issued** until Paper A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Paper B should not be attached to the corresponding paper A script. For Paper A only the candidates' printed answer books, in the same order as the attendance register, should be sent for marking; the question paper should be retained in the centre or recycled. For Paper B only the question papers, on which the candidates have written their answers, should be sent for marking; the insert should be retained in the centre or recycled. Any additional sheets used must be carefully attached to the correct paper.

For Paper B (Comprehension) only.

A standard English dictionary is allowed for the comprehension.
(Dictionaries to be used in the examination must be thoroughly checked before the examination.) Full regulations are in the JCQ Regulations and Guidance booklet.

This notice must be on the Invigilator's desk at all times during the morning of Thursday 14 June 2012.

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RECOGNISING ACHIEVEMENT

Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4715690612

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

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Section A (36 marks)

1 Solve the equation $\dfrac{4x}{x+1} - \dfrac{3}{2x+1} = 1$. **[5]**

2 Find the first four terms in the binomial expansion of $\sqrt{1+2x}$. State the set of values of x for which the expansion is valid. **[5]**

3 The total value of the sales made by a new company in the first t years of its existence is denoted by £V. A model is proposed in which the rate of increase of V is proportional to the square root of V. The constant of proportionality is k.

(i) Express the model as a differential equation.

Verify by differentiation that $V = (\frac{1}{2}kt + c)^2$, where c is an arbitrary constant, satisfies this differential equation. **[4]**

(ii) The value of the company's sales in its first year is £10 000, and the total value of the sales in the first two years is £40 000. Find V in terms of t. **[4]**

4 Prove that $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \operatorname{cosec}^2\theta$. **[4]**

5 Given the equation $\sin(x + 45°) = 2\cos x$, show that $\sin x + \cos x = 2\sqrt{2}\cos x$.

Hence solve, correct to 2 decimal places, the equation for $0° \leqslant x \leqslant 360°$. **[6]**

6 Solve the differential equation $\dfrac{dy}{dx} = \dfrac{y}{x(x+1)}$, given that when $x=1$, $y=1$. Your answer should express y explicitly in terms of x. **[8]**

Section B (36 marks)

7 Fig. 7a shows the curve with the parametric equations

$$x = 2\cos\theta, \quad y = \sin 2\theta, \quad -\tfrac{\pi}{2} \leqslant \theta \leqslant \tfrac{\pi}{2}.$$

The curve meets the x-axis at O and P. Q and R are turning points on the curve. The scales on the axes are the same.

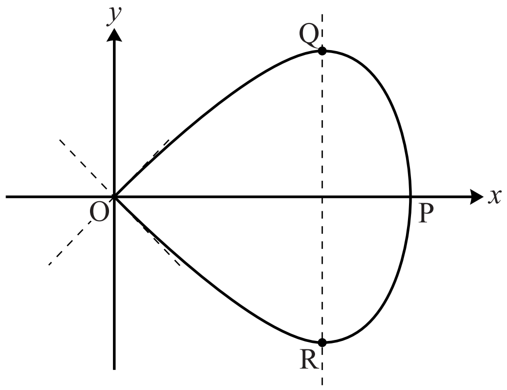

Fig. 7a

(i) State, with their coordinates, the points on the curve for which $\theta = -\frac{\pi}{2}$, $\theta = 0$ and $\theta = \frac{\pi}{2}$. **[3]**

(ii) Find $\frac{dy}{dx}$ in terms of θ. Hence find the gradient of the curve when $\theta = \frac{\pi}{2}$, and verify that the two tangents to the curve at the origin meet at right angles. **[5]**

(iii) Find the exact coordinates of the turning point Q. **[3]**

When the curve is rotated about the x-axis, it forms a paperweight shape, as shown in Fig. 7b.

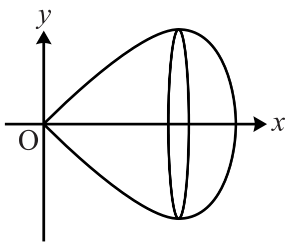

Fig. 7b

(iv) Express $\sin^2\theta$ in terms of x. Hence show that the cartesian equation of the curve is $y^2 = x^2(1 - \frac{1}{4}x^2)$. **[4]**

(v) Find the volume of the paperweight shape. **[4]**

8 With respect to cartesian coordinates Oxyz, a laser beam ABC is fired from the point A(1, 2, 4), and is reflected at point B off the plane with equation $x + 2y - 3z = 0$, as shown in Fig. 8. A′ is the point (2, 4, 1), and M is the midpoint of AA′.

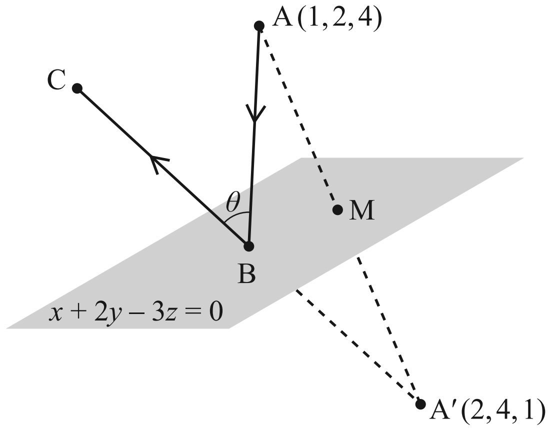

Fig. 8

(i) Show that AA′ is perpendicular to the plane $x + 2y - 3z = 0$, and that M lies in the plane. **[4]**

The vector equation of the line AB is $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

(ii) Find the coordinates of B, and a vector equation of the line A′B. **[6]**

(iii) Given that A′BC is a straight line, find the angle θ. **[4]**

(iv) Find the coordinates of the point where BC crosses the Oxz plane (the plane containing the x- and z-axes). **[3]**

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Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754A Applications of Advanced Mathematics (C4) Paper A

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:
- Question Paper 4754A (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (36 marks)

1	

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2

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3 (i)	

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3 (ii)

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4	

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5

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6

(answer space continued on next page)

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6	(continued)

Section B (36 marks)

7 (i)	

7 (ii)

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7 (iii)

7 (iv)

(answer space continued overleaf)

7 (iv)	**(continued)**
7 (v)	

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8 (i)	

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8 (ii)

(answer space continued on next page)

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8 (ii)	**(continued)**
8 (iii)	

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8 (iv)

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RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4754A:** Applications of Advanced Mathematics: Paper A

Mark Scheme for June 2012

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he / she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			$\frac{4x}{x+1} - \frac{3}{2x+1} = 1$		
			$\Rightarrow \quad 4x(2x+1) - 3(x+1) = (x+1)(2x+1)$	M1	Multiplying throughout by $(2x + 1)(x + 1)$ or combining fractions and multiplying up oe (eg can retain denominator throughout) Condone a single numerical error, sign error or slip provided that there is no conceptual error in the process involved Do not condone omission of brackets unless it is clear from subsequent work that they were assumed eg $4x(2x + 1) - 3(x + 1) = (x + 1)(2x \underline{- 1})$ gets M1 $4x(2x + 1) - 3(x + 1) = 1$ gets M0 $4x(x + 1)(2x + 1) - 3(x + 1)(2x + 1) = (x + 1)(2x + 1)$ gets M0 $4x(2x + 1) - 3(x + 1) = (x + 1)$ gets M1, just, for slip in omission of $(2x + 1)$
			$\Rightarrow \quad 8x^2 + 4x - 3x - 3 = 2x^2 + 3x + 1$ $\Rightarrow \quad 6x^2 - 2x - 4 = 0$	DM1	Multiplying out, collecting like terms and forming quadratic = 0. Follow through from their equation provided the algebra is not significantly eased and it is a quadratic. Condone a further sign or numerical error or minor slip when rearranging.
			$\Rightarrow \quad 3x^2 - x - 2 = 0$	A1	or $6x^2 - 2x - 4 = 0$ oe www, (not fortuitously obtained - check for double errors)
			$\Rightarrow \quad (3x + 2)(x - 1) = 0$	M1	Solving their three term quadratic provided $b^2 - 4ac \geq 0$. Use of <u>correct</u> quadratic equation formula (can be an error when substituting into correct formula) or factorising (giving their correct x^2 and constant terms when factors multiplied out) or comp the square oe. soi
			$\Rightarrow \quad x = -2/3$ or 1	A1	cao for both obtained www (accept $-4/6$ oe, or exact decimal equivalent (condone -0.667or better))
				[5]	SC B1 $x = 1$ with or without any working

Question			Answer	Marks	Guidance
2			$(1+2x)^{1/2} = 1 + \frac{1}{2}(2x) + \frac{\frac{1}{2}.(-\frac{1}{2})}{2!}(2x)^2 + \frac{\frac{1}{2}.(-\frac{1}{2})(-\frac{3}{2})}{3!}(2x)^3 + \ldots$	M1	Do not MR for $n \neq 1/2$ All four correct binomial coeffs (not nCr form) soi Accept **unsimplified** coefficients if a subsequent error when simplifying.
				B1	Condone absence of brackets only if followed by correct work eg $2x^2 = 4x^2$ must be soi for second B mark. $1 + x$ www
			$= 1 + x - ½x^2 + ½x^3 + \ldots$	B1 B1	… $-½x^2$ www … $+½x^3$ www If there is an error in say the third coeff of the expansion, M0, B1, B0, B1 can be scored
			Valid for $\|x\| < 1/2$ or $-1/2 < x < 1/2$	B1	Independent of expansion $\|x\| \leq 1/2$ and $-1/2 \leq x \leq 1/2$ are actually correct in this case so we will accept them. Condone a combination of inequalities. Condone also, say $-1/2 < \|x\| < 1/2$ but not $x < 1/2$ or $-1 < 2x < 1$ or $-1/2 > x > 1/2$
				[5]	

Question			Answer	Marks	Guidance
3	(i)		$\mathrm{d}V/\mathrm{d}t = k\sqrt{V}$	B1	cao condone different k (allow MR B1 for $= kV^2$)
			$V = (\frac{1}{2}kt + c)^2$ $\Rightarrow \mathrm{d}V/\mathrm{d}t = 2(\frac{1}{2}kt + c).\frac{1}{2}k$	M1	$2(1/2\ kt + c) \times$ constant multiple of k (or from multiplying out oe; or implicit differentiation)
			$= k(\frac{1}{2}kt + c)$	A1	cao www any equivalent form (including unsimplified)
			$= k\sqrt{V}$	A1	Allow SCB2 if $V=(1/2\ kt + c)^2$ fully obtained by integration including convincing change of constant if used Can score B1 M0 SCB2
				[4]	
	(ii)		$(\frac{1}{2}k + c)^2 = 10\ 000 \Rightarrow \frac{1}{2}k + c = 100$	B1	substituting any one from $t = 1$, $V = 10{,}000$ or $t = 0$, $V = 0$ or $t = 2$, $V = 40{,}000$ into squared form or rooted form of equation (Allow $-/\pm 100$ or $-/\pm 200$)
			$(k + c)^2 = 40\ 000 \Rightarrow k + c = 200$	B1	substituting any other from above
			$\Rightarrow \frac{1}{2}k = 100$ $\Rightarrow k = 200,\ c = 0$	M1	Solving correct equations for both www (possible solutions are (200,0), (−200,0), (600, −400), (−600,400) (some from –ve root))
			$\Rightarrow V = (100t)^2 = 10000t^2$	A1	either form www SC B2 for $V = (100t)^2$ oe stated without justification SCB4 if justification eg showing substitution SC those working with $(k + c)^2 = 30{,}000$ can score a maximum of B1B0 M1A0 (leads to $k \approx 146$, $c \approx 26.8$)
				[4]	

Question			Answer	Marks	Guidance
4			LHS = $\sec^2\theta + \csc^2\theta$		
			$= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$	M1	**Use** of $\sec\theta = 1/\cos\theta$ and $\csc\theta = 1/\sin\theta$ not just stating
			$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}$	M1	adding
			$= \dfrac{1}{\cos^2\theta\sin^2\theta}$	A1	use of $\cos^2\theta + \sin^2\theta = 1$ soi
			$= \sec^2\theta\csc^2\theta$	A1	**AG**
			…………………………………………		
			OR		
			$\sec^2\theta+\csc^2\theta=\tan^2\theta+1+\cot^2\theta+1=\sin^2\theta/\cos^2\theta+\cos^2\theta/\sin^2\theta+2$	M1	correct formulae oe
			$= \dfrac{\cos^4\theta+\sin^4\theta+2\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$	M1	adding
			$= \dfrac{(\cos^2\theta+\sin^2\theta)^2}{\sin^2\theta\cos^2\theta} = \dfrac{1}{\sin^2\theta\cos^2\theta} = \sec^2\theta\csc^2\theta$	A1 A1	use of Pythagoras **AG**
			…………………………………………		
			OR working with both sides **Eg** LHS $\sec^2\theta+\csc^2\theta=\tan^2\theta+1+\cot^2\theta+1=\tan^2\theta+\cot^2\theta+2$ RHS $=(1+\tan^2\theta)(1+\cot^2\theta)= 1+\tan^2\theta+\cot^2\theta+\tan^2\theta\cot^2\theta$ $=\tan^2\theta+\cot^2\theta+2=$LHS	M1 M1 A1 A1	Correct formulae used on one side Use of same formulae on other side Use of $\tan\theta\cot\theta = 1$ oe, dependent on both method marks Showing equal
				[4]	

Question			Answer	Marks	Guidance
5			$\sin(x + 45°) = \sin x \cos 45° + \cos x \sin 45°$	M1	Use of **correct** compound angle formula
			$= \sin x .1/\sqrt{2} + \cos x . 1/\sqrt{2}$	A1	
			$= (1/\sqrt{2})(\sin x + \cos x) = 2\cos x$		
			$\Rightarrow \sin x + \cos x = 2\sqrt{2}\cos x$ *	A1	**Since AG ,** $\sin x \cos 45° + \cos x \sin 45° = 2\cos x$ $\sin x + \cos x = 2\sqrt{2} \cos x$ only gets M1 need the second line or statement of $\cos 45° = \sin 45° = 1/\sqrt{2}$ oe as an intermediate step to get A1 A1
			$\Rightarrow \sin x = (2\sqrt{2} - 1) \cos x$		
			$\Rightarrow \tan x = 2\sqrt{2} - 1$	M1	terms collected and $\tan x = \sin x / \cos x$ used
			$\Rightarrow x = 61.32°$,	A1	for first correct solution
			$241.32°$	A1	for second correct solution and no others in the range 2dp but allow overspecification ignore solutions outside the range SC A1 for both 61.3° and 241.3° SC A1 for both 1.07 and 4.21 radians (or better) SC A1 for incorrect answers that round to 61.3° and 180° + their ans eg 61.33° and 241.33° Do not award SC marks if there are extra solutions in the range.
				[6]	

Question			Answer	Marks	Guidance
6			$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y}{x(x+1)}$		
			$\Rightarrow \int\frac{1}{y}\mathrm{d}y=\int\frac{1}{x(x+1)}\mathrm{d}x$	M1	correctly separating variables and intending to integrate (ie need to see attempt at integration or integral signs)
			$\frac{1}{x(x+1)}=\frac{A}{x}+\frac{B}{x+1}$	M1	partial fractions soi
			$\Rightarrow 1 = A(x + 1) + Bx$		
			$x = 0 \Rightarrow A = 1$	A1	$A = 1$ www
			$x = -1 \Rightarrow 1 = -B \Rightarrow B = -1$	A1	$B = -1$ www
			$\Rightarrow \ln y=\int(\frac{1}{x}-\frac{1}{x+1})\mathrm{d}x=\ln x-\ln(x+1)+c$	A1	ft their A,B condone absence of c or ln$c$
			$x = 1, y = 1 \Rightarrow 0 = 0 - \ln 2 + c \Rightarrow c = \ln 2$	B1	evaluating their c **at any stage** dependent on x and y terms all being logs of correct form but **do not award following incorrect log rules,** ft their A,B**.** c could be say a decimal. (eg $y = x/(x + 1) + c$ then c being found is B0)
			$\Rightarrow \ln y = \ln x - \ln(x + 1) + \ln 2 = \ln (2x/(x + 1))$	M1	correctly combining lns and antilogging throughout (must have included the constant term). Apply this strictly. Do not allow if c is included as an afterthought unless completely convinced. ft A,B Logs must be of correct form ie not following say $\int\frac{1}{x(x+1)}\mathrm{d}x=\ln(x^2+x)$ unless ft from partial fractions and $B = 1$
			$\Rightarrow y = 2x/(x + 1)$	A1	cao www ($y=e^{.693}\left(\frac{x}{x+1}\right)$ loses final A1) **NB** evaluating c and log work can be in either order. eg $y = cx/(x + 1)$, at $x = 1$, $y = 1$, $c = 2$
				[8]	

Question			Answer	Marks	Guidance
7	(i)		$\theta = -\pi/2$: O (0, 0) $\theta = 0$: P (2, 0) $\theta = \pi/2$: O (0, 0)	B1 B1 B1 **[3]**	Origin or O, condone omission of (0, 0) or O Or, say at P $x = 2$, $y = 0$, need P stated Origin or O, condone omission of (0,0) or O
7	(ii)		$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{2\cos 2\theta}{-2\sin\theta} = -\frac{\cos 2\theta}{\sin\theta}$ When $\theta = \pi/2$ $dy/dx = -\cos\pi/\sin\pi/2 = 1$ When $\theta = -\pi/2$ $dy/dx = -\cos(-\pi)/\sin(-\pi/2) = -1$ **Either** $1\times -1 = -1$ so perpendicular **Or** gradient tangent $=1 \Rightarrow$ meets axis at 45° , similarly, gradient $= -1$ $\Rightarrow$ meets axis at 45° oe	M1 A1 M1 A1 A1 **[5]**	their $dy/d\theta$ /$dx/d\theta$ any equivalent form www (not from $-2\cos2\theta/2\sin\theta$) subst $\theta= \pi/2$ in their equation Obtaining $dy/dx = 1$,and $dy/dx = -1$ **shown** (or explaining using symmetry of curve) **www** justification that tangents are perpendicular **www** dependent on previous A1
7	(iii)		At Q, $\sin 2\theta = 1 \Rightarrow 2\theta = \pi/2$, $\theta = \pi/4$ $\Rightarrow$ coordinates of Q are $(2\cos\pi/4, \sin\pi/2)$ $= (\sqrt{2}, 1)$	M1 A1 A1 **[3]**	or, using the derivative, $\cos 2\theta = 0$ soi or their $dy/dx = 0$ to find θ. If the only error is in the sign or the coeff of the derivative in (ii), allow full marks in this part (condone $\theta = 45°$) www (exact only) accept $2/\sqrt{2}$
7	(iv)		$\sin^2\theta = (1 - \cos^2\theta) = 1 - \frac{1}{4}x^2$ $\Rightarrow$ $y = \sin 2\theta = 2\sin\theta\cos\theta$ $= (\pm)\, x\sqrt{(1 - \frac{1}{4}x^2)}$ $\Rightarrow$ $y^2 = x^2(1 - \frac{1}{4}x^2)$*	B1 M1 A1 A1 **[4]**	oe, eg may be $x^2 =$….. **Use** of $\sin2\theta = 2\sin\theta\cos\theta$ subst for x **or** $y^2 = 4\sin^2\theta\cos^2\theta$ (squaring) either order oe squaring **or** subst for x either order oe **AG**

Question			Answer	Marks	Guidance
7	(v)		$V=\int_0^2 \pi x^2(1-\frac{1}{4}x^2)\,\mathrm{d}x$	M1	integral including correct limits but ft their '2' from (i) (limits may appear later) condone omission of dx if intention clear
			$=\int_0^2(\pi x^2-\frac{1}{4}\pi x^4)\,\mathrm{d}x$		
			$=\pi\left[\frac{1}{3}x^3-\frac{1}{20}x^5\right]_0^2$	B1	$\left[\frac{1}{3}x^3-\frac{1}{20}x^5\right]$ ie allow if no π and/or incorrect/no limits (or equivalent by parts)
			$=\pi\left[\frac{8}{3}-\frac{32}{20}\right]$	A1	substituting limits into correct expression (including π) ft their '2'
			$= 16\pi/15$	A1 **[4]**	cao oe, 3.35 or better (any multiple of π must round to 3.35 or better)
8	(i)		$\overrightarrow{AA'}=\begin{pmatrix}2\\4\\1\end{pmatrix}-\begin{pmatrix}1\\2\\4\end{pmatrix}=\begin{pmatrix}1\\2\\-3\end{pmatrix}$	B1	finding $\overrightarrow{AA'}$ or $\overrightarrow{A'A}$ by subtraction, **subtraction must be seen** B0 if $\overrightarrow{AA'}$, $\overrightarrow{A'A}$ confused Assume they have found $\overrightarrow{AA'}$ if no label
			This vector is normal to $x + 2y - 3z = 0$	B1	reference to normal or ***n***, or perpendicular to $x + 2y - 3z = 0$, or statement that vector matches coefficients of plane and is therefore perpendicular, or showing AA' is perpendicular to **two** vectors in the plane
			M is (1½, 3, 2½) $x + 2y - 3z = 1½ + 6 - 7½ = 0$	M1	for finding M correctly (can be implied by two correct coordinates)
			⇒ M lies in plane	A1 **[4]**	showing numerical subst of M in plane = 0

Question			Answer	Marks	Guidance
8	(ii)		$\mathbf{r} = \begin{pmatrix}1\\2\\4\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\2\end{pmatrix} = \begin{pmatrix}1+\lambda\\2-\lambda\\4+2\lambda\end{pmatrix}$ meets plane when		
			$1+\lambda+2(2-\lambda)-3(4+2\lambda)=0$	M1	subst of **AB** in the plane
			$\Rightarrow$ $-7-7\lambda=0,\ \lambda=-1$	A1	
			So B is (0, 3, 2)	A1	cao
			$\overrightarrow{A'B} = \begin{pmatrix}0\\3\\2\end{pmatrix} - \begin{pmatrix}2\\4\\1\end{pmatrix} = \begin{pmatrix}-2\\-1\\1\end{pmatrix}$	M1	or $\overrightarrow{BA'}$, ft **only** on their B (condone $\overrightarrow{A'B}$ used as $\overrightarrow{BA'}$ or no label) (can be implied by two correct coordinates)
			Eqn of line A'B is $\mathbf{r} = \begin{pmatrix}2\\4\\1\end{pmatrix} + \lambda\begin{pmatrix}-2\\-1\\1\end{pmatrix}$	B1 ft	$\begin{pmatrix}2\\4\\1\end{pmatrix}$ or their B +……
				A1 ft **[6]**	… λ × their $\overrightarrow{A'B}$ (or $\overrightarrow{BA'}$) ft only their B correctly
8	(iii)		Angle between $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}-2\\-1\\1\end{pmatrix}$	M1	correct vectors but ft their $\overrightarrow{A'B}$.Allow say, $\begin{pmatrix}-1\\1\\-2\end{pmatrix}$ and/or $\begin{pmatrix}2\\1\\-1\end{pmatrix}$ condone a minor slip if intention is clear
			$\Rightarrow$ $\cos\theta = \dfrac{1.(-2)+(-1).(-1)+2.1}{\sqrt{6}.\sqrt{6}}$	M1	correct formula (including cosθ) for their direction vectors from (ii) condone a minor slip if intention is clear
			= 1/6	A1	±1/6 or 99.6º from appropriate vectors only soi **Do not allow either A mark if the correct *B* was found fortuitously in (ii)**
			$\Rightarrow$ $\theta = 80.4°$	A1 **[4]**	cao or better

Question			Answer	Marks	Guidance
8	(iv)		Equation of BC is $\mathbf{r}=\begin{pmatrix}2\\4\\1\end{pmatrix}+\lambda\begin{pmatrix}-2\\-1\\1\end{pmatrix}=\begin{pmatrix}2-2\lambda\\4-\lambda\\1+\lambda\end{pmatrix}$		**NB this is not unique** eg $\begin{pmatrix}0\\3\\2\end{pmatrix}+\mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$
			Crosses Oxz plane when $y=0$	M1	For putting $y=0$ in their line BC and solving for λ
			$\Rightarrow \lambda=4$	A1	Do not allow either A mark if B was found fortuitously in (ii) for A marks need fully correct work only **NB this is not unique** eg $\begin{pmatrix}0\\3\\2\end{pmatrix}+\mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$ leads to $\mu=-3$
			$\Rightarrow \mathbf{r}=\begin{pmatrix}-6\\0\\5\end{pmatrix}$ so $(-6,0,5)$	A1	cao
				[3]	

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Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754B Applications of Advanced Mathematics (C4) Paper B: Comprehension

Candidates answer on the Question Paper.

OCR supplied materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Rough paper

Duration: Up to 1 hour

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES
- The Insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- The Insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES
- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You may find it helpful to make notes and to do some calculations as you read the passage.
- You are **not** required to hand in these notes with your Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
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1 Use Fig. 4 to estimate the number of 50–54 year olds in the UK in 2001. (These were born in the post World War 2 baby boom.) **[1]**

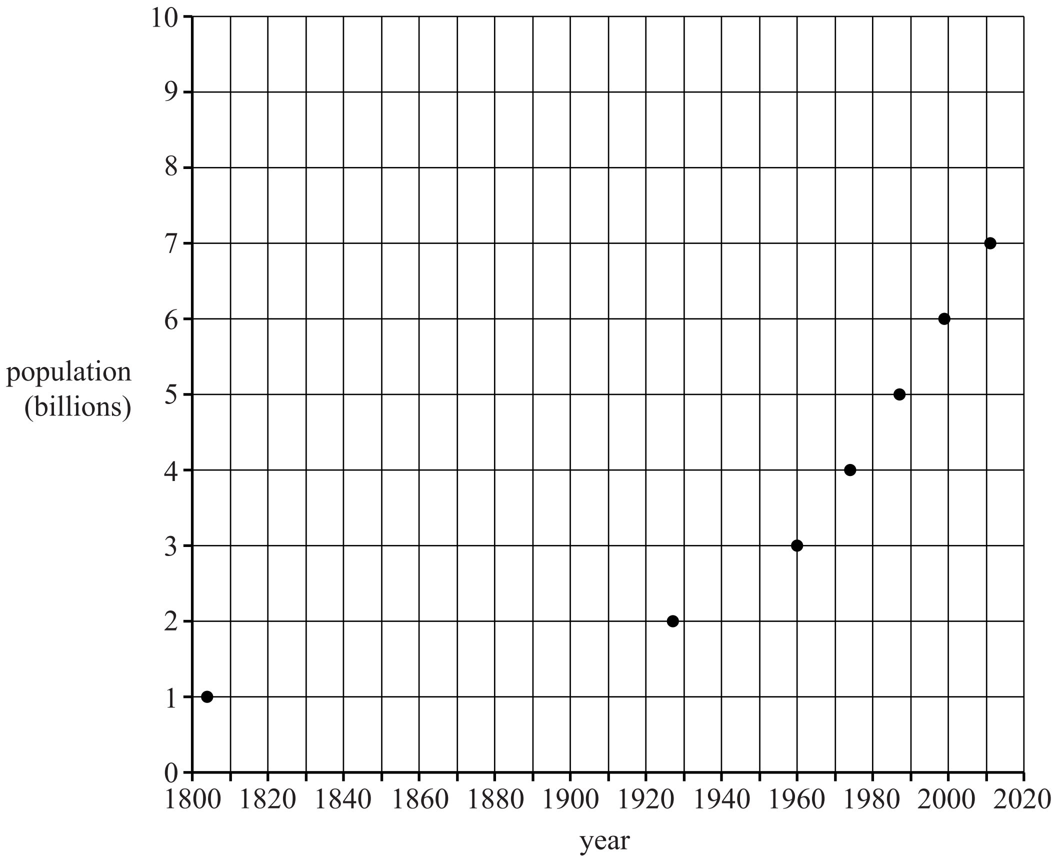
1	

2 A copy of Fig. 2 is given below.

(i) Join the points with a curve and hence estimate the rate of population growth in the year 1927 in people per year. **[3]**

(ii) Estimate this rate as a percentage of the population at that time. **[2]**

2 (i)



2 (ii)

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3 **(i)** In line 21, the solution of the differential equation $\frac{dp}{dt} = kp$ is stated to be $p = p_0 e^{kt}$.

Use integration to derive this result. **[3]**

(ii) The article then goes on to say

"If a model is to be valuable in this context, it must be possible to use it to predict the size of the world population in the future. So, as a test case, the first two data points in Table 1 should allow the later values to be predicted. These data points are

1804 $\quad t = 0,\ p = p_0 = 10^9,$

1927 $\quad t = 123,\ p = 2\times10^9,$

and these correspond to $k = 0.005\,63\ldots$."

Show how this value of k is obtained. **[2]**

3 (i)	
3 (ii)	

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4 In Table 6, the population profile of an imaginary country was predicted. Complete the table subject to the same general assumptions except that, after 2010:

- the average number of children per female is 2.2;
- 60% of those in the 40–59 age group survive into the 60–79 age group;
- 20% of those in the 60–79 age group survive into the 80+ age group. **[3]**

4

Age group	2010	2030	2050	2070
80+	1			
60–79	10			
40–59	20			
20–39	20			
0–19	20			
Total	**71**			

As in Table 6, the figures are in millions.

5 In constructing Table 6, some assumptions were made about the proportion of people surviving from one age group to the next. Use Table 6 to find

(i) the proportion of people in the 40–59 age group surviving into the 60–79 age group, **[1]**

(ii) the proportion of those in the 60–79 age group surviving into the 80+ age group. **[1]**

5 (i)

5 (ii)

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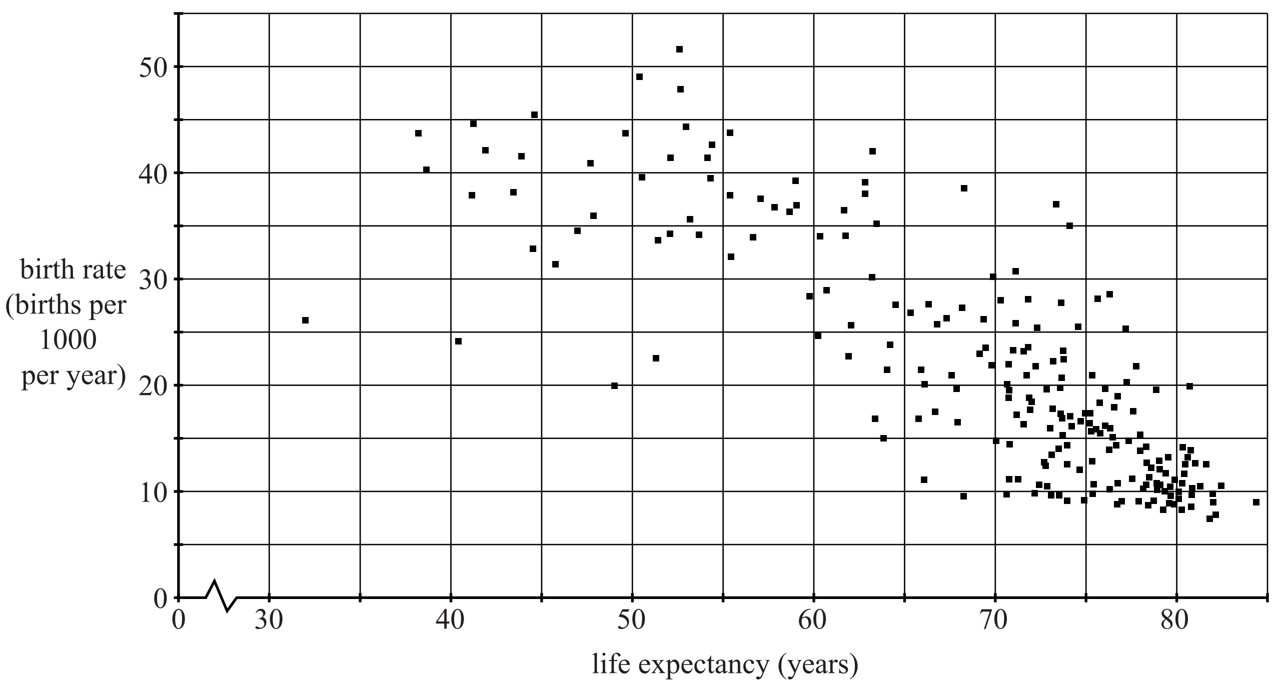
6 This table refers to the UK. It gives life expectancy and birth rate every 20 years from 1901 to 2001.

Year	Life expectancy	Birth rate (births / 1000)
1901	47	28.5
1921	58	22.7
1941	64	14.5
1961	71	17.8
1981	74	12.9
2001	78	12.0

Explain how these data relate to the conclusions of the article. **[2]**

[A copy of Fig. 7 is given below. You do not need to use it but may find it helpful.]

6



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Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754B Applications of Advanced Mathematics (C4) Paper B: Comprehension

INSERT

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The world population

Population pressure on our planet

During the last 200 years, the human population has increased by a factor of about 7. Table 1 gives the years when it reached 1, 2, 3 and so on billions of people, where 1 billion is 10^9.

Year	1804	1927	1960	1974	1987	1999	2011
Population (billions)	1	2	3	4	5	6	7

Table 1 World population

The increase in population is placing rising demands on the resources of our planet and on the whole eco-system that supports us. This raises very important questions.

- Is the world's population going to continue to increase indefinitely or will there be a limit?
- Will the world's population reach a level that the planet is unable to support?
- Should we be taking measures to restrict the world's population, and if so what?

The first two of these questions require mathematical modelling of the situation. The third involves political and ethical decisions which should be informed by that modelling.

The modelling involved is complicated; this article introduces some of the issues involved.

The exponential model

A simple mathematical model is that the world's population is increasing at a rate which is directly proportional to its existing size,

$$\frac{dp}{dt} = kp\ ,$$

where p is the number of people,
t is time, measured in years,
k is a constant.

The solution of this differential equation is

$$p = p_0 e^{kt}$$

where p_0 is the population at the time from which t is measured.

If a model is to be valuable in this context, it must be possible to use it to predict the size of the world population in the future. So, as a test case, the first two data points in Table 1 should allow the later values to be predicted. These data points are

$$1804 \qquad t = 0,\ p = p_0 = 10^9,$$
$$1927 \qquad t = 123,\ p = 2\times10^9,$$

and these correspond to $k = 0.005\,63\ldots$.

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With this value of k, this model would predict that the population in 2011 would be 3.2 billion but in fact it was 7 billion. This model, based on the first two data points, is clearly not suitable.

In fact no exponential model fits the data in Table 1 well. You can see this just by looking at the graph of the data in Fig. 2. The graph of an exponential function is a curve which gets steeper and steeper but for the last 50 years this graph is virtually a straight line, indicating a constant rate of growth.

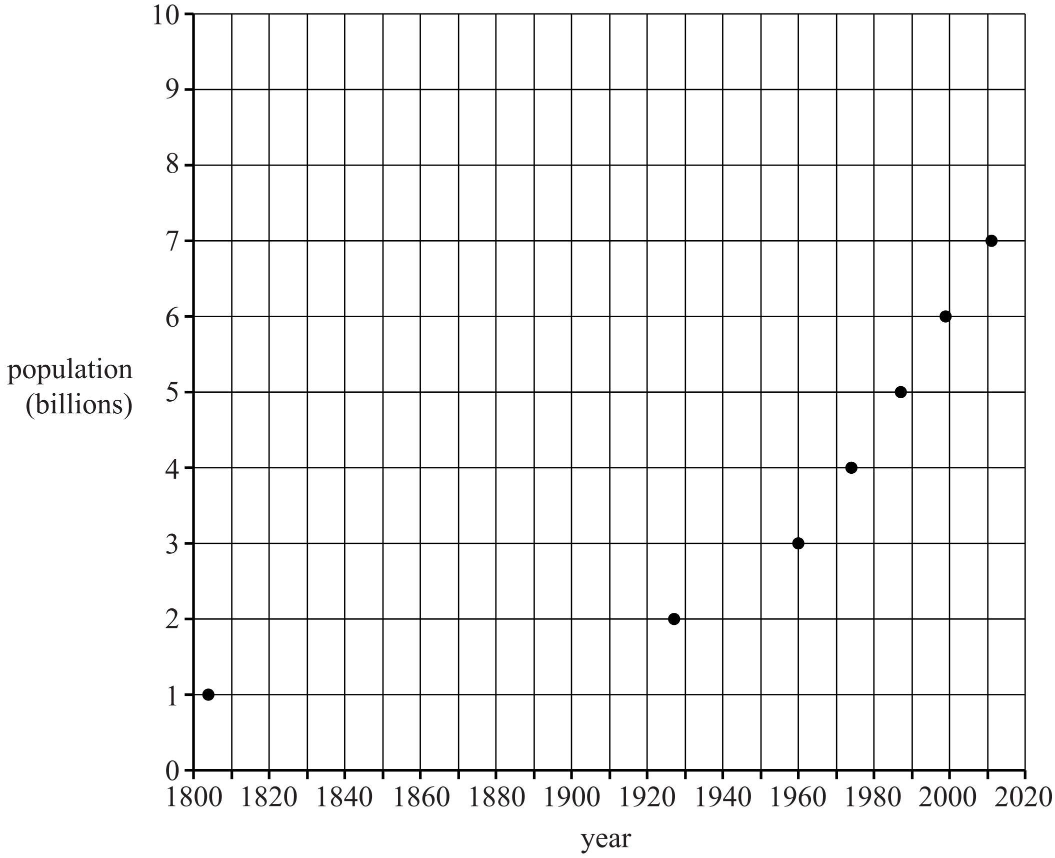

Fig. 2 World population from 1800 to the present

The logistic model

A standard mathematical model for a population which increases towards a limiting value of m is given by the differential equation

$$\frac{dp}{dt} = kp(m-p) .$$

This is known as the *logistic equation*. A typical solution curve is shown in Fig. 3.

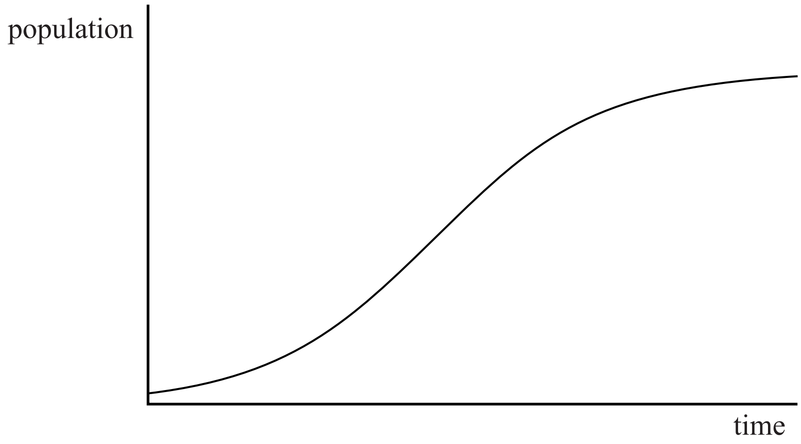

Fig. 3 The logistic model

While this looks as though it starts with the same sort of shape as a curve through the data points in Fig. 2, the resemblance is only superficial; it is not actually possible to find values of k and m that produce anything like a good fit. So this model is also unsatisfactory.

Like the previous model, this is an attempt to find a simple, neat solution to a very complicated problem. A different approach is needed and a starting point is provided by population profiles.

Population profiles

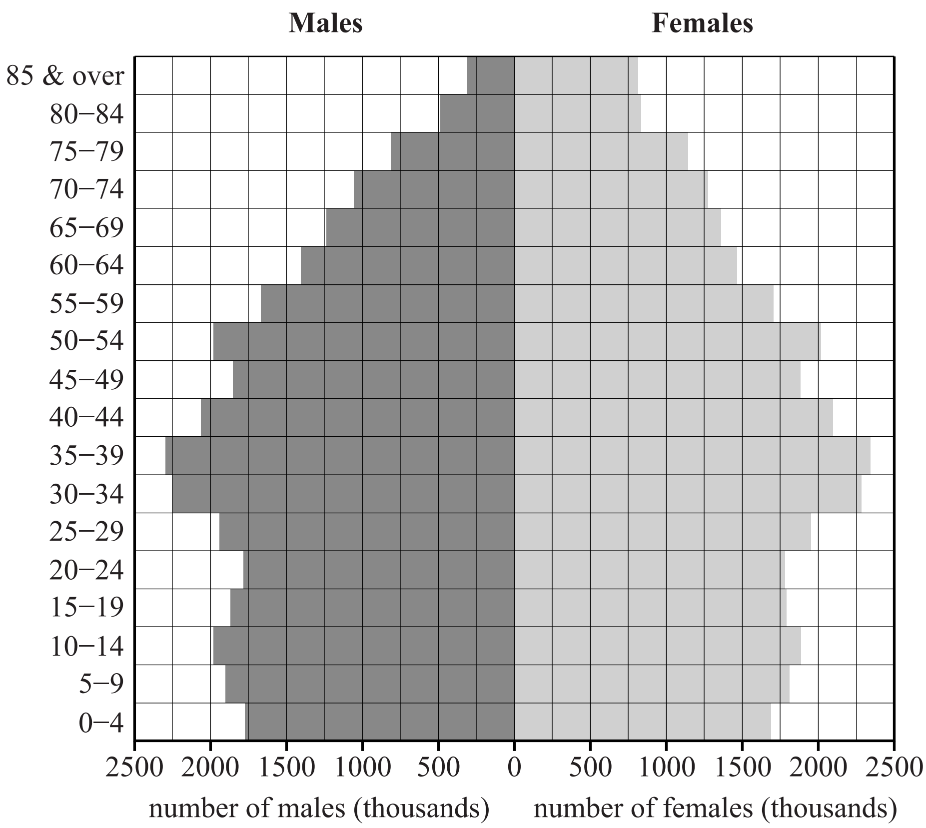

Fig. 4 Population profile of the UK in 2001

Population profiles are often illustrated by population pyramids, like that in Fig. 4. The lengths of the horizontal bars indicate the numbers of males and females in the UK population in 2001, in 5-year age intervals. In this case, the numbers on the horizontal scale are in thousands. Those on the vertical scale refer to age in completed years so that, for example, 10–14 means from 10 years 0 days to 14 years 364 days.

The UK population profile shows that in 2001 the number of children in the 0–4 age range was among the lowest for 50 years. Because there are fewer people in that age group, they in turn can be expected to have fewer children.

The shapes of the population profiles vary considerably between countries. In some countries the profiles have very wide bases, indicating large numbers of children.

It is worth noting that population figures for the UK are often affected by emigration and immigration.

Modelling using population profiles

If the individual population profiles of the large number of countries in the world are combined, a profile for the whole world emerges. It is possible to predict the changes in any country's profile in the years ahead, and hence the changes in the world's population. Each country is different and so needs to be looked at separately before combining the profiles.

The following model, for an imaginary country, is designed to highlight the key factors. Table 5 illustrates its profile in 2010 and part of that for 2030.

Age group	2010	2030
80+	1	?
60–79	10	?
40–59	20	20
20–39	20	20
0–19	20	?
Total	**71**	**?**

Table 5 Population profile of an imaginary country (in millions)

In 2010, this country has a stable population with the same numbers in the youngest three age groups, up to the age of 60; however, life expectancy is quite low with very few people reaching the age of 80.

Two of the figures for 2030 have also been filled in. The 20 million people in the 0–19 age group in 2010 will move into the 20–39 group. Similarly those in the 20–39 group will move into the 40–59 group. (It is assumed, for simplicity, that no one in these age groups dies.) What will the other figures for 2030 be?

Two different factors are involved: the birth rate and the life expectancy.

The 2010 profile in Table 5 was constructed using a number of assumptions:

- that those in the 0–19 age group are all children of females in the 20–39 group;
- that 50% of those in the 20–39 age group are female;
- that on average each female has 2 children;
- that there is no immigration or emigration.

While these assumptions are obviously somewhat artificial, particularly with regard to the age at which women have children, they are good enough to demonstrate the key features of a country's population.

Throughout the world, life expectancy is rising. The proportion of the population in Table 5 who reach the age of 80 could be expected to increase.

In Table 6, the population profile of the country in Table 5 is predicted for the next 100 years, on the basis of the following new assumptions about the birth rate and life expectancy.

- Every 20 years, each group of people moves up a level.
- The average figure of 2 children per female is assumed to fall to 1.8, from 2010 onwards.
- The proportion of those in the 40–59 age group surviving into the 60–79 group increases from the 2010 figure of 50%; similarly there is an increase in survival from the 60–79 group into the 80+ group.

The figures used in these assumptions have been chosen to illustrate the modelling process. Their use does not mean that they will actually apply to the population of any real country.

Age group	2010	2030	2050	2070	2090	2110
80+	1	4	6.4	6.4	6.4	5.76
60–79	10	16	16	16	14.4	12.96
40–59	20	20	20	18	16.2	14.58
20–39	20	20	18	16.2	14.58	13.12
0–19	20	18	16.2	14.58	13.12	11.81
Total	**71**	**78**	**76.6**	**71.18**	**64.7**	**58.23**

Table 6 Population profile of an imaginary country (in millions, to 4 significant figures)

The figures in Table 6 show the total population rising quite sharply to a maximum and then reducing, initially rather slowly but then more quickly. Patterns like this are observed in many countries. In some, like Japan, the population has passed its maximum and is declining. In most, however, it is still increasing and consequently the population of the world as a whole is still increasing.

An important feature of Table 6 is that it is based on a low birth rate of 1.8 children per female. In many countries the birth rate is much higher than the stable level of 2 children per female.

Birth rate and life expectancy

Two key factors that determine the change in a country's population have been identified as its birth rate and its life expectancy. Data show that these are closely associated. Countries with high birth rates tend to have low life expectancy and those with low birth rates have high life expectancy. This is illustrated in Fig. 7 for all 221 countries; the data were drawn from the CIA World Factbook for 2009.

Notice that in Fig. 7 the birth rate is the number of births per 1000 of the population per year. It is thus a different measure from that used so far in this article which is mean births per female over her lifetime. So, for example, a country with a population of 80 million people and 1.2 million births per year has a birth rate of 15 births per 1000 per year.

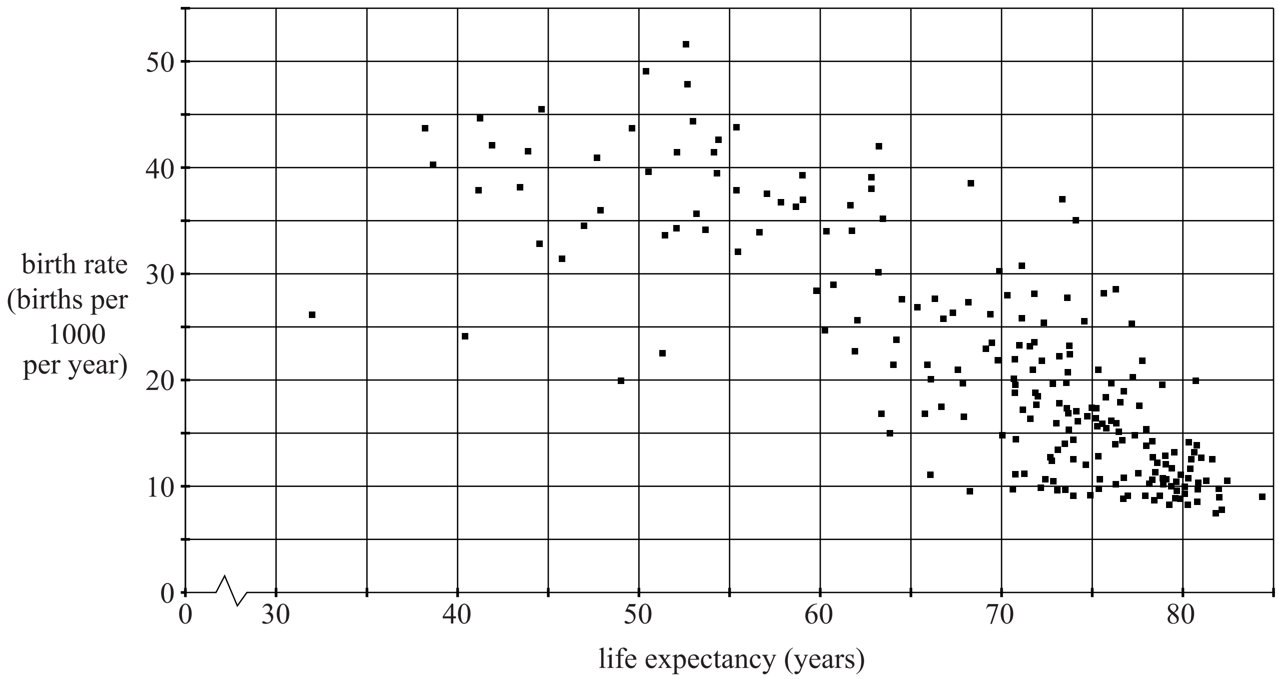

Fig. 7 Scatter diagram showing birth rate against life expectancy for the countries of the world

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Conclusion

Table 8 gives the data for some selected countries in 2009.

Country	Life expectancy	Birth rate
Japan	82.12	7.64
Sweden	80.86	10.13
Italy	80.20	8.18
UK	79.01	10.65
USA	78.11	13.82
Tunisia	75.78	15.42
Jamaica	73.53	19.68
China	73.47	14.00
Brazil	71.99	18.43
India	69.89	21.76
Bangladesh	60.25	24.68
Ghana	59.85	28.58
Uganda	52.72	47.84
Afghanistan	44.64	45.46

Table 8 Life expectancy and birth rates of selected countries (2009)

The data in Table 8 illustrate the observation that countries with low birth rate and high life expectancy tend to be those with developed economies. Studies over time indicate that as they develop, countries follow a path from high birth rate and low life expectancy to low birth rate and high life expectancy. So it is reasonable to expect that at some time in the future, the world's population will attain a maximum value and then start to decline.

When that maximum occurs, and how great the population then is, will depend on how quickly countries progress along that path. Consequently modelling the world's population requires an understanding of the factors involved. Then it will be possible to determine what can be done to match the population to the planet's resources.

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GCE

Mathematics (MEI)

Advanced GCE

Unit **4754B:** Applications of Advanced Mathematics: Paper B

Mark Scheme for June 2012

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

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OCR Publications
PO Box 5050
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NOTTINGHAM
NG15 0DL

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E-mail: publications@ocr.org.uk

Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

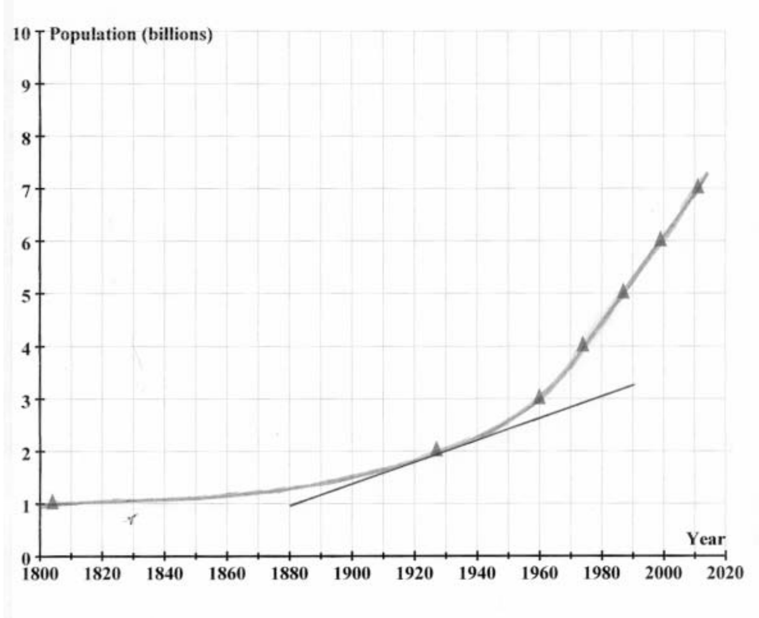
g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he / she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

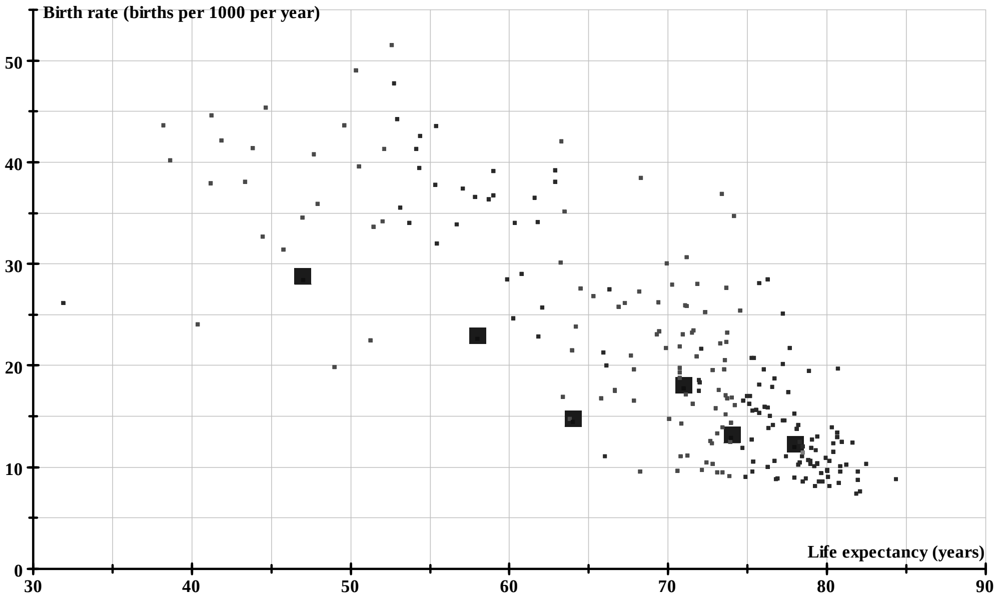
h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			Males 1.95 million, Females 2 million: Total 3.95 million	B1 **[1]**	accept 3.9-4 million allow 4000 thousand oe
2		(i)	 Gradient 0.018×10^{9} giving 18 000 000 **(** people per year)	B1 M1 A1 **[3]**	curve and tangent drawn do not accept a polygon accept any reasonable **tangent** at the correct point (ie touches, not crosses) (NB B0M1A1 is possible if a full curve is not drawn) use of gradient (**<u>from tangent only</u>**) accept 12-28,000,000 do not accept unreasonable accuracy eg no more than 3sf (0.018 or 180 000, say, can score M1 A0) without tangent is M0 A0
		(ii)	$\frac{0.018\times10^{9}}{2\times10^{9}}\times100\% = 0.9\%$	M1 A1 **[2]**	allow follow through from previous part **for both marks** ie (**their (i)**/ 2×10^{9}) × 100%, for A mark do not allow more than 3sf could get M1A1 from say 0.018/2 × 100% without having scored A1 in (i)

Question			Answer	Marks	Guidance
3		(i)	$\frac{\mathrm{d}p}{\mathrm{d}t} = kp$ $\int \frac{\mathrm{d}p}{p} = k\int \mathrm{d}t$ $\ln p = kt + c$	M1 A1	separating variables correctly and intending to integrate solving correctly, any form, need a constant
			When $t = 0, p = p_0 \Rightarrow c = \ln p_0$ $\ln\left(\frac{p}{p_0}\right) = kt$ $p = p_0 \mathrm{e}^{kt}$	A1 **[3]**	**AG,** fully correct derivation of given result including **explicitly using** initial condition (condone t = 0, p = 10^9 = $\boldsymbol{p_0}$) SC1 for verifying the given result correctly ie differentiating p=… and substituting for p
		(ii)	$p_0 = 10^9$ so the 1927 figures give $2\times10^9 = 10^9 \times e^{k\times(1927-1804)}$ $\Rightarrow 123k = \ln 2$ $\Rightarrow k = 0.00563...$*	M1 A1 **[2]**	the equation must be correct (soi) (10^9 could be cancelled) cao **AG** so must **SHOW** enough, eg k = ln2/123 or 0.005635….
4			(table below)	B1 B1 B1 **[3]**	2030 column 2050 column 2070 column (need 2dp) SCB2 for columns correct but no totals

Age group	2010	2030	2050	2070
80^+	1	**2**	**2.4**	**2.4**
60-79	10	**12**	**12**	**12**
40-59	20	**20**	**20**	**22**
20-39	20	**20**	**22**	**24.2**
0-19	20	**22**	**24.2**	**26.62**
Total	**71**	**76**	**80.6**	**87.22**

Question			Answer	Marks	Guidance
5		(i)	The proportion of people in the 40-59 age group surviving into the 60-79 group is 80%.	B1 **[1]**	cao oe in fractions or decimals
		(ii)	The proportion of those in the 60-79 group surviving in to the 80+ group is 40%.	B1 **[1]**	cao oe in fractions or decimals
6			The data show that, over the last 100 years, the birth rate in the UK has declined and the life expectancy has increased. This pattern is consistent with the UK having a developing economy.	B1 B1 **[2]**	1 mark for correct comments on changing (over time) birth rate and life expectancy. Not just negative correlation if no link (soi) to time 1 mark for linking to **development**
					This diagram does not need to be seen but some candidates may use it to help in their explanations.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

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Head office
Telephone: 01223 552552
Facsimile: 01223 552553

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4754 Applications of Advanced Mathematics (C4)

General Comments

This paper proved to be of a similar standard to that of previous years. The questions were accessible to candidates of all abilities, the students seemed well prepared and there were very few low marks.

This Paper B was more accessible than most and most candidates scored well here, although many failed to draw a tangent in question 2.

In Paper A most candidates scored highly on the straightforward, well practised questions. Marks were more frequently lost where candidates had to think for themselves in less familiar questions.

It was pleasing to see that, unlike on previous papers, almost all candidates included a constant of integration-particularly in question 6.

Algebraic errors still caused an unnecessary loss of marks. In question 1, $-3(x+1) = -3x-1$ or $-3x+3$ or $-3x+1$ were all familiar errors. In question 4 some candidates felt that if $\sin^2\theta + \cos^2\theta = 1$ then $1/\sin^2\theta + 1/\cos^2\theta = 1/1 = 1$ and others felt that in question 5, if $\sin x + \cos x = 2\sqrt{2}\cos x$ then $\sin^2 x + \cos^2 x = 8\cos^2 x$.

The less structured form of question 6 caused some candidates to lose many marks as they did not realise that they needed to use partial fractions.

Candidates should be advised to read questions carefully and to show all working when establishing given answers or when asked to 'show' results.

Centres are reminded that Papers A and B are no longer marked together and so additional sheets must be attached to the correct paper.

Comments on Individual Questions

Paper A

1) The majority of candidates understood the method needed to add the fractions and solve the quadratic equation. Most errors were algebraic, the most common being the incorrect expansion of $-3(x+1)$. Those who continued were able to gain marks for solving their quadratic equation provided that 'b^2-4ac' ≥ 0. Those with a negative discriminant should have realised they had made an error and checked their work.

2) This was probably the most successful question on paper A and few errors were seen. A few omitted the set of values for which the expansion was valid.

3) (i) Most candidates scored the first mark for writing down the differential equation. Those who differentiated often scored full marks. Common errors included, incorrectly differentiating the inside of the bracket- instead of $1/2k$, a variety of errors were seen, including functions of t, and, for those who did differentiate correctly, failing to equate this to $k\sqrt{V}$ at the final stage.

Quite a number omitted this differentiation. Some others decided to ignore the instruction given and integrate instead in order to derive the given result instead of verifying it. Very few of these attempts gained any further credit as they failed to deal with the change in constant. Those who integrated to reach $2\sqrt{V}=kt+c$ then, too often, gave $\sqrt{V}$= [$1/2(kt+c)$ =] $1/2kt+c$ when trying to establish the given result and obtained no marks unless they explained the change of constant.

(ii) The majority scored two marks for writing down two correct equations. Those who then square rooted say, $(1/2k+c)^2 = 10{,}000$ to reach $1/2k+c = 100$, and the other equation to obtain $k+c=200$ usually obtained full marks. Those who did not square root the equations were sometimes successful but more often made errors or abandoned their attempts.
Some felt that $(1/2k+c)^2 = 1/4k^2+c^2$

4) The most common and most successful method seen was from those that changed $\sec^2\theta+\text{cosec}^2\theta$ to $1/\cos^2\theta + 1/\sin^2\theta$ and added these fractions together and then used $\sin^2\theta + \cos^2\theta=1$.
There were other successful methods including changing both sides to $\tan^2\theta+\cot^2\theta+2$ or starting with
$\sec^2\theta\text{cosec}^2\theta=\sec^2\theta(1+\cot^2\theta)=\sec^2\theta+\sec^2\theta\cot^2\theta=\sec^2\theta+1/\cos^2\theta \text{x} \cos^2\theta/\sin^2\theta$
$=\sec^2\theta+1/\sin^2\theta=\sec^2\theta+\text{cosec}^2\theta$
Some candidates seemed to write down every relevant trig identity they could think of and make multiple starts of attempts without any clear structure to their methods. Some attempts included taking reciprocals term by term. In general, candidates need to be encouraged to produce more structured responses when proving identities.

5) Most candidates correctly expanded the double angle formula, substituted the values for sin 45º and cos 45º and gained the first three marks.
Many candidates then proceeded correctly to obtain full marks. Others squared term by term and lost the last three marks.
There were few instances of additional solutions in the range being given although not all gave their final solutions to the required degree of accuracy.

6) Almost all separated the variables correctly with the intention of integrating.
Partial fractions is always a well answered part of this paper, but on this occasion candidates had to realise for themselves that they needed to use partial fractions. Those that did usually gained at least the first 5 marks. The others used a wide variety of incorrect methods in order to try to integrate $1/x(x+1)$. These included, $1/(2x+1) \ln (x^2+x)$ and $\ln x \text{ x } \ln(x+1)$.
For those who proceeded correctly, $\ln y=\ln x - \ln(x+1) +c$ was almost always obtained. Those who substituted first were usually successful in gaining the mark for finding the constant.
For some, the laws of logarithms were not applied correctly. Such errors as $y=x/(x+1) +c$ being common.

7 (i) The majority of candidates failed to read the question carefully and, although they obtained the correct coordinates, lost a mark by failing to name the points, particularly P.

(ii) Nearly all candidates understood the principle of finding $\mathrm{d}y/\mathrm{d}\theta$ and $\mathrm{d}x/\mathrm{d}\theta$ in order to find $\mathrm{d}y/\mathrm{d}x$. There were errors including wrong signs and ½ instead of 2 but there were many fully correct expressions. A pleasing number continued correctly throughout the question, substituting both $\pi/2$ and $-\pi/2$ into $\mathrm{d}y/\mathrm{d}x$ and explaining that since the gradients multiplied to give -1 they must be perpendicular lines.
Some had either the correct expression for $\mathrm{d}y/\mathrm{d}x$ but cancelled it incorrectly or made errors in $\mathrm{d}y/\mathrm{d}x$ and so forfeited the last two marks. Others failed to show their substitution of $-\pi/2$, and merely stated that the other gradient must be -1 since 1x-1=-1.

(iii) Many successfully solved $\sin2\theta=1$or $\cos2\theta=0$ (full marks were available even if the coefficient in (ii) had been incorrect), and proceeded to obtain full marks. Others thought that $\cos2\theta/\sin\theta=0$ implied that $\cos2\theta=\sin\theta$ and tried to solve that. Others obtained more than one solution for θ and chose the wrong one.
Almost all gave their answers in exact form as required.

(iv) Those who used y=sin2θ=2sinθcosθ usually squared and gained at least two marks although some failed to square the 2.Those who answered the question and expressed $\sin^2\theta$ in terms of x usually obtained full marks. Some candidates did not explicitly state the identities or seemed to be working backwards from the answer.

(v) Many candidates obtained full marks for this integration. Those who did not either did not use the correct limits (despite having usually found them correctly in (i)) or made various errors when attempting integration by parts instead of multiplying out the brackets. A few lost the π.

8) (i) Most candidates found AA' but did not always show the subtraction and they were asked to 'Show'. They also often failed to make reference to the normal. Some, unnecessarily, calculated scalar products at this stage. Many correctly found the point M and showed that it lay in the plane.

(ii) There were many completely correct solutions in this part. Other candidates made errors in the algebra when finding the coordinates of B and their point was then followed through for the following marks.
The main error in this part was that candidates felt that B= (1,2,4) +(1,-1,2) =(2,1,6).
Others found B apparently correctly as (0,3,2) but having used just the x coordinate as $1+\lambda=0$ hence $\lambda= -1$ and thus found B fortuitously.
The follow through marks helped many candidates here.

(iii) Most candidates knew the correct method here but did not always have the correct vectors and so obtained the method marks.

(iv) Many candidates made no response to this part. Full marks was only obtained in a minority of cases. The most common error from those who attempted this being an attempt at solving $x+z$ =0 rather than y=0.

Paper B

1) This was often correct. In some cases 4 000 000 being given as 4000 thousand. The most common error was to give the answer as 4000 (or 3950). Some only gave the number of males or females.

2) (i) The majority did not draw a tangent. Those who did usually scored full marks although a few gave too many significant figures in their answer.

(ii) This was marked as a follow through whatever their answer in part (i) and most scored marks here. Some, again, over-specified their answer, others failed to multiply by 100 and some were confused by how many 0s there were in a billion.

3) (i) Most candidates integrated correctly and obtained ln$p$=kt+c. Few then explicitly used the initial condition to find their constant and merely stated the given answer.

ii) Most successfully used ln2/123 to find k.

4) This was less successful than expected. There were many completely correct solutions but also a wide range of incorrect values were seen in the table.

5) Both parts of this question were usually correct.

6) Questions requiring explanations in the comprehension paper usually cause the most problems for candidates. This was less so on this occasion. Most candidates scored the first mark for saying that as the birth rate declined over time the life expectancy increased.
A few only gave the values at the end points, or did not link it to change over time.
Fewer candidates linked this change with the developing economy in the UK in order to obtain the second mark.

GCE Mathematics (MEI)

			Max Mark	90% cp	a	b	c	d	e	u
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	66	60	53	47	41	34	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	16	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	16	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	90	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	73	65	57	50	43	36	0
		UMS	100	90	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	66	61	53	46	39	32	0
		UMS	100	90	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	61	54	47	40	34	28	0
		UMS	100	90	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	68	63	57	51	45	39	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	16	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	16	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	90	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	65	58	51	44	38	32	0
		UMS	100	90	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	67	63	56	50	44	38	0
		UMS	100	90	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	63	56	49	42	35	29	0
		UMS	100	90	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	66	61	55	49	43	38	0
		UMS	100	90	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	65	58	51	44	38	32	0
		UMS	100	90	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	63	56	49	42	35	28	0
		UMS	100	90	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	62	56	50	44	39	34	0
		UMS	100	90	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	52	46	40	34	29	24	0
		UMS	100	90	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	63	55	47	39	32	25	0
		UMS	100	90	80	70	60	50	40	0