

**ADVANCED SUBSIDIARY GCE UNIT
MATHEMATICS (MEI)**

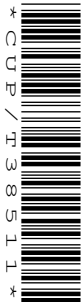
Decision Mathematics 1

TUESDAY 22 JANUARY 2008

4771/01

Afternoon
Time: 1 hour 30 minutes

Additional materials: Printed Answer Book (Enclosed)
MEI Examination Formulae and Tables (MF2)



INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

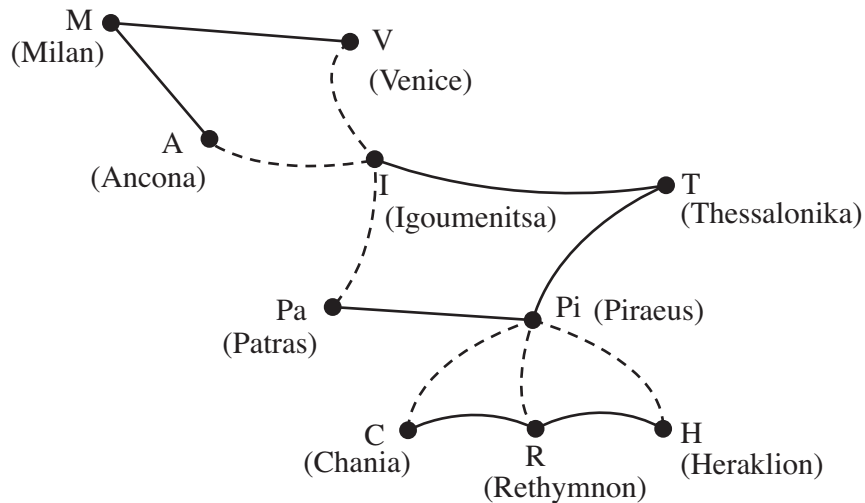
- The number of marks for each question is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **7** printed pages and **1** blank page.

Answer all the questions in the printed answer book provided.

Section A (24 marks)

- 1 The graph shows routes that are available to an international lorry driver. The solid arcs represent motorways and the broken arcs represent ferry crossings.



- (i) Give a route from Milan to Chania involving exactly two ferry crossings.
How many such routes are there? [2]
- (ii) Give a route from Milan to Chania involving exactly three ferry crossings.
How many such routes are there? [2]
- (iii) Give a route from Milan to Chania using as many ferry crossings as possible, without repeating any arc. [1]
- (iv) Give a route leaving Piraeus and finishing elsewhere which uses every arc once and only once. [3]
- 2 Consider the following linear programming problem.

$$\text{Maximise } P = 6x + 7y$$

$$\begin{aligned} \text{subject to } & 2x + 3y \leq 9 \\ & 3x + 2y \leq 12 \\ & x \geq 0 \\ & y \geq 0 \end{aligned}$$

- (i) Use a graphical approach to solve the problem. [6]
- (ii) Give the optimal values of x , y and P when x and y are integers. [2]

- 3 The following algorithm (J. M. Oudin, 1940) claims to compute the date of Easter Sunday in the Gregorian calendar system.

The algorithm uses the year, y , to give the month, m , and day, d , of Easter Sunday.

All variables are integers and **all remainders from division are dropped**. For example, 7 divided by 3 is 2 remainder 1. The remainder is dropped, giving the answer 2.

$$c = y / 100$$

$$n = y - 19 \times (y / 19)$$

$$k = (c - 17) / 25$$

$$i = c - (c / 4) - (c - k) / 3 + (19 \times n) + 15$$

$$i = i - 30 \times (i / 30)$$

$$i = i - (i / 28) \times (1 - (i / 28)) \times (29 / (i + 1)) \times ((21 - n) / 11)$$

$$j = y + (y / 4) + i + 2 - c + (c / 4)$$

$$j = j - 7 \times (j / 7)$$

$$p = i - j$$

$$m = 3 + (p + 40) / 44$$

$$d = p + 28 - 31 \times (m / 4)$$

For example, for 2008:

$$y = 2008$$

$$c = 2008 / 100 = 20$$

$$n = 2008 - 19 \times (2008 / 19) = 2008 - 19 \times (105) = 13, \text{ etc.}$$

Complete the calculation for 2008.

[8]

Section B (48 marks)

- 4 In a population colonizing an island 40% of the first generation (parents) have brown eyes, 40% have blue eyes and 20% have green eyes. Offspring eye colour is determined according to the following rules.

<u>Eye colours of parents</u>	<u>Eye colour of offspring</u>
① both brown	brown
② one brown and one blue	50% brown and 50% blue
③ one brown and one green	blue
④ both blue	25% brown, 50% blue and 25% green
⑤ one blue and one green	50% blue and 50% green
⑥ both green	green

- (i) Give an efficient rule for using 1-digit random numbers to simulate the eye colour of a parent randomly selected from the colonizing population. [3]
- (ii) Give an efficient rule for using 1-digit random numbers to simulate the eye colour of offspring born of parents both of whom have blue eyes. [4]

The table in your answer book shows an incomplete simulation in which parent eye colours have been randomly selected, but in which offspring eye colours remain to be determined or simulated.

- (iii) Complete the table using the given random numbers where needed. (You will need your own rules for cases ② and ⑤.)
Each time you use a random number, explain how you decide which eye colour for the offspring. [9]

- 5 The table shows some of the activities involved in building a block of flats. The table gives their durations and their immediate predecessors.

Activity		Duration (weeks)	Immediate Predecessors
A	Survey sites	8	–
B	Purchase land	22	A
C	Supply materials	10	–
D	Supply machinery	4	–
E	Excavate foundations	9	B, D
F	Lay drains	11	B, C, D
G	Build walls	9	E, F
H	Lay floor	10	E, F
I	Install roof	3	G
J	Install electrics	5	G

- (i) Draw an activity on arc network for these activities. [4]
- (ii) Mark on your diagram the early and late times for each event. Give the minimum completion time and the critical activities. [6]

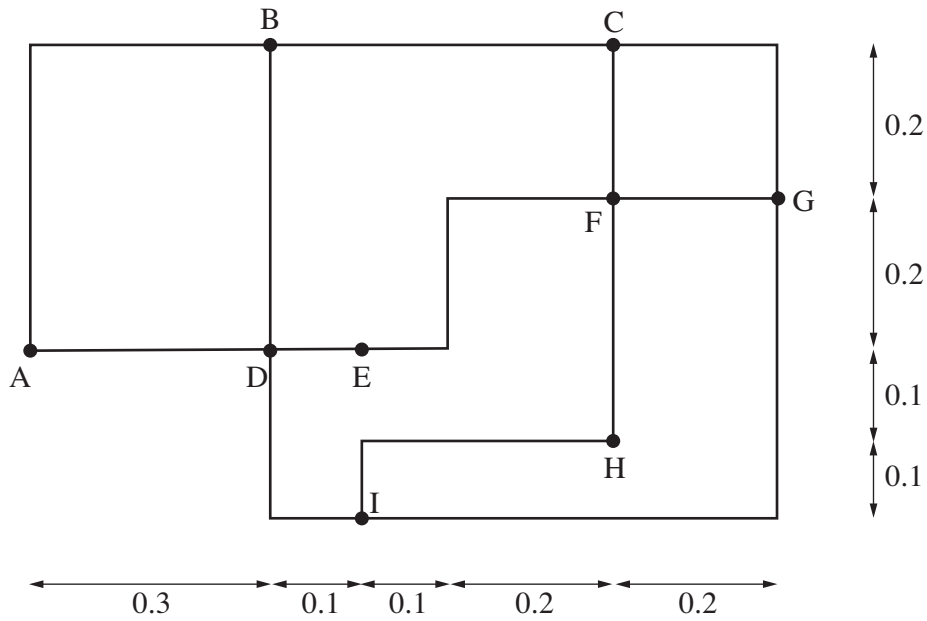
Each of the tasks E, F, H and J can be speeded up at extra cost. The maximum number of weeks by which each task can be shortened, and the extra cost for each week that is saved, are shown in the table below.

Task	E	F	H	J
Maximum number of weeks by which task may be shortened	3	3	1	3
Cost per week of shortening task (in thousands of pounds)	30	15	6	20

- (iii) Find the new shortest time for the flats to be completed. [1]
- (iv) List the activities which will need to be speeded up to achieve the shortest time found in part (iii), and the times by which each must be shortened. [4]
- (v) Find the total extra cost needed to achieve the new shortest time. [1]

[Question 6 is printed overleaf.]

- 6 The diagram shows routes between points in a town. The distances are in kilometres.



- (i) Use an appropriate algorithm to find a set of connecting arcs of minimum total length. Indicate your connecting arcs on the copy of the diagram in your answer book, and give their total length. [7]
- (ii) Give the name of the algorithm you have used, and describe it briefly. [2]
- (iii) Using the second diagram in your answer book, apply Dijkstra's algorithm to find the shortest distances from A to each of the other points.

List the connections that are used, and give their total length. [7]

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**ADVANCED SUBSIDIARY GCE UNIT
 MATHEMATICS (MEI)**

Decision Mathematics 1

PRINTED ANSWER BOOK

TUESDAY 22 JANUARY 2008

4771/01

Afternoon
 Time: 1 hour 30 minutes



Candidate
 Forename

Candidate
 Surname

Centre
 Number

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Candidate
 Number

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INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the boxes above.
- Write your answers in the spaces provided on the printed answer book. If extra paper is required use a 4 page answer booklet making sure that you label your work clearly.

FOR EXAMINER'S USE	
Qu.	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

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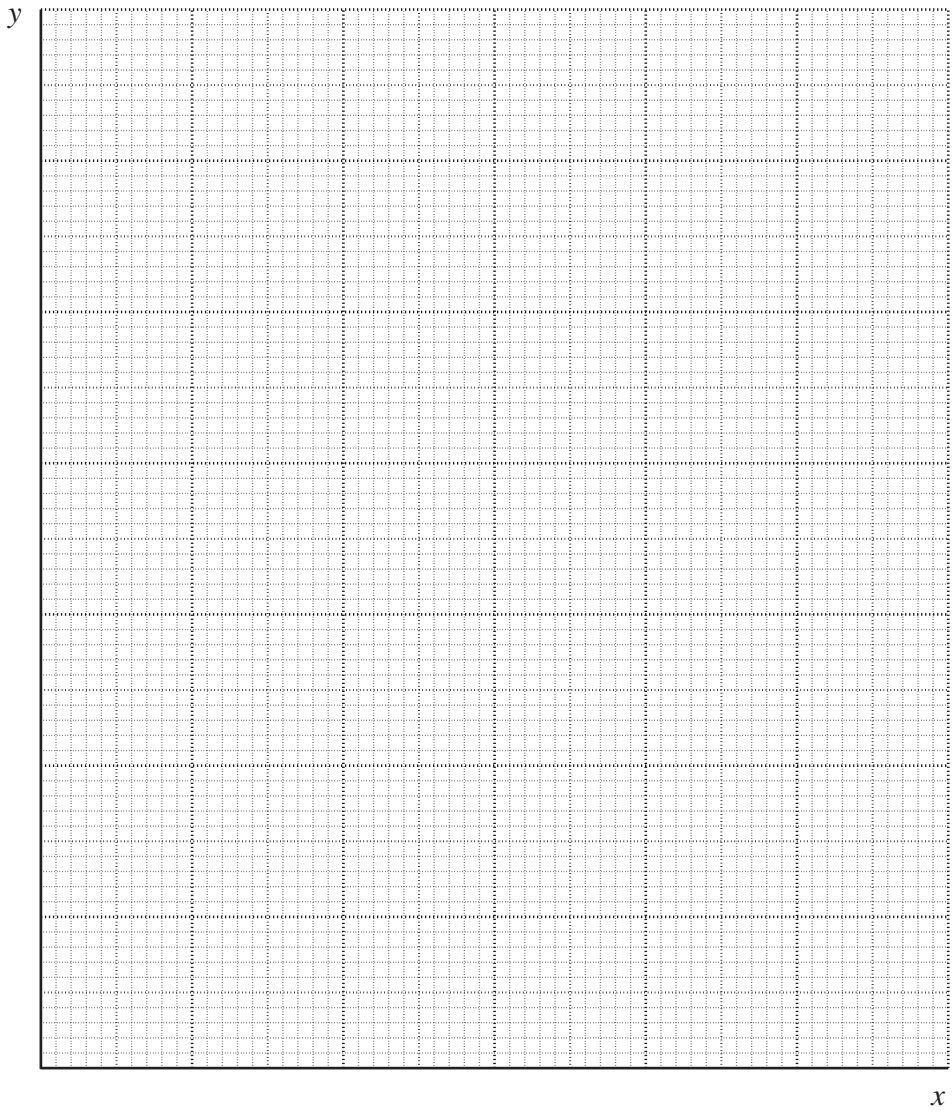
1 (i)

(ii)

(iii)

(iv)

2 (i)



(ii)

$$3 \quad y = 2008$$

$$c = 2008/100 = 20$$

$$n = 2008 - 19 \times (2008/19) = 2008 - 19 \times (105) = 13$$

$$k =$$

$$i =$$

$$i =$$

$$i =$$

$$j =$$

$$j =$$

$$p =$$

$$m =$$

$$d =$$

So Easter Sunday, 2008 will be on

4 (i)

(ii)

(iii) Random numbers: 3 9 0 1 3 5 8 8 0 9 7

Eye colours

Parent 1	brown	brown	brown	blue	brown	green	blue	green	brown	brown
Parent 2	brown	blue	brown	blue	brown	blue	brown	green	brown	green
Offspring										

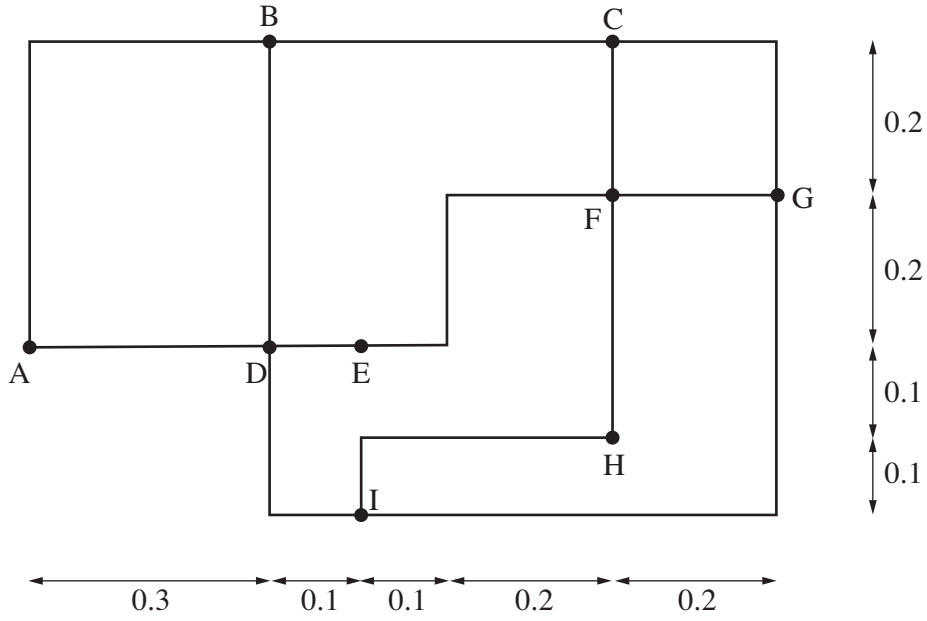
5 (i) & (ii)

(iii)

(iv)

(v)

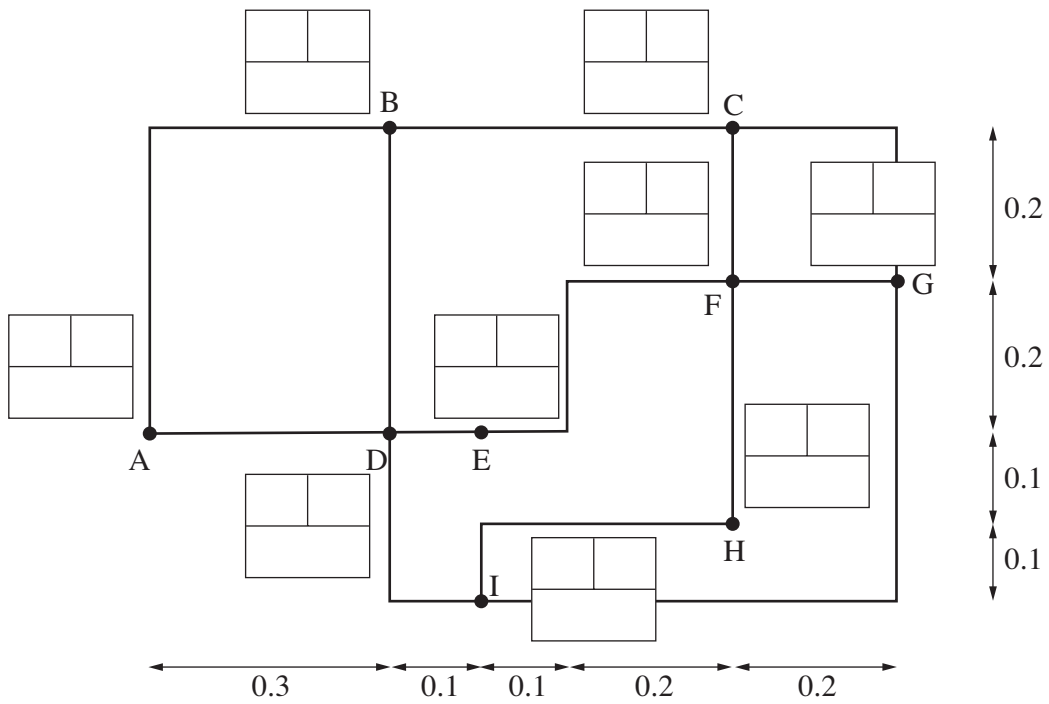
6 (i)



Total length of minimum connector:

(ii)

(iii)

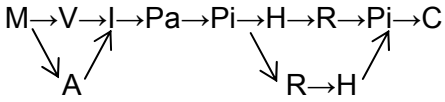


Arcs used:

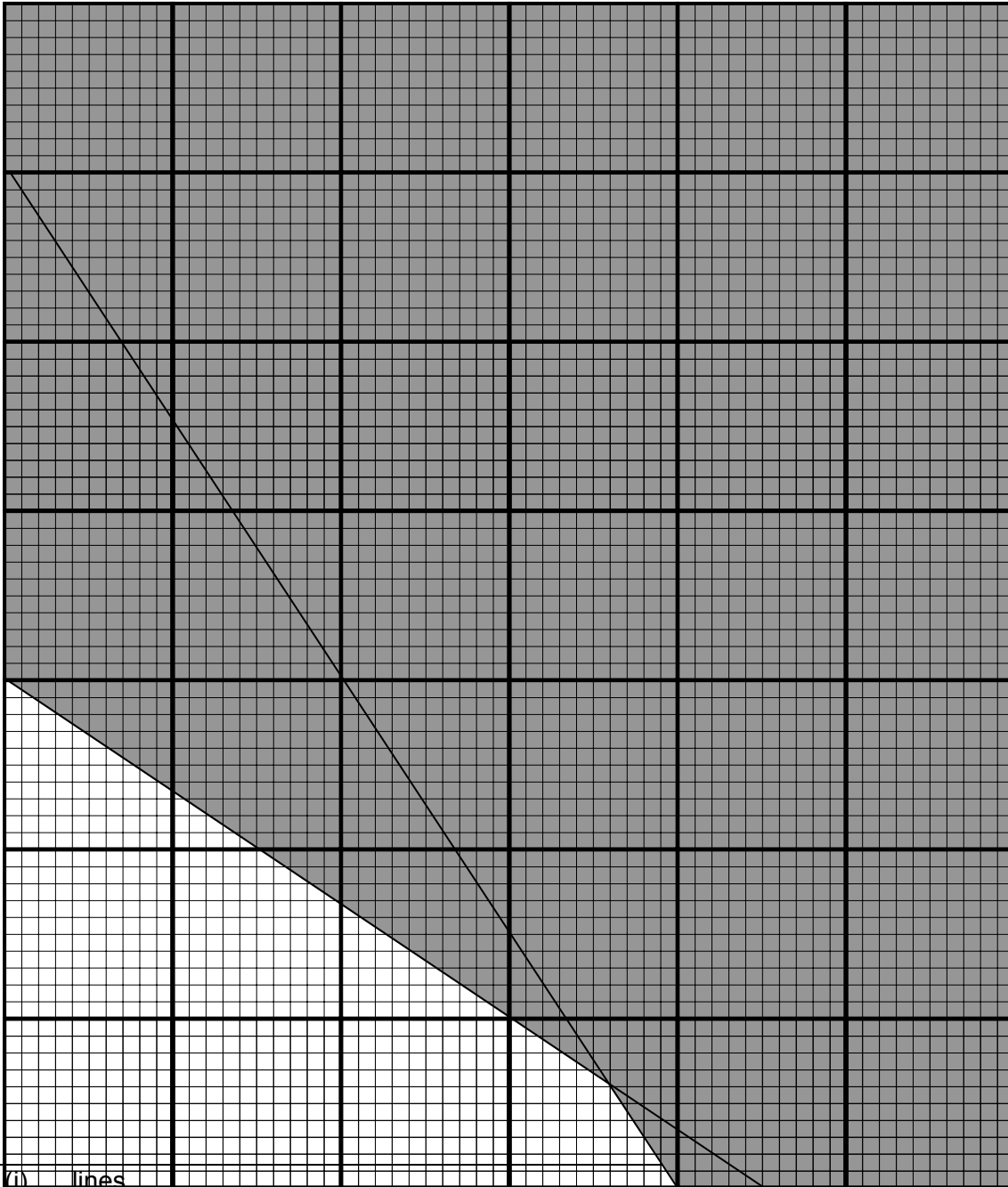
Total length of arcs used:

You may write on this page if required. Make sure you label your work clearly.

1

<p>(i) 6 routes</p> <p>$M \rightarrow A \rightarrow I \rightarrow T \rightarrow Pi \rightarrow C$</p> <p>$M \rightarrow A \rightarrow I \rightarrow T \rightarrow Pi \rightarrow R \rightarrow C$</p> <p>$M \rightarrow A \rightarrow I \rightarrow T \rightarrow Pi \rightarrow H \rightarrow R \rightarrow C$</p> <p>$M \rightarrow V \rightarrow I \rightarrow T \rightarrow Pi \rightarrow C$</p> <p>$M \rightarrow V \rightarrow I \rightarrow T \rightarrow Pi \rightarrow R \rightarrow C$</p> <p>$M \rightarrow V \rightarrow I \rightarrow T \rightarrow Pi \rightarrow H \rightarrow R \rightarrow C$</p>	<p>B1</p> <p>B1</p>
<p>(ii) 6 routes</p> <p>$M \rightarrow A \rightarrow I \rightarrow Pa \rightarrow Pi \rightarrow C$</p> <p>$M \rightarrow A \rightarrow I \rightarrow Pa \rightarrow Pi \rightarrow R \rightarrow C$</p> <p>$M \rightarrow A \rightarrow I \rightarrow Pa \rightarrow Pi \rightarrow H \rightarrow R \rightarrow C$</p> <p>$M \rightarrow V \rightarrow I \rightarrow Pa \rightarrow Pi \rightarrow C$</p> <p>$M \rightarrow V \rightarrow I \rightarrow Pa \rightarrow Pi \rightarrow R \rightarrow C$</p> <p>$M \rightarrow V \rightarrow I \rightarrow Pa \rightarrow Pi \rightarrow H \rightarrow R \rightarrow C$</p>	<p>B1</p> <p>B1</p>
<p>(iii)</p> 	<p>B1</p>
<p>(iv) e.g.</p> <p>$P \rightarrow T \rightarrow I \rightarrow V \rightarrow M \rightarrow A \rightarrow I \rightarrow Pa \rightarrow P \rightarrow H \rightarrow R \rightarrow C \rightarrow P \rightarrow R$</p>	<p>M1 ends at R</p> <p>A2 (-1 each error/omission)</p>

2. y



<p>(i) lines shading (3.6, 0.6) 25.8 at (3.6, 0.6) versus 21 and 24 (or profit line)</p>	<p style="text-align: right;">x</p> <p>B1 B1 graph or sim. eqns M1 A1</p>
<p>(ii) 25 at (3, 1)</p>	<p>B1 B1</p>

3.

$y = 2008$ $c = 2008/100 = 20$ $n = 2008 - 19 \times (2008/19) = 2008 - 19 \times (105) = 13$ $k = 3/25 = 0$ $i = 20 - 5 - 20 / 3 + 19 \times 13 + 15 = 271$ $i = 1$ $i = 1 - 0 = 1$ $j = 2008 + 502 + 1 + 2 - 20 + 5 = 2498$ $j = 6$ $p = -5$ $m = 3$ $d = 23$ So 23 rd March	 B1 B1 B1 B1 B1 B1 B1 B1
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4.

(i) e.g. 0-3→brown 4-7→blue 8-9→green	M1 A1 proportions OK A1 efficient																		
(ii) e.g. 0-1→brown 2-5→blue 6-7→green 8-9→reject	M1 some rejected A2 proportions OK (-1 each error) A1 efficient																		
(iii) e.g.																			
Eye colours																			
<table border="1"> <tbody> <tr> <td>Parent 1</td> <td>brown</td> <td>brown</td> <td>brown</td> <td>blue</td> </tr> <tr> <td>Parent 2</td> <td>brown</td> <td>blue</td> <td>brown</td> <td>blue</td> </tr> <tr> <td>Offspring</td> <td>brown</td> <td>brown</td> <td>brown</td> <td>brown</td> </tr> </tbody> </table>	Parent 1	brown	brown	brown	blue	Parent 2	brown	blue	brown	blue	Offspring	brown	brown	brown	brown	B1 br/br→br (4 times) B1 br/gr→bl B1 gr/gr→gr			
Parent 1	brown	brown	brown	blue															
Parent 2	brown	blue	brown	blue															
Offspring	brown	brown	brown	brown															
<table border="1"> <tbody> <tr> <td>brown</td> <td>green</td> <td>blue</td> <td>green</td> <td>brown</td> <td>brown</td> </tr> <tr> <td>brown</td> <td>blue</td> <td>brown</td> <td>green</td> <td>brown</td> <td>green</td> </tr> <tr> <td>brown</td> <td>blue</td> <td>brown</td> <td>green</td> <td>brown</td> <td>blue</td> </tr> </tbody> </table>	brown	green	blue	green	brown	brown	brown	blue	brown	green	brown	green	brown	blue	brown	green	brown	blue	M1 br/bl rule A1 application A1 application
brown	green	blue	green	brown	brown														
brown	blue	brown	green	brown	green														
brown	blue	brown	green	brown	blue														
	B1 bl/bl application																		
	M1 gr/bl rule A1 application																		

5.

(i)&(ii) e.g.

time – 55 weeks
critical – A; B; F; G; J

(iii) 50 weeks (49 weeks if G crashed rather than H)

(iv) E – 1 week
F – 3 weeks
J – 2 weeks
(G – 1 week, if crashed)

(v) £115000 (£121000)

M1 sca (activity on arc)	
A1 dummy activities + E and F	
A1 A, B, C, D	
A1 G, H, I, J	
M1 forward pass	
A1	
M1 backward pass	
A1	
B1 cao	
B1 cao	
B1	
M1	
A3	
A1	

6.

(i) e.g.

Total length = 2.2 km

(ii) Prim: connect in nearest to connected set
 Kruskal: Shortest arc s.t. no cycles

(iii)

Arcs used: AD, DE, EF, FG, DI, IH, AB or DB, FC or BC
 Total length = 2.7 km (AB&FC) or 2.9 km (AB&BC) or 2.4 km (DB&FC) or 2.6 km (DB&BC)

M1 connecting tree
 A1 DE
 A1 FC, FG
 A1 AD, DI, FH
 A1 2 of length 0.4

M1 A1

M1 name
 A1 description

M1 Dijkstra
 A1 working values (see vertex G)
 A1 order of labelling
 A1 labels

M1 arcs counted
 A1 only once
 A1

4771: Decision Mathematics 1

General Comments

Candidates generally performed well on this paper, with the exception of Question 3. In that question few were able to work through the algorithm.

The prepared answer books are now well-understood and seem to be helpful to candidates.

Comments on Individual Questions

1) Graphs

Most candidates scored well on this question. Not all were able to count the six possibilities in each of parts (i) and (ii). It was pleasing to see most candidates dealing with the semi-Eulerian issue in part (iv).

2) LP

Most candidates were able to draw the graph and identify the feasible region, and many were able to identify the point (3.6, 0.6). However, very many failed to complete their answers to part (i) – they did not solve the problem. It seemed that candidates were distracted from finishing part (i) by the requirements of part (ii).

3) Algorithms

It was rare to see a correct answer to this question. Candidates either were unable to master integer division, or to take due note of brackets, or both. This inability correctly to perform calculations was worrying, and it led to many candidates battling on with obviously silly values.

4) Simulation

(i) (ii) The specification of simulation rules was done well in both parts.

(iii) In the brown/blue cases and the blue/green case candidates needed both to describe their rule and indicate which random number they were using. In the blue/blue case they needed to use their rule from part (ii) and to identify the random number, or numbers, which they used. Whilst many candidates produced complete explanations of their simulations, many others failed to indicate which random numbers were being used.

5) CPA

(i) (ii) Most candidates scored well in parts (i) and (ii). A common difficulty was the failure to use a dummy when two activities would otherwise have the same i node as each other and the same j node as each other.

(iii) It is very much regretted that there was a typographical error in the question,

(iv) (v) with G being printed instead of H in the stem to these parts. Only a few candidates seemed to notice this, and whether H was speeded up, or G, made no difference to the difficulty. Both were allowed in the marking scheme.

6) **Networks**

- (i) Most candidates found, if not the minimum connector, then at least a connecting tree, but a significant minority ended up either with a cycle or with two disconnected sets of arcs. Many made mistakes with the arithmetic.
- (ii) A surprisingly small number of candidates collected both marks in part (ii). There were those who failed to match description to name, and many who “hedged their bets” – e.g. by giving a description of Prim and then adding in for supposed good measure a requirement to avoid cycles. These answers did not gain the second mark.
- (iii) As always, there were many candidates who failed to convince examiners that they were applying Dijkstra – the onus is on the candidate to be convincing. However, the final part was done well more often than we had anticipated.