OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **4772**

Decision Mathematics 2

Wednesday **25 MAY 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

[Turn over

1 The switching circuit in Fig. 1.1 shows switches, s for a car's sidelights, h for its dipped headlights and f for its high-intensity rear foglights. It also shows the three sets of lights.

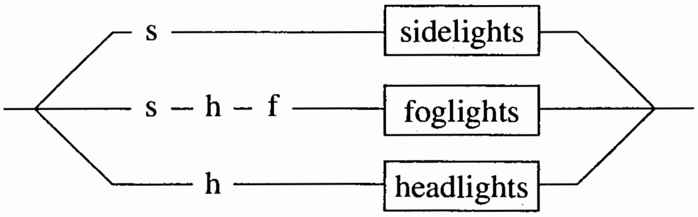


Fig. 1.1

(Note: s and h are each "ganged" switches. A ganged switch consists of two connected switches sharing a single switch control, so that both are either on or off together.)

(a) **(i)** Describe in words the conditions under which the foglights will come on. [2]

Fig. 1.2 shows a combinatorial circuit.

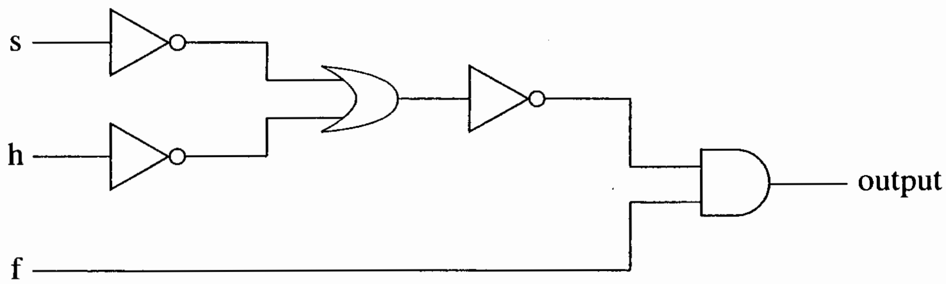


Fig. 1.2

(ii) Write the output in terms of a Boolean expression involving s, h and f. [2]

(iii) Use a truth table to prove that $s \wedge h \wedge f = \sim(\sim s \vee \sim h) \wedge f$. [3]

(b) A car's first gear can be engaged (g) if either both the road speed is low (r) and the clutch is depressed (d), or if both the road speed is low (r) and the engine speed is the correct multiple of the road speed (m).

(i) Draw a switching circuit to represent the conditions under which first gear can be engaged. Use two ganged switches to represent r, and single switches to represent each of d, m and g. [2]

(ii) Draw a combinatorial circuit to represent the Boolean expression $r \wedge (d \vee m) \wedge g$. [4]

(iii) Use Boolean algebra to prove that $r \wedge (d \vee m) \wedge g = ((r \wedge d) \vee (r \wedge m)) \wedge g$. [2]

(iv) Draw another switching circuit to represent the conditions under which first gear can be selected, but without using a ganged switch. [1]

2 Karl is considering investing in a villa in Greece. It will cost him 56 000 euros (€ 56 000). His alternative is to invest his money, £35 000, in the United Kingdom.

He is concerned with what will happen over the next 5 years. He estimates that there is a 60% chance that a house currently worth € 56 000 will appreciate to be worth € 75 000 in that time, but that there is a 40% chance that it will be worth only € 55 000.

If he invests in the United Kingdom then there is a 50% chance that there will be 20% growth over the 5 years, and a 50% chance that there will be 10% growth.

(i) Given that £1 is worth € 1.60, draw a decision tree for Karl, and advise him what to do, using the EMV of his investment (in thousands of euros) as his criterion. [4]

In fact the £/€ exchange rate is not fixed. It is estimated that at the end of 5 years, if there has been 20% growth in the UK then there is a 70% chance that the exchange rate will stand at 1.70 euros per pound, and a 30% chance that it will be 1.50. If growth has been 10% then there is a 40% chance that the exchange rate will stand at 1.70 and a 60% chance that it will be 1.50.

(ii) Produce a revised decision tree incorporating this information, and give appropriate advice. [3]

A financial analyst asks Karl a number of questions to determine his utility function. He estimates that for x in cash (in thousands of euros) Karl's utility is $x^{0.8}$, and that for y in property (in thousands of euros), Karl's utility is $y^{0.75}$.

(iii) Repeat your computations from part **(ii)** using utility instead of the EMV of his investment. Does this change your advice? [3]

(iv) Using EMVs, find the exchange rate (number of euros per pound) which will make Karl indifferent between investing in the UK and investing in a villa in Greece. [2]

(v) Show that, using Karl's utility function, the exchange rate would have to drop to 1.277 euros per pound to make Karl indifferent between investing in the UK and investing in a villa in Greece. [4]

3 The distance and route matrices shown in Fig. 3.1 are the result of applying Floyd's algorithm to the incomplete network on 4 vertices shown in Fig. 3.2.

Distance Matrix

	1	**2**	**3**	**4**
1	4	2	3	9
2	2	2	1	7
3	3	1	2	6
4	9	7	6	12

Route Matrix

	1	**2**	**3**	**4**
1	2	2	2	2
2	1	3	3	3
3	2	2	2	4
4	3	3	3	3

Fig. 3.1

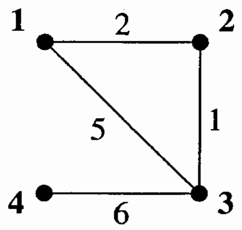


Fig. 3.2

(i) Draw the complete network of shortest distances. [2]

(ii) Explain how to use the route matrix to find the shortest route from vertex 4 to vertex 1 in the original incomplete network. [2]

A new vertex, vertex **5**, is added to the original network. Its distances from vertices to which it is connected are shown in Fig. 3.3.

	1	**2**	**3**	**4**
5	–	3	–	1

Fig. 3.3

(iii) Draw the extended network and the complete 5-node network of shortest distances. (You are not required to use an algorithm to find the shortest distances.) [3]

(iv) Produce the shortest distance matrix and the route matrix for the extended 5-node network. [3]

(v) Apply the nearest neighbour algorithm to your 5×5 distance matrix, starting at vertex **1**. Give the length of the cycle produced, together with the actual cycle in the original 5-node network. [3]

(vi) By deleting vertex **1** and its arcs, and by using Prim's algorithm on the reduced distance matrix, produce a lower bound for the solution to the practical travelling salesperson problem in the original 5-node network. Show clearly your use of the matrix form of Prim's algorithm. [4]

(vii) In the original 5-node network find a shortest route starting at vertex **1** and using each of the 6 arcs at least once. Give the length of your route. [3]

4 Kassi and Theo are discussing how much oil and how much vinegar to use to dress their salad. They agree to use between 5 and 10ml of oil and between 3 and 6ml of vinegar and that the amount of oil should not exceed twice the amount of vinegar.

Theo prefers to have more oil than vinegar. He formulates the following problem to maximise the proportion of oil:

$$\begin{aligned} &\text{Maximise} && \frac{x}{x+y} \\ &\text{subject to} && 0 \leqslant x \leqslant 10, \\ & && 0 \leqslant y \leqslant 6, \\ & && x - 2y \leqslant 0. \end{aligned}$$

(i) Explain why this problem is not an LP. [1]

(ii) Use the simplex method to solve the following LP.

$$\begin{aligned} &\text{Maximise} && x - y \\ &\text{subject to} && 0 \leqslant x \leqslant 10, \\ & && 0 \leqslant y \leqslant 6, \\ & && x - 2y \leqslant 0. \end{aligned}$$

[7]

(iii) Kassi prefers to have more vinegar than oil. She formulates the following LP.

$$\begin{aligned} &\text{Maximise} && y - x \\ &\text{subject to} && 5 \leqslant x \leqslant 10, \\ & && 3 \leqslant y \leqslant 6, \\ & && x - 2y \leqslant 0. \end{aligned}$$

Draw separate graphs to show the feasible regions for this problem and for the problem in part **(ii)**. [5]

(iv) Explain why the formulation in part **(ii)** produced a solution for Theo's problem, and why it is more difficult to use the simplex method to solve Kassi's problem in part **(iii)**. [2]

(v) Produce an initial tableau for using the two-stage simplex method to solve Kassi's problem.

Explain briefly how to proceed. [5]

Mark Scheme 4772
June 2005

Instructions to markers

M marks are for method and are dependent on correct numerical substitution/correct application. Method marks can only be awarded if the method used would have led to the correct answer had not an arithmetic error occurred.
M marks may be awarded following evidence of an **sca** (substantially correct attempt).

M marks can be implied by correct answers.

A marks are for accuracy, and are dependent upon the immediately preceding **M** mark. They cannot be awarded unless the **M** mark is awarded.

B marks are for specific results or statements, and are independent of method.

√ marks are for follow-through. This applies to **A** marks for answers which follow correctly from a previous incorrect result. Whilst mark schemes will occasionally emphasise a follow-through requirement, the default will be to apply follow-through whenever possible. The exception to this are **A** marks which are labelled **cao** (correct answer only).

MR Where a candidate misreads all or part of a question, and where the integrity/difficulty of the question is not affected, a penalty (of –1, –2 or –3) can be applied (according to the extent of the work affected), and the question marked as read.
Note that it is **not** a misread if a candidate makes an error in copying his own work.

SC special case

1.

Answer	Marks
(a)	
(i) If sidelights and headlights are on, and if the foglights are switched on.	B1 B1
(ii) $\sim(\sim s \vee \sim h) \wedge f$	M1 A1
(iii) see truth table below	M1 8 rows A1 $s \wedge h \wedge f$ A1 $\sim(\sim s \vee \sim h) \wedge f$

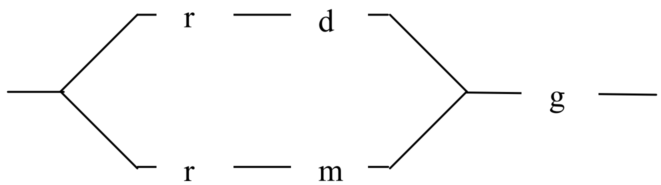
(s	∧	h)	∧	f	⇔	~	(~	s	∨	~	h)	∧	f
0	0	0	0	0	1	0	1	0	1	1	0	0	0
0	0	0	0	1	1	0	1	0	1	1	0	0	1
0	0	1	0	0	1	0	1	0	1	0	1	0	0
0	0	1	0	1	1	0	1	0	1	0	1	0	1
1	0	0	0	0	1	0	0	1	1	1	0	0	0
1	0	0	0	1	1	0	0	1	1	1	0	0	1

Accept t/table showing $s \wedge h = \sim(\sim s \vee \sim h)$

alternative: B1 comment re $\wedge f$; M1 4 lines; A1

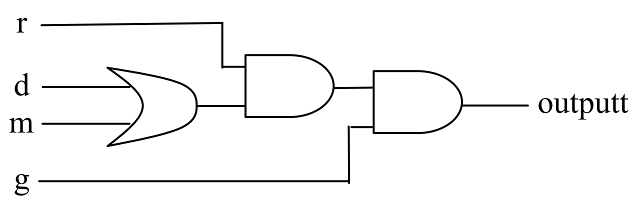
(b)

(i)



M1
A1

(ii)



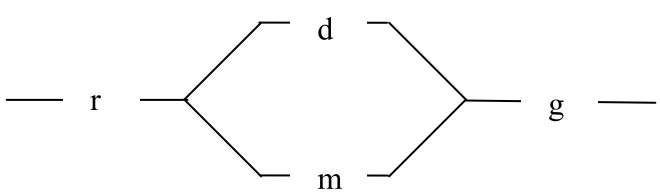
M1
A1 "or"
A1 first "and"
A1 second "and"

(iii) $r \wedge (d \vee m) \wedge g = (r \wedge (d \vee m)) \wedge g$ by associativity

$= ((r \wedge d) \vee (r \wedge m)) \wedge g$ by distributivity

M1 distributive law
A1 handling brackets
(law names not needed)

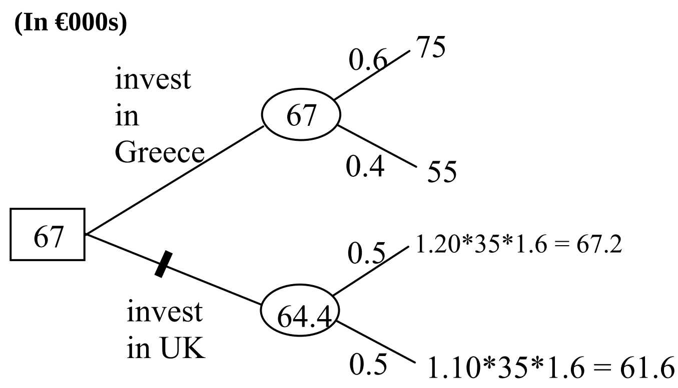
(iv)



B1

2.

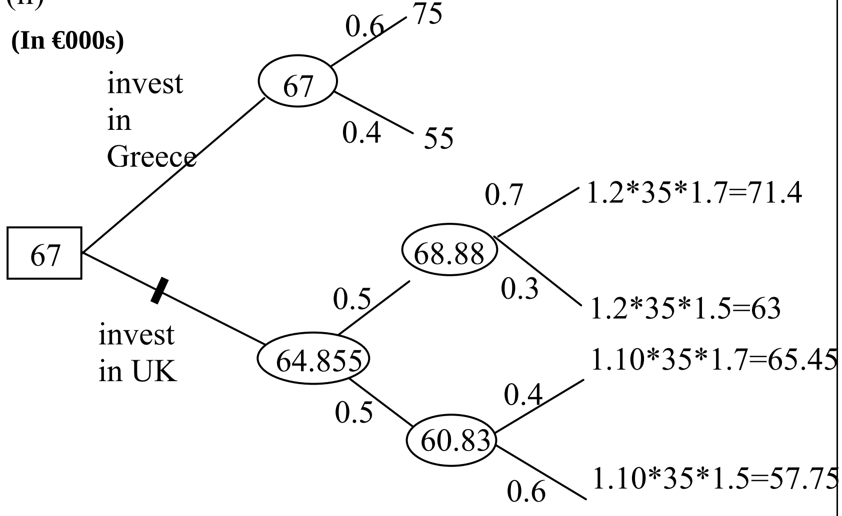
(i)



M1
A1 chance nodes
A1 choice node

B1√ invest in Greece

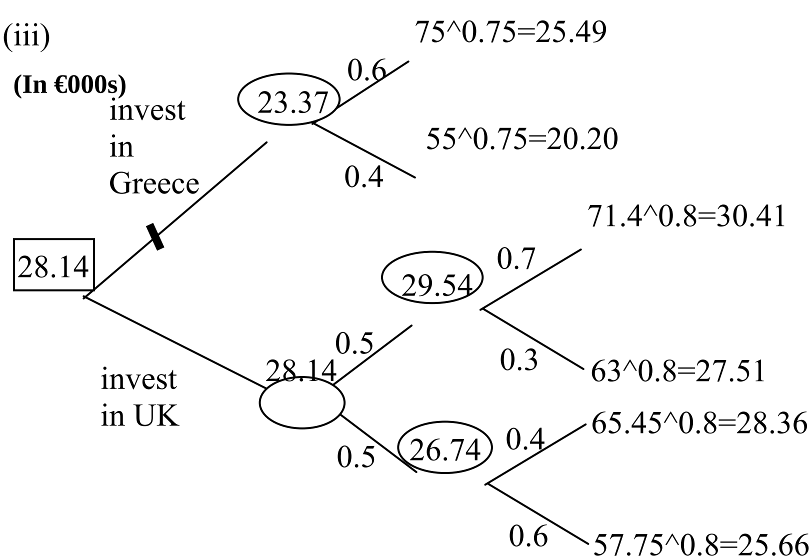
(ii)



M1 new chance nodes
A1 64.855 or .86 or .85

B1 invest in Greece

(iii)



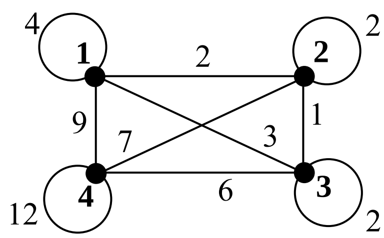
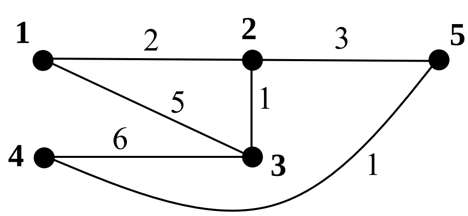
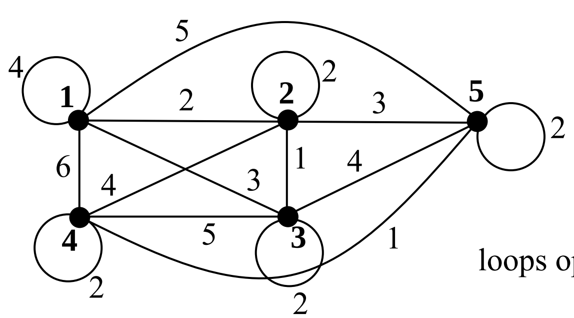
M1 utilities
A1 23.37 and 28.14

B1 invest in UK

2 (cont)

(iv) Require $\frac{1.2+1.1}{2} \times 35 \times x = 67$,	M1
giving x = 1.665	A1 cao
(v) Require $\frac{(1.2 \times 35 \times y)^{0.8} + (1.1 \times 35 \times y)^{0.8}}{2} = 23.37$.	M1 cash M1 house
Trying y = 1.277:	
$(1.2 \times 35 \times 1.277)^{0.8} = 24.185$ $(1.1 \times 35 \times 1.277)^{0.8} = 22.559$	A1 one bracket evaluated correctly
(24.185+22.559)/2 = 23.37	A1

3.

(i)  loops optional	M1 A1
(ii) First vertex en route is 3. First vertex en route from 3 to 1 is 2. First vertex en route from 2 to 1 is 1.	M1 A1
(iii) 	B1
 loops optional	M1 A1

(iv)

	1	2	3	4	5
1	4	2	3	6	5
2	2	2	1	4	3
3	3	1	2	5	4
4	6	4	5	2	1
5	5	3	4	1	2

	1	2	3	4	5
1	2	2	2	2	2
2	1	3	3	5	5
3	2	2	2	2	2
4	5	5	5	5	5
5	2	2	2	4	4

B1 distance matrix

M1 route matrix
A1 cao

(v) 1 2 3 5 4 1
14
1 2 3 2 5 4 5 2 1

M1
A1
A1

(vi)

	1	2	3	4	5
		1	2	4	3
1	4	2	3	6	5
2	2	2	1	4	3
3	3	(1)	2	5	4
4	6	4	5	2	(1)
5	5	(3)	4	1	2

Lower bound is 5 + 2 + 3 = 10

M1 Prim on matrix
A1

B1 B1

(vii) e.g.
1 2 5 4 <u>3 2 3</u> 1
19

M1 A1 cao
B1

4.

(i) The objective is nonlinear. — B1

(ii)

P	x	y	S1	S2	S3	RHS
1	–1	1	0	0	0	0
0	1	0	1	0	0	10
0	0	1	0	1	0	6
0	1	–2	0	0	1	0
1	0	–1	0	0	1	0
0	0	2	1	0	–1	10
0	0	1	0	1	0	6
0	1	–2	0	0	1	0
1	0	0	1/2	0	1/2	5
0	0	1	1/2	0	–1/2	5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

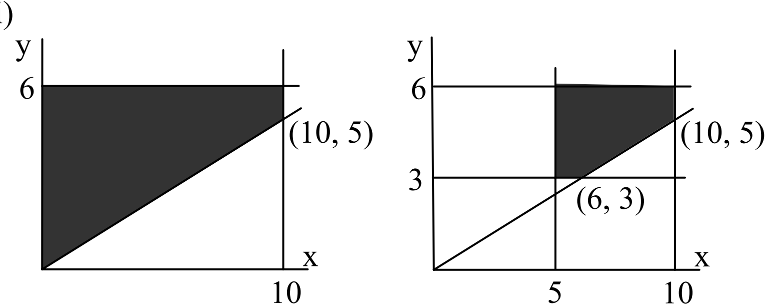
M1 tableau
A1

M1 pivot choice
A1 pivot

M1 pivot choice
A1 pivot

10 ml of oil and 5 ml of vinegar — B1

(iii)



B1 x ≤ 10 and y ≤ 6
B1 5 ≤ x and 3 ≤ y

B1 proportion line

B1 region 1
B1 region 2

(iv) Omitted constraints non-active — B1
(0, 0) not in feasible region. — B1

(v)

C	P	x	y	s1	s2	s3	s4	s5	a1	a2	RHS
1	0	1	1	0	–1	0	–1	0	0	0	8
0	1	1	–1	0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0	10
0	0	1	0	0	–1	0	0	0	1	0	5
0	0	0	1	0	0	1	0	0	0	0	6
0	0	0	1	0	0	0	–1	0	0	1	3
0	0	1	–2	0	0	0	0	1	0	0	0

B1 > constraints

B1 artificial columns

B1 new objective

Minimise C, hopefully to zero. — B1
Thereafter delete C row and a1/a2 columns, and proceed as usual. — B1

4772 - Decision Mathematics 2

General Comments

This was the first presentation of the new unit. Questions were extended versions of those set for 2621. Performances were clustered towards the mean, with few very poor performances and few very high scores. Whilst most candidates attempted all 4 questions there was some evidence of time pressure.
It was clear that candidates had been well prepared for the paper.

Comments on Individual Questions

1) **Logic**

This question was answered well. Only part (iii) caused any difficulty. Some candidates thought it so obvious that they could not see what needed to be written down.

2) **Decision Analysis**

Most candidates were able to complete part (i) and gain at least some credit on part (ii). Few gained much on part (iii) however, the concept of utility completely passing most of the candidates by.
This question also revealed a significant difficulty in work on Decision Analysis. Alternative approaches are possible to the accounting, but some have the potential for causing problems. The safest is to work with final payoffs. Thus in part (i) candidates who worked with profits came to the correct answer with effectively the same computations as those using payoffs, but that was not the case in part (ii). The problem here is that, if r(t) is the exchange rate and v(t) is the value of the investment, then

$$r(t) \times (v(t) - v(0)) \neq (r(t) \times v(t)) - (r(0) \times v(0))$$

The left hand side of the above expression is what many candidates used – it results from working with profits. The right hand side is correct, and is consistent with the answer obtained by working with payoffs.
Whilst this error is not obvious, working with profits rather than payoffs in part (iii) is a fundamental mistake. Utility functions give the utilities of positions not changes.
The definition of the utility function created some problems ("… **thousands** of euros."), but those using euros instead of thousands of euros were not heavily penalised.
Part (iv) was answered by few, and part (v) by very few.

3) **Networks**

Most candidates found some success in this question. A few went into knee-jerk routine in part (iii) and attempted to apply Floyd, wasting quite a lot of time in the process.

4) **LP**

As in Q2, this question revealed a fundamental flaw in candidates' approaches to part of the question. A majority chose the wrong first pivot in part (ii). Choosing the wrong pivot always leads to a negative element appearing in the last column. It is that which the ratio test, when applied correctly, avoids. Candidates making this error carried on with their negative RHS, blissfully unaware that there was a problem.

Another slight difficulty in the question occurred in part (iv). The first mark here was asking why it is that Theo's formulation, though incomplete, leads to the correct solution. The answer looked for was that the constraints he omitted are (clearly) not active at the solution. Candidates did not recognise the issue.

Apart from those two difficulties, and the fact that some candidates were short of time for this last question, it was answered well.