RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4772

Decision Mathematics 2

Monday **19 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- There is an **insert** for use in Question **2**.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages and an insert.

1 **(i)** Use a truth table to prove $\sim(\sim T \Rightarrow \sim S) \Leftrightarrow (\sim T \wedge S)$. [8]

(ii) Prove that $(A \Rightarrow B) \Leftrightarrow (\sim A \vee B)$ and hence use Boolean algebra to prove that

$$\sim(\sim T \Rightarrow \sim S) \Leftrightarrow (\sim T \wedge S).$$ [5]

(iii) A teacher wrote on a report "It is not the case that if Joanna doesn't try then she won't succeed." He meant to say that if Joanna were to try then she would have a chance of success. By letting T be "Joanna will try" and S be "Joanna will succeed", find the real meaning of what the teacher wrote. [3]

2 **Answer this question on the insert provided.**

Fig. 2 shows a network in which the weights on the arcs represent distances.

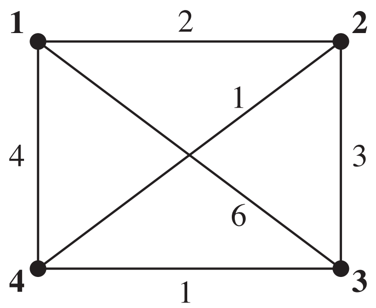

Fig. 2

(i) Apply Floyd's algorithm on the insert provided to find the complete network of shortest distances. [8]

(ii) Show how to use your final matrices to find the shortest route from vertex **1** to vertex **3**, together with the length of that route. [4]

(iii) Use the nearest neighbour algorithm, starting at vertex **1**, to find a Hamilton cycle in the complete network of shortest distances.

Give the corresponding cycle in the original network, together with its length. [4]

3 Emma has won a holiday worth £1000. She is wondering whether or not to take out an insurance policy which will pay out £1000 if she should fall ill and be unable to go on the holiday. The insurance company tells her that this happens to 1 in 200 people. The insurance policy costs £10. Thus Emma's monetary value if she buys the insurance and does not fall ill is £990.

(i) Draw a decision tree for Emma's problem. Use the EMV criterion in your calculations. [6]

(ii) Interpret your tree and say what the maximum cost of the insurance would have to be for Emma to consider buying it if she uses the EMV criterion. [2]

Suppose that Emma's utility function is given by $utility = \sqrt[3]{monetary\ value}$.

(iii) Using expected utility as the criterion, should Emma purchase the insurance?

Under this criterion what is the cost at which she will be indifferent to buying or not buying it? [3]

Emma could pay for a blood pressure check to help her to make her decision. Statistics show that 75% of checks are positive, and that when a check is positive the chance of missing a holiday through ill heath is 0.001. However, when a check is negative the chance of cancellation through ill health is 0.017.

(iv) Draw a decision tree to help Emma decide whether or not to pay for the check. Use EMV, not expected utility, in your calculations and assume that the insurance policy costs £10.

What is the maximum amount that she should pay for the blood pressure check? [9]

[Question 4 is printed overleaf.]

4 The "Cuddly Friends Company" produces soft toys. For one day's production run it has available $11\,\text{m}^2$ of furry material, $24\,\text{m}^2$ of woolly material and 30 glass eyes. It has three soft toys which it can produce:

The "Cuddly Aardvark", each of which requires $0.5\,\text{m}^2$ of furry material, $2\,\text{m}^2$ of woolly material and two eyes. Each sells at a profit of £3.

The "Cuddly Bear", each of which requires $1\,\text{m}^2$ of furry material, $1.5\,\text{m}^2$ of woolly material and two eyes. Each sells at a profit of £5.

The "Cuddly Cat", each of which requires $1\,\text{m}^2$ of furry material, $1\,\text{m}^2$ of woolly material and two eyes. Each sells at a profit of £2.

An analyst formulates the following LP to find the production plan which maximises profit.

$$\begin{aligned} \text{Maximise} \quad & 3a + 5b + 2c \\ \text{subject to} \quad & 0.5a + b + c \leqslant 11, \\ & 2a + 1.5b + c \leqslant 24, \\ & 2a + 2b + 2c \leqslant 30. \end{aligned}$$

(i) Explain how this formulation models the problem, and say why the analyst has not simplified the last inequality to $a + b + c \leqslant 15$. [4]

(ii) The final constraint is different from the others in that the resource is integer valued. Explain why that does not impose an additional difficulty for this problem. [1]

(iii) Solve this problem using the simplex algorithm.

Interpret your solution and say what resources are left over. [9]

On a particular day an order is received for two Cuddly Cats, and the extra constraint $c \geqslant 2$ is added to the formulation.

(iv) Set up an initial simplex tableau to deal with the modified problem using either the big-M approach or two-phase simplex. Do not perform any iterations on your tableau. [3]

(v) Show that the solution given by $a = 8$, $b = 2$ and $c = 5$ uses all of the resources, but that $a = 6$, $b = 6$ and $c = 2$ gives more profit.

What resources are left over from the latter solution? [3]

Candidate Name	Centre Number	Candidate Number

OCR RECOGNISING ACHIEVEMENT

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MEI STRUCTURED MATHEMATICS 4772

Decision Mathematics 2

INSERT

Monday **19 JUNE 2006** Morning 1 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- This **insert** should be used in Question **2.**
- Write your name, centre number and candidate number in the spaces provided at the top of this page and attach it to your answer booklet.

This insert consists of 2 printed pages.

2 **(i)**

Initial matrices

Distance matrix

	1	2	3	4
1				
2				
3				
4				

Route matrix

	1	2	3	4
1				
2				
3				
4				

Iteration 1

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

Iteration 2

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

Iteration 3

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

Iteration 4

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

(ii) ____

____

(iii) ____

____

Mark Scheme 4772
June 2006

1.

(i)

~ (	~	T	⇒	~	S)	⇔	~	T	∧	S
0	1	0	1	1	0	1	1	0	0	0
1	1	0	0	0	1	1	1	0	1	1
0	0	1	1	1	0	1	0	1	0	0
0	0	1	1	0	1	1	0	1	0	1

M1 4 lines
A1 T and S
A1 ~T (twice) and ~S
A1 ⇒
A1 ∧
A1 ~on LHS
M1
A1 result

(ii)

A	⇒	B	⇔	~	A	∨	B
0	1	0	1	1	0	1	0
0	1	1	1	1	0	1	1
1	0	0	1	0	1	0	0
1	1	1	1	0	1	1	1

or a correct verbal argument

M1
A1

$$\sim(\sim T \Rightarrow \sim S) \Leftrightarrow \sim(T \vee \sim S) \Leftrightarrow \sim T \wedge S$$

M1 Boolean
A1 applying result
A1 correct negating

(iii) Joanna will not try and will succeed

B1 not try
B1 and
B1 succeed

2.

(i)

	1	2	3	4
1	∞	2	6	4
2	2	∞	3	1
3	6	3	∞	1
4	4	1	1	∞

	1	2	3	4
1	1	2	3	4
2	1	2	3	4
3	1	2	3	4
4	1	2	3	4

M1 sca Floyd
A1 distance
A1 route

	1	2	3	4
1	∞	2	6	4
2	2	4	3	1
3	6	3	12	1
4	4	1	1	8

	1	2	3	4
1	1	2	3	4
2	1	1	3	4
3	1	2	1	4
4	1	2	3	1

A1

	1	2	3	4
1	4	2	5	3
2	2	4	3	1
3	5	3	6	1
4	3	1	1	2

	1	2	3	4
1	2	2	2	2
2	1	1	3	4
3	2	2	2	4
4	2	2	3	2

A1

	1	2	3	4
1	4	2	5	3
2	2	4	3	1
3	5	3	6	1
4	3	1	1	2

	1	2	3	4
1	2	2	2	2
2	1	1	3	4
3	2	2	2	4
4	2	2	3	2

A1 no change

A1 circled element
A1 rest

	1	2	3	4
1	4	2	4	3
2	2	2	2	1
3	4	2	2	1
4	3	1	1	2

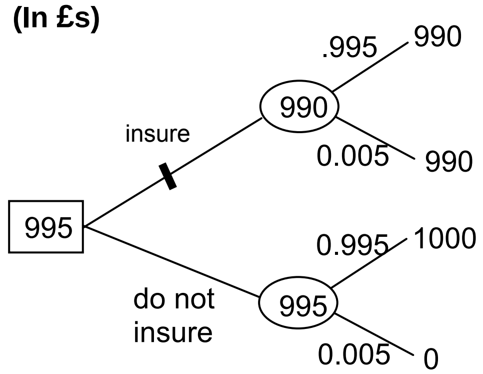
	1	2	3	4
1	2	2	(2)	2
2	1	4	4	4
3	4	4	4	4
4	2	2	3	2

(ii) distance = 4 (row 1, col 3 of dist matrix) — M1 A1
route = 1, 2, 4, 3 (1 – r1c3 – r2c3 – r4c3 of route matrix) — M1 A1

(iii) **1**, **2**, **4**, **3**, **1** — B1
1, **2**, **4**, **3**, **4**, **2**, **1** — M1 A1
8 — B1

3.

(i) **(In £s)**



M1 pay–offs
A1

M1 chance nodes
A1

M1 decision node
A1

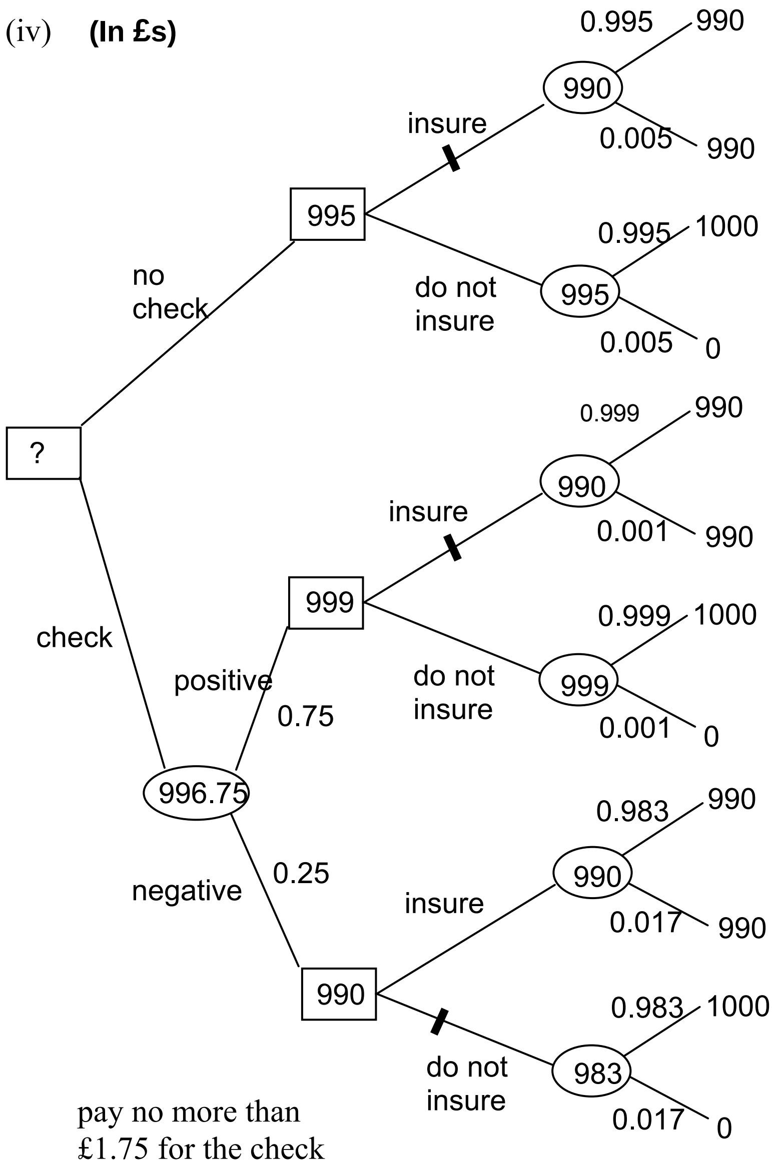
(ii) Do not insure. — B1

Pay no more than £5 for it. — B1

(iii) Yes $\left(\left(\sqrt[3]{990}\times(0.995+0.005)\right) v \left(0.995\times\sqrt[3]{1000}\right)\right)$ — B1

$\sqrt[3]{1000-x} = 9.95$ giving $x = £14.93$ — M1 A1

(iv) **(In £s)**



pay no more than
£1.75 for the check

M1 check/no check
A1

M1 positive/negative
A1

M1 insure/not insure
A1

M1 go/no go
A1

B1

4. (i) a is the number of aardvarks, etc. — B1
First inequality models the furry material constraint — M1
Second inequality models the woolly material constraint — A1
Third inequality models the glass eyes constraint

That would model a "pairs of glass eyes" constraint. — B1

(ii) The problem is an IP, so the number of eyes used will be integer anyway. — B1

(iii) e.g.

P	a	b	c	s1	s2	s3	RHS
1	−3	–5	–2	0	0	0	0
0	0.5	1	1	1	0	0	11
0	2	1.5	1	0	1	0	24
0	2	2	2	0	0	1	30
1	–0.5	0	3	5	0	0	55
0	0.5	1	1	1	0	0	11
0	1.25	0	-0.5	–1.5	1	0	7.5
0	1	0	0	–2	0	1	8
1	0	0	2.8	4.4	0.4	0	58
0	0	1	1.2	1.6	–0.4	0	8
0	1	0	–0.4	–1.2	0.8	0	6
0	0	0	0.4	0.8	0.8	1	2

M1, A1 (initial tableau)
M1 pivot choice, A1 pivot (second tableau)
M1 pivot choice, A1 pivot (third tableau)

Make 6 aardvarks and 8 bears giving £58 profit. — B1 B1
2 eyes are left over. — B1

(iv)

P	a	b	c	s1	s2	s3	su4	a	RHS
1	−3	–5	–(2+M)	0	0	0	M	0	–2M
0	0.5	1	1	1	0	0	0	0	11
0	2	1.5	1	0	1	0	0	0	24
0	2	2	2	0	0	1	0	0	30
0	0	0	1	0	0	0	–1	1	2

or

C	P	a	b	c	s1	s2	s3	su4	a	RHS
1	0	0	0	1	0	0	0	–1	0	2
0	1	−3	–5	–2	0	0	0	0	0	0
0	0	0.5	1	1	1	0	0	0	0	11
0	0	2	1.5	1	0	1	0	0	0	24
0	0	2	2	2	0	0	1	0	0	30
0	0	0	0	1	0	0	0	–1	1	2

B1 new constraint
M1 objective
A1

(v) $8\times0.5 + 2\times1 + 5\times1 = 11$
$8\times2 + 2\times1.5 + 5\times1 = 24$ — B1
$8\times2 + 2\times2 + 5\times2 = 30$

$3\times8 + 5\times2 + 2\times5 = 44$ but $3\times6 + 5\times6 + 2\times2 = 52$ — B1
1 m^2 of woolly material and 2 eyes left. — B1

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General Comments

Performances on this paper were mostly at least good. Candidates were able and well-prepared.

Comments on Individual Questions

Q 1 **Logic**

Most candidates collected the first 10 marks. After that the difficulty level rose in the second requirement of part (ii), but a pleasing number still managed a sophisticated two step proof. Against that, far too many candidates started with what they were required to prove and deduced a true statement, the "howler" of logic.
In part (iii) very many candidates insisted on inserting their own linguistic flourishes, and often revealed their own prejudices.

Q 2 **Networks**

This question worked well in that candidates who did make slips, understandable in applying Floyd, were able to recover.
Not much space was provided on the insert for parts (ii) and (iii), since long essays were not required. However, long essays were often provided!
The most difficult aspect of part (i) was ending with a "2" in the first row and third column of the route matrix. In part (ii) this gives the "2" in the route 1–2–4–3. Many candidates, as expected, ended up with a "4" in that position of the route matrix, yet still gave the correct route in part (ii). Markers, however, followed through the incorrect "4" to the route 1–4–3 in part (ii).

Q 3 **Decision Analysis**

Many candidates worked with very strange payoffs in part (i), sometimes delivering negative EMVs. Many also thought that finding the value and marking the decision at the decision node on their diagram in part (i) provided an answer to the interpretation part of part (ii). It did not.
More candidates than has been the case in the past were able to handle utilities in part (iii), though there were still some who applied the utility function to calculated EMVs instead of to payoffs.
There were some good answers seen to part (iv), but some candidates got into a terrible tangle with the order of their branching.
Some candidates overcomplicated their computations in part (iii) in allowing for the cost of a blood pressure test by subtracting "x" from the relevant payoffs. Had the computations involved utilities that would have been necessary, but in the question as posed all that was needed was the difference in EMVs between having the check and not having the check.

Q 4 **LP**

Candidates were very well prepared for this question, with many succeeding in accurate implementations of the simplex algorithm.
In part (i) too many candidates echoed the error seen often in D1 – failing to identify variables.
The question as to why the analyst did not simplify the "eyes" inequality turned out to be the most difficult on the paper. Very few answered it correctly. Many thought that dividing both sides by two would allow toys to be provided with an odd number of eyes.
Contrastingly it was very pleasing to see quite a number of candidates correctly answering part (ii) – examiners had thought that this would be as difficult.
As mentioned above, the algorithm was well applied in part (iii), but interpretation was less good. Listing the values taken by the variables does not constitute interpretation – examiners needed to know what was to be made at what profit, and what would be left over. Similar comments could be applied to part (v), though marking was more lenient at that point.
Part (iv) was answered well, although it seemed to consume quite a lot of energy in setting it up.