

**ADVANCED GCE UNIT
MATHEMATICS (MEI)**

Decision Mathematics 2

MONDAY 16 JUNE 2008

4772/01

Afternoon
Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)



INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

- 1 (a) The Plain English Society presents an annual “Foot in Mouth” award for “a truly baffling comment”. In 2004 it was presented to Boris Johnson MP for a comment on the 12th December 2003 edition of “Have I Got News For You”:

“I could not fail to disagree with you less.”

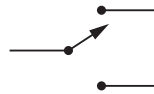
- (i) Explain why this can be rewritten as:

“I could succeed in agreeing with you more.” [2]

- (ii) Rewrite the comment more simply in your own words without changing its meaning. [2]

- (b) Two switches are to be wired between a mains electricity supply and a light so that when the state of either switch is changed the state of the light changes (i.e. from off to on, or from on to off). Draw a switching circuit to achieve this.

The switches are both 2-way switches, thus:



[5]

- (c) Construct a truth table to show the following.

$$[(a \wedge b) \vee ((\sim a) \wedge (\sim b))] \Leftrightarrow [((\sim a) \vee b) \wedge (a \vee (\sim b))] \quad [7]$$

- 2 Jane has a house on a Mediterranean island. She spends eight weeks a year there, either visiting twice for four weeks each trip or four times for two weeks each trip. Jane is wondering whether it is best for her to fly out and rent a car, or to drive out.

Flights cost £500 return and car rental costs £150 per week.

Driving out costs £900 for ferries, road tolls, fuel and overnight expenses.

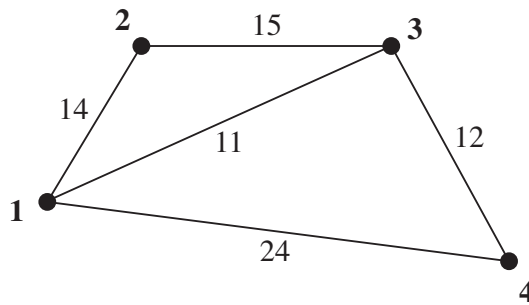
- (i) Draw a decision tree to model this situation. Advise Jane on the cheapest option. [6]

As an alternative Jane considers buying a car to keep at the house. This is a long-term alternative, and she decides to cost it over 10 years. She has to cost the purchase of the car and her flights, and compare this with the other two options.

In her costing exercise she decides that she will not be tied to two trips per year nor to four trips per year, but to model this as a random process in which she is equally likely to do either.

- (ii) Draw a decision tree to model this situation. Advise Jane on how much she could spend on a car using the EMV criterion. [8]
- (iii) Explain what is meant by “the EMV criterion” and state an alternative approach. [2]

3 The weights on the network represent distances.



- (a) (i) Apply Floyd's algorithm to the network to find the complete network of shortest distances, showing that the final matrices are as follows.

	1	2	3	4
1	22	14	11	23
2	14	28	15	27
3	11	15	22	12
4	23	27	12	24

	1	2	3	4
1	3	2	3	3
2	1	1	3	3
3	1	2	1	4
4	3	3	3	3

Draw the complete network of shortest distances. [8]

- (ii) Starting at vertex **1**, apply the nearest neighbour algorithm to the complete network of shortest distances to find a Hamilton cycle. Give the length of your cycle and interpret it in the original network. [4]
- (iii) By temporarily deleting vertex **1** and its connecting arcs from the complete network of shortest distances, find a lower bound for the solution to the Travelling Salesperson's Problem in that network. Say what this implies in the original network. [4]
- (b) Solve the route inspection problem in the **original** network, and say how you can be sure that your solution is optimal. [4]

[Question 4 is printed overleaf]

- 4 A factory's output includes three products. To manufacture a tonne of product A, 3 tonnes of water are needed. Product B needs 2 tonnes of water per tonne produced, and product C needs 5 tonnes of water per tonne produced.

Product A uses 5 hours of labour per tonne produced, product B uses 6 hours and product C uses 2 hours.

There are 60 tonnes of water and 50 hours of labour available for tomorrow's production.

- (i) Formulate a linear programming problem to maximise production within the given constraints. [5]
- (ii) Use the simplex algorithm to solve your LP, **pivoting first on your column relating to product C**. [7]
- (iii) An extra constraint is imposed by a contract to supply at least 8 tonnes of A. Use either two stage simplex or the big M method to solve this revised problem. [8]

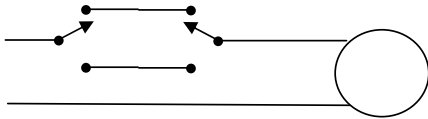
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1.

(a)(i) "not fail" \rightarrow "succeed"
 "disagree less" \rightarrow "agree more"

(ii) e.g. "I don't entirely agree with you".

(b) e.g.



(c)

$((a \wedge b) \vee (\sim a \wedge \sim b)) \Leftrightarrow ((\sim a \vee b) \wedge (a \vee \sim b))$	a	b	$\sim a$	$\sim b$	$(a \wedge b)$	$(\sim a \wedge \sim b)$	$((\sim a \vee b) \wedge (a \vee \sim b))$
1	1	1	0	0	1	0	1
1	0	0	0	1	0	1	1
0	0	1	1	0	0	0	0
0	0	0	1	1	0	1	1

B1
 B1

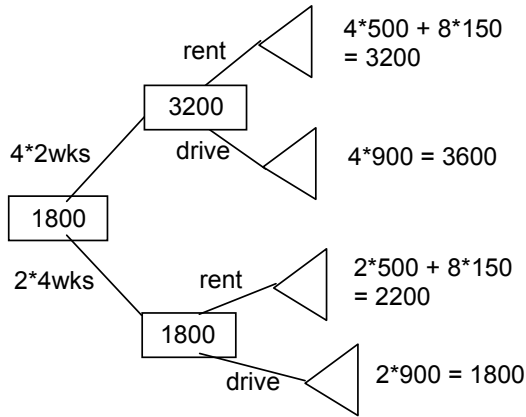
M1 same meaning
 A1 simpler

M1 2 switches +
 light in a circuit
 A4 one for each
 correct setting

M1 4 lines
 A1 a's and b's
 A1 negations
 A1 level 1 and's
 A1 level 1 or's
 A1 level 2
 A1 result

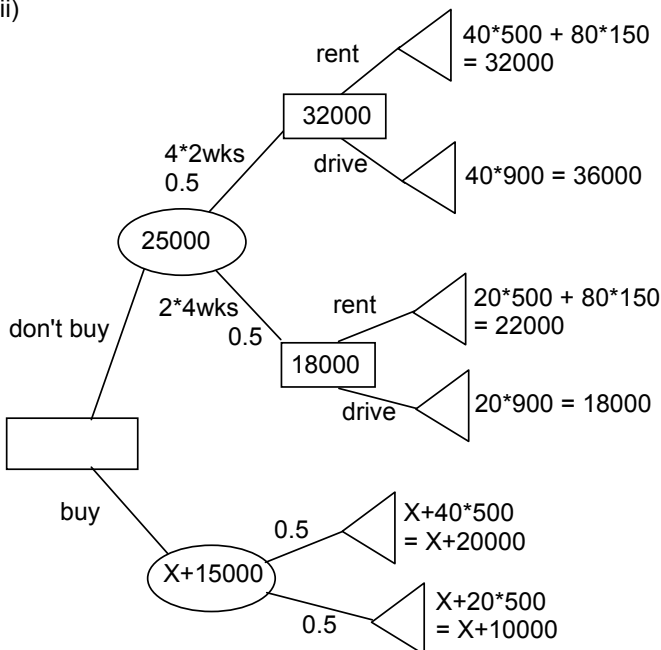
2.

(i) e.g. (Decisions could be in other order.)



Drive down for 2 lots of 4 weeks

(ii)



Jane could save money if she spent less than £10000 on a car

(iii) EMV – expected monetary value – probabilistically weighted cash values
Utility measure is an alternative.

- M1 4*2/2*4
- M1 rent/drive
- A1
- M1 costs
- A1
- B1 advice
- B1 buy/don't buy
- M1 don't buy analysis
- A1 costings
- M1 chance node
- A1 buy analysis
- M1 buy costings
- A1
- B1
- B1
- B1

3.

(a) (i)

	1	2	3	4
1	∞	14	11	24
2	14	∞	15	∞
3	11	15	∞	12
4	24	∞	12	∞

	1	2	3	4
1	1	2	3	4
2	1	2	3	4
3	1	2	3	4
4	1	2	3	4

	1	2	3	4
1	∞	14	11	24
2	14	28	15	38
3	11	15	22	12
4	24	38	12	48

	1	2	3	4
1	1	2	3	4
2	1	1	3	1
3	1	2	1	4
4	1	1	3	1

	1	2	3	4
1	28	14	11	24
2	14	28	15	38
3	11	15	22	12
4	24	38	12	48

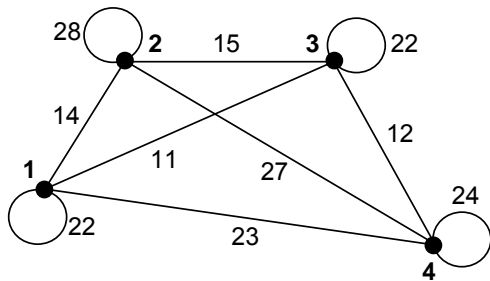
	1	2	3	4
1	2	2	3	4
2	1	1	3	1
3	1	2	1	4
4	1	1	3	1

	1	2	3	4
1	22	14	11	23
2	14	28	15	27
3	11	15	22	12
4	23	27	12	24

	1	2	3	4
1	3	2	3	3
2	1	1	3	3
3	1	2	1	4
4	3	3	3	3

	1	2	3	4
1	22	14	11	23
2	14	28	15	27
3	11	15	22	12
4	23	27	12	24

	1	2	3	4
1	3	2	3	3
2	1	1	3	3
3	1	2	1	4
4	3	3	3	3



(ii) 1 3 4 2 1
64
⇒ 1 3 4 3 2 1

M1 sca Floyd
A1 distance
A1 route

A1

A1

A1

B1 loops
B1 rest

M1 A1
B1
B1

<p>(iii) $27 + 11 + 14 = 52$ TSP solution has length between 52 and 64</p> <p>(b) e.g. 1 3 1 2 3 4 1 length = 87 One repeated arc \rightarrow Eulerian</p>	<p>M1 A1 M1 A1</p> <p>M1 A1 A1 B1</p>
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4.

(i) Let a be the number of tonnes of A produced ...

$$\begin{aligned} \text{Max} & \quad a+b+c \\ \text{st} & \quad 3a+2b+5c < 60 \\ & \quad 5a+6b+2c < 50 \end{aligned}$$

M1 A1

B1
B1
B1

(ii) e.g.

P	a	b	c	s ₁	s ₂	RHS
1	-1	-1	-1	0	0	0
0	3	2	5	1	0	60
0	5	6	2	0	1	50
1	-0.4	-0.6	0	0.2	0	12
0	0.6	0.4	1	0.2	0	12
0	3.8	5.2	0	-0.4	1	26
1	>0	0	0	>0	>0	15
0		0	1			10
0	19/26	1	0	-2/26	5/26	5

M1 initial tableau
A1

M1 pivot
A1

M1
A1

Make 5 tonnes of B and 10 tonnes of C

B1 interpretation

(iii) & (iv) e.g.

A	P	a	b	c	s ₁	s ₂	s ₃	art	RHS
1	0	1	0	0	0	0	-1	0	8
0	1	-1	-1	-1	0	0	0	0	0
0	0	3	2	5	1	0	0	0	60
0	0	5	6	2	0	1	0	0	50
0	0	1	0	0	0	0	-1	1	8
1	0	0	0	0	0	0	0	-1	0
0	1	0	-1	-1	0	0	-1	1	8
0	0	0	2	5	1	0	3	-3	36
0	0	0	6	2	0	1	5	-5	10
0	0	1	0	0	0	0	-1	1	8
1	0	0	0	0	0	0	0	-1	0
	1	0	2	0	0	0.5	1.5		13
	0	0	-13	0	1	-2.5	-4.5		11
	0	0	3	1	0	0.5	2.5		5
	0	1	0	0	0	0	-1		8

B1 new constraint
M1 surplus +
A1 artificial
B1 new objective

M1
A1

B1

Make 8 tonnes of A and 5 tonnes of C

B1 interpretation

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General Comments

Most candidates were able and well prepared, and gave good accounts of themselves. This report focuses on the difficulties encountered, and should not be allowed to detract from the overall success.

Comments on Individual Questions

1) Logic

- (a) Most candidates were able to use their training in propositional logic to clarify this phrasing (even though a full analysis requires predicate calculus).
- (b) There was some confusion here between switching circuits and logical circuits.
- (c) Most candidates were able to produce the required four-line truth table correctly.

2) Decision Analysis

The essence and value of Decision Analysis is the modelling of the structure of the process – identifying the decisions and the chance outcomes, and getting them in the correct order.

In part (i) there were two independent decisions to be taken, and their order was irrelevant.

In part (ii) one of those decisions was switched to a chance outcome and another decision was introduced. Most candidates failed to introduce an extra decision node to model this. They introduced an extra alternative to the existing decision node, thus failing to capture the logic.

With the changes there is only one correct order. First Jane has to decide whether or not to buy a car. Next comes the chance node for 2 trips or 4 trips. Thence there are the two costings in the event of having bought a car, or the decision between flying and driving if a car has not been purchased.

This is difficult modelling, and only very good candidates were successful with it.

3) Networks

- (a) Again, candidates were very well prepared for Floyd's algorithm. Most scored all 6 marks.

The most common response to the instruction to draw the complete network of shortest distances was to ignore the instruction.

Nearest neighbour was done well by most, as was the construction of a lower bound. Surprisingly few candidates were able to put into words what a bound means.

- (b) A surprising number of candidates ploughed on through this as if they were still being asked about the TSP. The question labelling indicates that this part is different.

Of those that were successful in constructing an inspection route of length 87, few were able to mount a convincing explanation of why it was optimal.

4)

LP

Candidates were very competent both in the basic techniques of simplex, and, pleasingly, in its extension to two-phase or big-M.

Less pleasing was the almost universal failure of candidates to identify and define their variables. This is the first and essential step of LP and of much other modelling. Furthermore, it is essential that they remind themselves that they are dealing with numerical algebra here by starting their definitions with "Let ... be the **number** of ..."

A similar point pertains to the interpretation of results. In interpreting a final tableau there are two steps. One is to identify the values of the variables, the second is to say what this means in the "real" world. Thus "b=5 and c=10" is only part way there. The remaining step may seem trivial but is vital in developing those modelling skills – "Make 5 tonnes of product B and 10 tonnes of product C."