OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4772**

Decision Mathematics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- Answer Booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 17 June 2009
Morning

Duration: 1 hour 30 minutes

OCE/T67864

4772

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(a)** The following was said in a charity appeal on Radio 4 in October 2006.

"It is hard to underestimate the effect that your contribution will make."

Rewrite the comment more simply in your own words without changing its meaning. **[2]**

(b) A machine has three components, A, B and C, each of which is either active or inactive.

- The machine is active if A and B are both active.
- The machine is active if A is inactive and C is active.
- The machine is active if B is inactive and C is active.
- Otherwise the machine is inactive.

The states (active or inactive) of the components and the machine are to be modelled by a combinatorial circuit in which "active" is represented by "true" and "inactive" is represented by "false".

Draw such a circuit. **[7]**

(c) Construct a truth table to show the following.

$$\left[\left(\left(a \wedge b\right) \vee \left(\left(\sim a\right) \wedge c\right)\right) \vee \left(\left(\sim b\right) \wedge c\right)\right] \Leftrightarrow \sim \left[\left(\left(\sim a\right) \wedge \left(\sim c\right)\right) \vee \left(\left(\sim b\right) \wedge \left(\sim c\right)\right)\right]$$ **[7]**

2 Zoe is preparing for a Decision Maths test on two topics, Decision Analysis (D) and Simplex (S). She has to decide whether to devote her final revision session to D or to S.

There will be two questions in the test, one on D and one on S. One will be worth 60 marks and the other will be worth 40 marks. Historically there is a 50% chance of each possibility.

Zoe is better at D than at S. If her final revision session is on D then she would expect to score 80% of the D marks and 50% of the S marks. If her final session is on S then she would expect to score 70% of the S marks and 60% of the D marks.

(i) Compute Zoe's expected mark under each of the four possible circumstances, i.e. Zoe revising D and the D question being worth 60 marks, etc. **[5]**

(ii) Draw a decision tree for Zoe. **[5]**

Michael claims some expertise in forecasting which question will be worth 60 marks. When he forecasts that it will be the D question which is worth 60, then there is a 70% chance that the D question will be worth 60. Similarly, when he forecasts that it will be the S question which is worth 60, then there is a 70% chance that the S question will be worth 60. He is equally likely to forecast that the D or the S question will be worth 60.

(iii) Draw a decision tree to find the worth to Zoe of Michael's advice. **[6]**

3 A farmer has 40 acres of land. Four crops, A, B, C and D are available.
Crop A will return a profit of £50 per acre. Crop B will return a profit of £40 per acre.
Crop C will return a profit of £40 per acre. Crop D will return a profit of £30 per acre.
The total number of acres used for crops A and B must not be greater than the total number used for crops C and D.

The farmer formulates this problem as:

Maximise $50a + 40b + 40c + 30d$,

subject to $a + b \leqslant 20$,
$a + b + c + d \leqslant 40$.

(i) Explain what the variables a, b, c and d represent.
Explain how the first inequality was obtained.
Explain why expressing the constraint on the total area of land as an inequality will lead to a solution in which all of the land is used. **[3]**

(ii) Solve the problem using the simplex algorithm. **[10]**

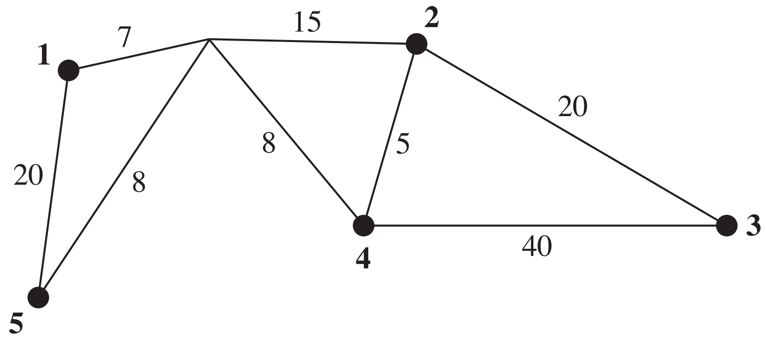
Suppose now that the farmer had formulated the problem as:

Maximise $50a + 40b + 40c + 30d$,

subject to $a + b \leqslant 20$,
$a + b + c + d = 40$.

(iii) Show how to adapt this problem for solution either by the two-stage simplex method or the big-M method. In either case you should show the initial tableau and describe what has to be done next. You should not attempt to solve the problem. **[7]**

4 The diagram shows routes connecting five cities. Lengths are in km.



(i) Produce the initial matrices for an application of Floyd's algorithm to find the complete network of shortest distances between the five cities. **[4]**

The following are the distance and route matrices after the third iteration of Floyd's algorithm.

	1	**2**	**3**	**4**	**5**
1	44	22	42	15	15
2	22	44	20	5	23
3	42	20	40	25	43
4	15	5	25	10	16
5	15	23	43	16	30

	1	**2**	**3**	**4**	**5**
1	2	2	2	4	5
2	1	1	3	4	5
3	2	2	2	2	2
4	1	2	2	2	5
5	1	2	2	4	1

(ii) Perform the fourth iteration. **[4]**

There are no changes on the fifth iteration, so your answer to part **(ii)** should give the complete network of shortest distances.

(iii) Use your matrices to find the shortest distance and route from vertex **3** to vertex **1**, and explain how you do it. **[5]**

(iv) Draw the complete network of shortest distances, not including the loops. **[2]**

(v) Apply the nearest neighbour algorithm to your network in part **(iv)**, starting at vertex **2**. Give the length of the Hamilton cycle that is produced.

Interpret the Hamilton cycle in terms of the original diagram and state what the algorithm has achieved. **[5]**

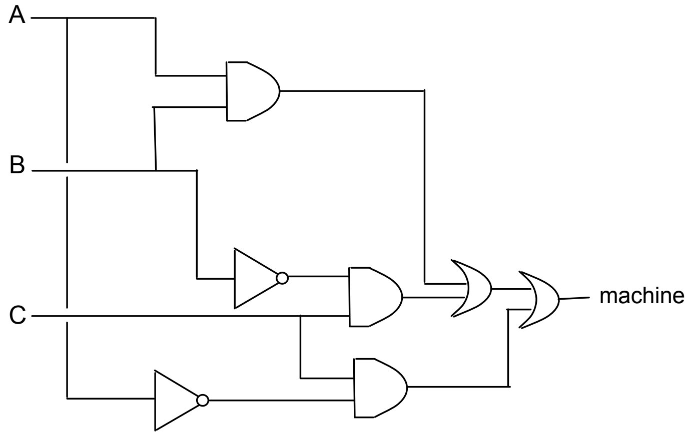
4772 Decision Mathematics 2

Question 1.

(a) e.g.
"It is easy to overestimate the effect that your contribution will make."

M1 remove double negatives
A1 same meaning

(b) e.g.



M1 combinatorial
A1 "ands"
A1 negations
A1 "ors"
A3 one for each alternative

(c) e.g.

((a	∧	b)	∨	(~a	∧	c))	∨	(~b	∧	c)
1	1	1	**1**	0	0	1	1	0	0	1
1	1	1	**1**	0	0	0	1	0	0	0
1	0	0	**0**	0	0	1	1	1	1	1
1	0	0	**0**	0	0	0	0	1	0	0
0	0	1	**1**	1	1	1	1	0	0	1
0	0	1	**0**	1	0	0	0	0	0	0
0	0	0	**1**	1	1	1	1	1	1	1
0	0	0	**0**	1	0	0	0	1	0	0

M1 8 lines
A1 a, b, c
A1 negations
A1 "and"s
A1 "or"s

~	((~a	∧	~c)	∨	(~b	∧	~c))
1	0	0	0	**0**	0	0	0
1	0	0	1	**0**	0	0	1
1	0	0	0	**0**	1	0	0
0	0	0	1	**1**	1	1	1
1	1	0	0	**0**	0	0	0
0	1	1	1	**1**	0	0	1
1	1	0	0	**0**	1	0	0
0	1	1	1	**1**	1	1	1

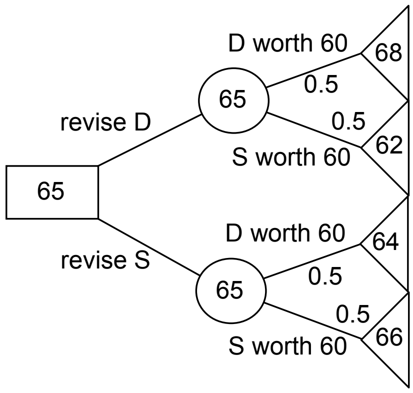
M1
A1

Question 2.

(i)

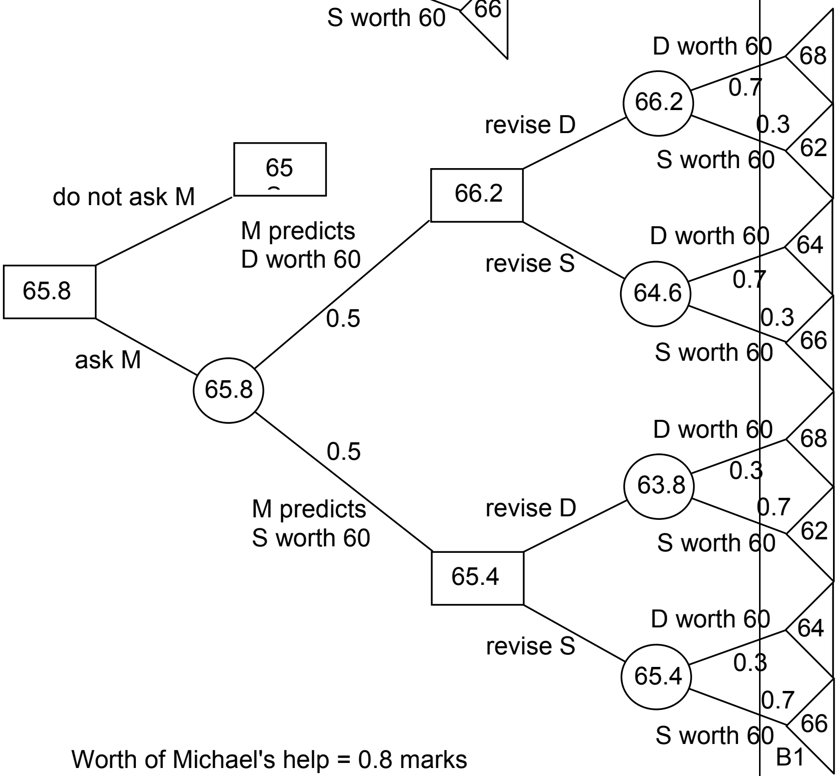
revised	60marks	score	
			M1
D	D	48+20 = 68	A1
D	S	32+30 = 62	A1
S	D	36+28 = 64	A1
S	S	24+42 = 66	A1

(ii)



M1 chance node
A1
A1

M1 decision node
A1



M1 chances
A1

M1decisions
A1 revise
A1 ask/don't

Worth of Michael's help = 0.8 marks B1

Question 3.

(i)	a is the number of acres of land put to crop A, etc a + b ≤ 20 is equivalent to a + b ≤ c + d Given that a + b + c + d ≤ 40, the maximisation will ensure that a + b + c + d = 40 (and it's easier to solve using simplex).	B1 B1 B1

(ii)

P	a	b	c	d	s1	s2	RHS	
1	–50	–40	–40	–30	0	0	0	M1 A1
0	1	1	0	0	1	0	20	A1
0	1	1	1	1	0	1	40	A1
1	0	10	–40	–30	50	0	1000	M1 A1
0	1	1	0	0	1	0	20	
0	0	0	1	1	–1	1	20	
1	0	10	0	10	10	40	1800	M1 A1
0	1	1	0	0	1	0	20	
0	0	0	1	1	–1	1	20	

20 acres to A and 20 acres to C, giving profit of £1800 — B1 B1

(iii) Max $50a + 40b + 40c + 30d$

st $a + b \leq 20$

$a + b + c + d \leq 40$

$a + b + c + d \geq 40$ — B1

A	P	a	b	c	d	s1	s2	sur	art	R	
1	0	1	1	1	1	0	0	–1	0	40	B1 new obj
0	1	–50	–40	–40	–30	0	0	0	0	0	B1 surplus
0	0	1	1	0	0	1	0	0	0	20	B1 artificial
0	0	1	1	1	1	0	1	0	0	40	
0	0	1	1	1	1	0	0	–1	1	40	B1 3 constraints

Minimise A (to zero) then drop A row and art column and continue normally — B1 B1

OR — **OR**

P	a	b	c	d	s1	s2	sur	art	R	
1	–50 –M	–40 –M	–40 –M	–30 –M	0	0	M	0	–40M	M1 A1
0	1	1	0	0	1	0	0	0	20	B1 surplus
0	1	1	1	1	0	1	0	0	40	B1 artificial
0	1	1	1	1	0	0	–1	1	40	

Proceed as per simplex, regarding M as a large fixed number. — B1 B1

Question 4.

(a) (i),(ii) and (iii)

	1	2	3	4	5			1	2	3	4	5
1	∞	22	∞	15	15		**1**	1	2	3	4	5
2	22	∞	20	5	23		**2**	1	2	3	4	5
3	∞	20	∞	40	∞		**3**	1	2	3	4	5
4	15	5	40	∞	16		**4**	1	2	3	4	5
5	15	23	∞	16	∞		**5**	1	2	3	4	5

M1 distance
A1 1 to 5 etc
A1 rest

B1 route

	1	2	3	4	5			1	2	3	4	5
1	∞	22	∞	15	15		**1**	1	2	3	4	5
2	22	44	20	5	23		**2**	1	1	3	4	5
3	∞	20	∞	40	∞		**3**	1	2	3	4	5
4	15	5	40	30	16		**4**	1	2	3	1	5
5	15	23	∞	16	30		**5**	1	2	3	4	1

Not part of the question

	1	2	3	4	5			1	2	3	4	5
1	44	22	42	15	15		**1**	2	2	2	4	5
2	22	44	20	5	23		**2**	1	1	3	4	5
3	42	20	40	25	43		**3**	2	2	2	2	2
4	15	5	25	10	16		**4**	1	2	2	2	5
5	15	23	43	16	30		**5**	1	2	2	4	1

Not part of the question

	1	2	3	4	5			1	2	3	4	5
1	44	22	42	15	15		**1**	2	2	2	4	5
2	22	44	20	5	23		**2**	1	1	3	4	5
3	42	20	40	25	43		**3**	2	2	2	2	2
4	15	5	25	10	16		**4**	1	2	2	2	5
5	15	23	43	16	30		**5**	1	2	2	4	1

Not part of the question

	1	2	3	4	5			1	2	3	4	5
1	30	20	40	15	15		**1**	4	4	4	4	5
2	20	10	20	5	21		**2**	4	4	3	4	4
3	40	20	40	25	41		**3**	2	2	2	2	2
4	15	5	25	10	16		**4**	1	2	2	2	5
5	15	21	41	16	30		**5**	1	4	4	4	1

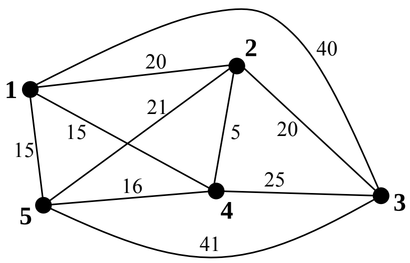
M1
A1 10 changed dists

M1 2's in r3 of route
A1 rest of route

	1	2	3	4	5			1	2	3	4	5
1	30	20	40	15	15		**1**	4	4	4	4	5
2	20	10	20	5	21		**2**	4	4	3	4	4
3	40	20	40	25	41		**3**	2	2	2	2	2
4	15	5	25	10	16		**4**	1	2	2	2	5
5	15	21	41	16	30		**5**	1	4	4	4	1

Shortest distance from **3** to **1** is 40
(1^{st} row and 3^{rd} column of distance matrix)

B1
B1

	Shortest route is **3 2 4 1** **3** followed by route matrix (3,1) = **2** followed by route matrix (2,1) = **4** followed by route matrix (4,1) = **1**	B1 M1 A1
(iv)		M1 A1
(v)	**2** (5) **4** (15) **1** (15) **5** (41) **3** (20) **2** Total length = 96 **2 4 1 5 (4 2) 3 2** Finds a (hopefully short) route visiting every vertex and returning to the start, **or**, upper bound to the TSP	B1 B1 M1 A1 B1

4772 Decision Mathematics 2

General Comments

This paper was accessible to most students but contained plenty of challenge to distinguish between top candidates and others. The tail of poor performers was less long than has sometimes been the case.

Comments on Individual Questions

1) **Logic**

(a) Very few scored both marks for this. A large number did get one mark by removing double negatives but did not get the exact meaning.

(b) A substantial number of candidates did not know what a combinatorial circuit was. Many drew switching circuits. Of those who drew the correct type of circuit, many did not get the two OR gates placed correctly.

(c) The majority did well on this part, and many scored the full 7 marks.

2) **Decision Analysis**

Performances on this question were rather patchy; very few candidates scored full marks.

(i) This part was generally well done, although a few made arithmetic errors!

(ii) Most could draw the decision tree but a substantial number of candidates failed to put the value in the decision node.

(iii) Many failed to construct the correct tree. Very few started with 'ask' and 'don't ask' options. The other main error was getting the 'Michael predicts' and the 'revise' branches in the wrong order, which gave the wrong solution. A number of candidates had only the 'predict' branches, and did not have the 'revise' branches.

3) **Linear Programming (Simplex)**

This question was done well, with a significant number of candidates getting full marks.

(i) There are still a number of candidates who define variables as 'a is crop A' etc, and several took the objective function as the first constraint. Many were able to explain adequately why the inequalities were as they were.

(ii) This was done well, with many getting the basic simplex all correct. Most even remembered to interpret the solution.

(iii) This was designed to test understanding – and it did so! While some got it right, many did not realise that the equality should be expressed as two inequalities, and so their tableau had too few constraints. Those who chose the big M method often had sign errors in their objective.
In spite of being told not to solve the problem, a small number did so!

4) **Networks**

(i) Quite a few were confused by there not being a vertex between 1 and 2 where the roads meet; having 1 to 5 as 20 rather than 15 was a common error. Most could set up the initial tableaus more or less correctly.

(ii) As with the simplex, a small number tired to work through the whole of the Floyd, in spite of the fact that the third iteration was given. Most successfully got the distance matrix, but a number did not know how to complete the route matrix correctly and had '4 2 2 2 4' in the third row.

(iii) In spite of being told there were no changes, a few wrote out the fifth iteration! Most were able to read the distance and route from their matrices but not all tired to explain how they had done it. A significant majority found the distance by adding the distances on the steps corresponding to the route matrix rather than simply reading from the distance matrix.

(v) While many got the Hamilton cycle correct, a number did not return to the start so did not have a cycle. Some failed to interpret their result in the light of the original network. The majority knew that this related to the travelling salesperson problem, and the best candidates stated that it was an upper bound or a feasible tour. Some stated that it gave the shortest tour around the vertices. A small number thought it was the route inspection problem, and some stated that they had found a Eulerian cycle, in spite of being told it was Hamiltonian!