OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4773

Decision Mathematics Computation

Friday **24 JUNE 2005** Morning 2 hours 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Additional sheets, including computer print-outs, should be fastened securely to the answer booklet.
- You are permitted to use a graphical calculator in this paper.
- There is an **insert** for use in Questions **2** and **4**.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is 72.

This question paper consists of 6 printed pages, 2 blank pages and an insert.

HN/3

1 Jim needs to control his weight. His weight at time $n + 2$ (days) is related to his food intake from time n to time $n + 1$ by the recurrence relation

$$u_{n+2} = u_{n+1} + \tfrac{1}{144}(v_n - 32u_n), \qquad (n \text{ is a non-negative integer}),$$

where u_n is his weight in kg at time n, and where v_n is his intake of food in calories from time n to time $n + 1$.

(i) Jim weighs 80 kg. Find the intake in calories which will maintain his weight in equilibrium at 80 kg. [2]

Jim goes on holiday. At the start of his holiday (time = 0) he weighs 80kg. At the start of the next day (time = 1) he still weighs 80 kg. He consumes 3000 calories per day whilst on holiday.

(ii) Find Jim's equilibrium weight at an intake of 3000 calories per day. [2]

(iii) Solve the recurrence relation with $v_n = 3000$ (constant) to find a formula giving Jim's weight on subsequent days of his holiday. [10]

Some time after his holiday Jim decides to diet. At the start of his diet (time = 0) he weighs 90 kg. At the start of the next day (time = 1) he still weighs 90 kg. He restricts his food intake so that $v_n = 3000 - 8u_n$ whilst he is on his diet.

(iv) Build a spreadsheet to show how Jim's weight changes over time. Produce a printout showing his weight over the first two weeks of his diet, and say what happens in the long run. [3]

(v) Explain what will happen if instead of $3000 - 8u_n$, Jim restricts his daily intake to $2560 - 8u_n$ calories. [1]

2 **Answer parts (i) to (v) of this question on the insert provided.**

Bob and Lynne have a network of irrigation pipes in their garden. Pipes have not always been added in a logical fashion, and the current network is shown in Fig. 2.1. The system delivers water from a source marked S, through junctions at A, B, C and D, to locations E, F, G, H and I. The numbers on the arcs are the capacities of the pipes in litres per minute.

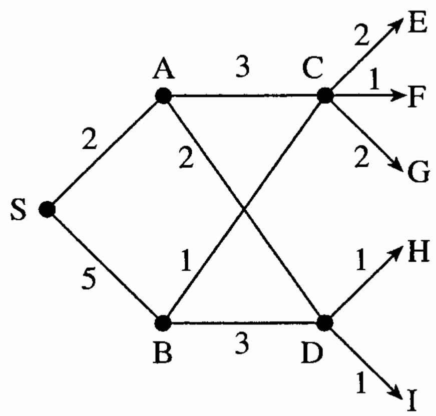

Fig. 2.1

(i) On the copy of the network in the insert, add a super sink, T, with arcs of appropriate capacities connecting it to E, F, G, H and I. [2]

Flows are established as shown in Table 2.2.

S→A	A→C	C→E	S→B	B→C	C→F	B→D	D→H	D→I
2	2	2	3	1	1	2	1	1

Table 2.2

(ii) For this flow, complete the labelling on the insert for the arc BC. [3]

(iii) Find and list a flow-augmenting path of capacity 1 litre per minute. [2]

(iv) Show on Fig. 2.3 on the insert a flow pattern giving a total flow of 6 litres per minute. [2]

(v) Give a cut of capacity 6. [2]

Bob lays a new pipe of capacity 1 from S to D. Lynne wants to know the effect of this on her olive tree at G.

(vi) Formulate an LP to find the maximum flow through this extended network. [4]

(vii) Run your LP and interpret your results. [3]

3 A railway station platform has an automatic exit barrier. When a train arrives, 100 passengers leave the train and have to pass through the barrier. The time taken for an individual to pass through the barrier is a random variable, and is specified in table 3.

Time (seconds)	1.5	2	3.5
Probability	0.15	0.75	0.1

Table 3

(i) Assume that all of the passengers arrive at the barrier at the same moment and form a single queue. Build a simulation model to find how long it takes for all of the passengers to pass through the barrier.

Include a printout of your simulation. [2]

(ii) Repeat 10 times your simulation from part **(i)** and calculate the mean time taken for all 100 passengers to pass through the barrier. From your results calculate how many repetitions are required to estimate the mean time with a maximum error of 0.5 seconds with 95% confidence.

Include a printout of your simulation. [5]

In fact passengers do not all arrive at the barrier at the same moment. The barrier is located near to the front of the train and passengers at the rear of the train take 2 minutes to walk to it.

(iii) In a column of a spreadsheet simulate 100 uniformly distributed random numbers between 0 and 120. **Copy** them and paste a fixed copy of them into a second column. To do this using EXCEL, use the **Paste Special** command from the **Edit** menu, selecting the **Paste Values** option. Now use the **Sort** command from the **Data** menu to sort them in ascending order into a third column.

This will give simulated arrival times (in seconds) of 100 passengers at the barrier. [3]

(iv) Complete a simulation of 100 passengers walking to the barrier and passing through it. Give the time taken for all 100 passengers to pass through the barrier.

Include a printout of your simulation. Show the formulae which you used for row 10 of your spreadsheet. [3]

A second identical exit barrier is installed.

(v) Using your arrival times from part **(iii)**, and assuming that a single queue is formed, simulate the 100 passengers passing through the two barriers. Give the time it takes for all 100 passengers to pass through the barriers.

Include a printout of your simulation. Show the formulae which you used for row 10 of your spreadsheet. [5]

4 Answer part (i) of this question on the insert provided.

Flyair needs to schedule a pilot for each of 12 flights between London, Berlin, Paris and Milan. Table 4.1 shows flight numbers, routes and flight times.

Flight		Departs	Arrives
101	London → Berlin	07.00	09.00
102	London → Berlin	19.00	21.00
201	Berlin → Paris	09.30	11.00
202	Berlin → Milan	10.30	11.30
203	Berlin → Paris	22.00	23.30
204	Berlin → Milan	22.30	23.30
301	Milan → Paris	12.00	13.00
302	Milan → London	15.30	17.00
303	Milan → Berlin	18.00	19.00
401	Paris → Berlin	07.00	08.30
402	Paris → Milan	12.00	13.00
403	Paris → London	14.00	15.00

Table 4.1

Fig. 4.2 represents this information diagrammatically.

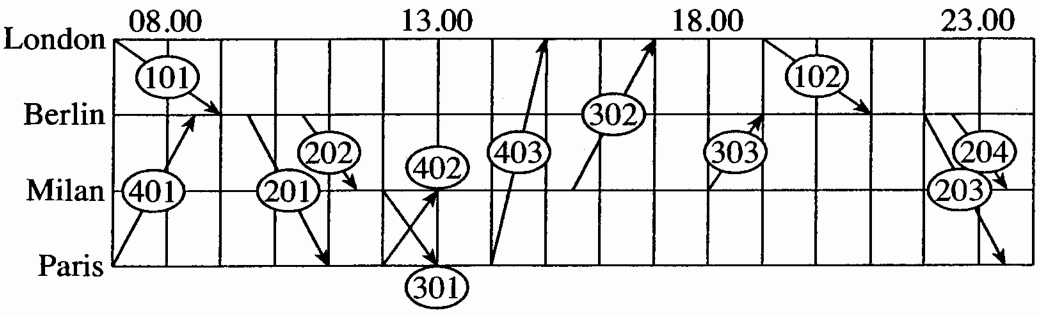

Fig. 4.2

Pilots are scheduled for up to 4 flights per day, and must end the day in their home city. Thus for a Berlin based pilot the following is a possible schedule:

$$202 \rightarrow 301 \rightarrow 403 \rightarrow 102$$

(i) Complete the table in the insert showing 20 possible schedules. [5]

(ii) Formulate an LP to choose the schedules to use so that all flights have at least one pilot, and so that the number of schedules used is minimised. Your LP should have 20 variables and 12 constraints, one for each flight. [6]

(iii) Run your LP, and interpret your solution. How many pilots are needed? [3]

(iv) Show that the optimal solution is unique. [3]

(v) Criticise the model. [1]

Candidate Name	Centre Number	Candidate Number

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INSERT

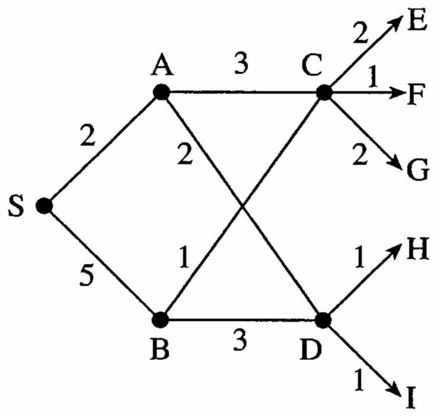
Friday **24 JUNE 2005** Morning 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- This insert should be used in Questions **2** and **4.**
- Write your name, centre number and candidate number in the spaces provided at the top of this page and attach it to your answer booklet.

This insert consists of 3 printed pages and 1 blank page.

2 **(i)**



(ii)

key

capacity	forward potential
flow	backward potential

(iii) Flow-augmenting path: ____________________

(iv)

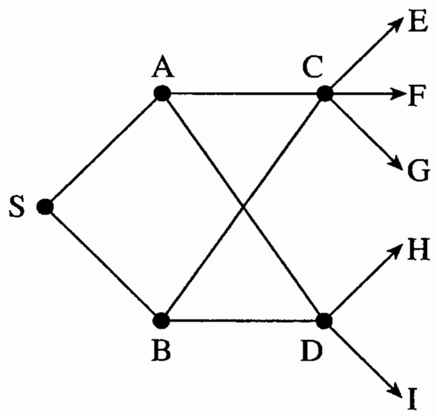


Fig. 2.3

(v) Cut: ____________________________

4

Schedule	City	Flight	City	Flight	City	Flight	City	Flight	City
S1	L	101	B	201	P	402	M	302	L
S2	L	101	B						
S3	L	101	B						
S4	L	101	B						
S5	B	201	P						
S6	B	201	P						
S7	B	201	P						
S8	B	202	M	301	P	403	L	102	B
S9	B	202	M	302	L	102	B		
S10	B	202	M						
S11	M	301	P						
S12	M	302	L						
S13	M	303	B						
S14	P	401	B						
S15	P	401	B						
S16	P	401	B						
S17	P	401	B						
S18	P	402	M						
S19	P	402	M						
S20	P	403	L						

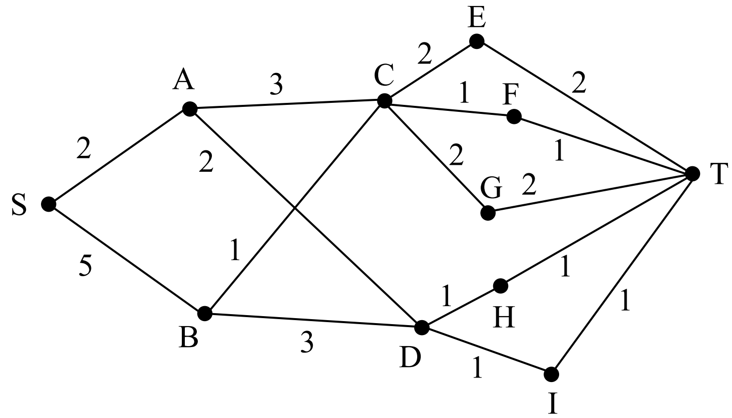
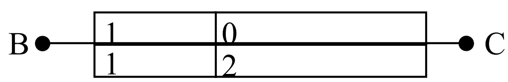
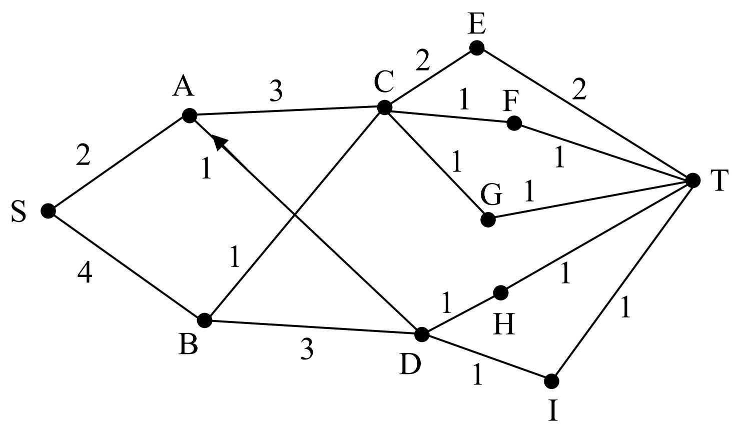
Mark Scheme 4773
June 2005

Qu. 1

(i) 32*80 = 2560 calories	M1 A1
(ii) 3000/32 = 93.75 kg	M1 A1
(iii) Auxiliary equation is $(3x-1)(3x-2) = 0$	M1 A1 M1 particular A1 93.75 or 3rd eqn M1 gen homogeneous
Solution is $u_n = 13.75(1/3)^n - 27.5(2/3)^n + 93.75$	A1 correct form B1 case 1 ($u_0 = 80$) + case 2 ($u_1 = 80$) M1 simultaneous A1 13.75 and −27.5 B1 final answer
(iv) 90 90 85.83333 81.66667 78.65741 76.80556 75.78961 75.28807 75.06873 74.98871 74.96962 74.97276 74.98119	M1 A1
(Oscillatory) convergence to 75 kg.	B1
(v) 90 90 82.77778 75.55556 70.33951 67.12963 65.36866 64.49931 64.11913 63.98043 63.94734 63.95278 63.9674	B1

63.98052

Qu. 2

(i) 	M1 A1
(ii) 	B1 1 and 1 B1 0 B1 2
(iii) S B D A C G (T)	M1 A1
(iv) 	M1 A1
(v) S B | A C D E F G H I T or S A B D | C E F G H I T	M1 a cut, properly specified A1 correct cut

(vi) Max SA + SB + SD st SA+CA+DA-AD-AC = 0 SB+CB+DB-BC-BD = 0 AC+BC-CA-CB-CE-CF-CG = 0 SD+BD+AD-DA-DB-DH-DI = 0 SA < 2 SB < 5 SD < 1 AD < 2 DA < 2 BC < 1 CB < 1 AC < 3 BD < 3 CA < 3 CE < 2 CF < 1 CG < 2 DB < 3 DH < 1 DI < 1 end	B1 M1 A1 B1
(vii) OBJECTIVE FUNCTION VALUE 1) 6.00000	M1 A1 B1

VARIABLE	VALUE	REDUCED COST
SA	2.000000	0.000000
SB	3.000000	0.000000
SD	1.000000	0.000000
CA	0.000000	1.000000
DA	1.000000	0.000000
AD	0.000000	0.000000
AC	3.000000	0.000000
CB	0.000000	1.000000
DB	0.000000	0.000000
BC	1.000000	0.000000
BD	2.000000	0.000000
CE	2.000000	0.000000
CF	0.000000	0.000000
CG	2.000000	0.000000
DH	1.000000	0.000000
DI	1.000000	0.000000

Flows are as listed in the "VALUE" column.	

Qu. 3

(i)	Simulating service times (=lookup(rand(),cum.probs,times)) Accumulating (expectation is 207.5 seconds)	B1 B1
(ii)	Repetitions Mean (not far off 207.5 seconds) sd (order of magnitude 5 seconds) $(2*1.96*s)^2$ = (about) 400 repetitions (assuming a 95% confidence interval half-width of 0.5s)	B1 B1 B1 M1 A1
(iii)	Rand()*120 fixed sorted	M1 A1 B1
(iv)	max(arrival time, gate available time) + service time finish time approx as in (i)	B1 B1 B1
(v)	Test barrier free times to see which barrier passenger uses. Computation of barrier free times, eg: =if(bar=1, max(arrival t + service t, bar t + service t), bar t) finish time approx 130s	M1 A1 M1 A1 B1

Qu. 4

(i)

Sched.	City	Flight	City	Flight	City	Flight	City	Flight	City
S1	L	101	B	201	P	402	M	302	L
S2	L	101	B	201	P	403	L		
S3	L	101	B	202	M	302	L		
S4	L	101	B	202	M	301	P	403	L
S5	B	201	P	402	M	302	L	102	B
S6	B	201	P	402	M	303	B		
S7	B	201	P	403	L	102	B		
S8	B	202	M	301	P	403	L	102	B
S9	B	202	M	302	L	102	B		
S10	B	202	M	303	B				
S11	M	301	P	403	L	102	B	204	M
S12	M	302	L	102	B	204	M		
S13	M	303	B	204	M				
S14	P	401	B	201	P				
S15	P	401	B	202	M	301	P		
S16	P	401	B	203	P				
S17	P	401	B	202	M	303	B	203	P
S18	P	402	M	302	L	102	B	203	P
S19	P	402	M	303	B	203	P		
S20	P	403	L	102	B	203	P		

M1
A1 London
A1 Berlin
A1 Milan
A1 Paris

(ii)

Min S1+S2+S3+S4+S5+S6+S7+S8+S9+S10+S11+S12+S13+S14+S15+S16+S17+S18+S19+S20

M1 A1 objective

st
S1+S2+S3+S4>1
S5+S7+S8+S9+S11+S12+S18+S20>1
S1+S2+S5+S6+S7+S14>1
S3+S4+S8+S9+S10+S15+S17>1
S16+S17+S18+S19+S20>1
S11+S12+S13>1
S4+S8+S11+S15>1
S1+S3+S5+S9+S12+S18>1
S6+S10+S13+S17+S19>1
S14+S15+S16+S17>1
S1+S5+S6+S18+S19>1
S2+S4+S7+S8+S11+S20>1

M1
A3 (−1 each error/omission)

(iii) OBJECTIVE FUNCTION VALUE 1) 3.000000 VARIABLE VALUE REDUCED COST S1 1.000000 0.000000 S2 0.000000 0.000000 S3 0.000000 1.000000 S4 0.000000 0.000000 S5 0.000000 0.000000 S6 0.000000 0.000000 S7 0.000000 0.000000 S8 0.000000 0.000000 S9 0.000000 1.000000 S10 0.000000 1.000000 S11 1.000000 0.000000 S12 0.000000 1.000000 S13 0.000000 1.000000 S14 0.000000 0.000000 S15 0.000000 0.000000 S16 0.000000 0.000000 S17 1.000000 0.000000 S18 0.000000 0.000000 S19 0.000000 0.000000 S20 0.000000 0.000000	M1 A1
3 pilots are used	B1
(iv) Three more runs, with S1=0, S11=0 and S17=0 in turn. All require 4 pilots	M1 A1 (3 runs) A1 (4 pilots)
(v) No account taken of pilot stress (workload/long day/short changeover)	B1

4773 - Decision Mathematics Computation

General Comments

This paper was substantially the same as the paper set for 2622, and this report overlaps greatly with that for 2622. On that paper each question was marked out of 20 and candidates were required to attempt 3 out of 4 questions. The questions were reduced slightly in content for 4773, and were worth 18 marks each, but candidates were required to attempt all 4.

Candidate performances on 4773 were good. There was evidence of some candidates running out of time. In a few other cases, Lindo appeared to have been used to generate a solution, but no evidence was included with the script.

Candidates need to take great care in labelling their computer printout pages, ensuring that they have the correct question number on them and that they are assembled in the correct order.

Comments on Individual Questions

1) **Recurrence relations**

(i) Most candidates got this right, although some computed u_2 as their answer.

(ii) This was a little more difficult than part (i), and a small but significant number of candidates failed on it.

(iii) A large proportion of candidates managed to find their way completely successfully through this intricate calculation.

(iv)(v) Most candidates succeeded in building correct spreadsheets. Not all of those achieved full marks, failing to make simple observations about convergence and limits.

2) **Networks**

(i) to (v) This work on network theory was completed on the insert. It was generally well done. Some candidates had difficulty with part (iii) which, for them, made part (iv) more difficult than it should have been. Nevertheless, they were able to recover since any flow pattern giving a total flow of 6 was acceptable.

(vi) & (vii) Most showed a good idea of how to construct the LP model, even if mistakes were made along the way. Again, there was a weakness in extracting results from the output.

3) **Simulation**

(i) Most could do this in principle, but a significant minority made one of two mistakes –

- using 0, 0.15 and 0.75 instead of 0, 0.15 and 0.9 in their lookup table
- failing to accumulate the service times.
-

(ii) A significant minority of candidates failed to compute the standard deviation of their 10 accumulated times. In some instances candidates tried to do the computation longhand, instead of using the spreadsheet function.
The majority of candidates could do the computation to determine approximately how many repetitions are required. However, it was quite common to see answers in error by a factor of 4.

(iii) Almost all candidates succeeded with this.

(iv) Most candidates could do this simulation, but many failed to compute the queuing times.

(v) A majority could build this two-server simulation but many failed with the queuing times. A number of candidates treated the barriers as being in series rather than parallel – this of course extends rather than reduces the exit time and thus defeats the purpose of this section of the question. Others had errors in their formulae and did not appear to check their computed values for reasonableness.

4) **LP modelling**

(i) It was expected that there would be errors made in this part of the question. In fact, many candidates got it completely right.

(ii) & (iii) Most succeeded in building and running the LP model. However, many of those who produced output failed to interpret it to say how many pilots were needed. A number of candidates missed one or two schedules from constraints and did not appear to check back through their work.

(iv) Very few candidates scored any marks on this. All that was required was a systematic suppression of each of the three solution schedules in turn. In each case more than 3 pilots are required, showing that there is no alternative solution. A few candidates produced alternative, logically reasoned arguments based on permutations of 4-flight schedules, whilst others tried to base their answer on their original Lindo output.

(v) Examiners do not expect candidates to show any detailed ab initio knowledge of the scenarios presented. Nothing is required beyond that which is given in the question. Thus they should not, for instance, have been worrying about the mechanics of refuelling and preparing aircraft. Indeed, there is no mention in the question about the aircraft which are used, so that can form no legitimate part of the answer. However, "turning around" a pilot certainly was relevant.