RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4773

Decision Mathematics Computation

Thursday **15 JUNE 2006** Afternoon 2 hours 30 minutes

Additional materials:
- 8 page answer booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Additional sheets, including computer print-outs, should be fastened securely to the answer booklet.
- You are permitted to use a graphical calculator in this paper.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

1 An investor is considering three investment opportunities over the next five years. He wishes to maximise the amount of money he has at the end of those five years.

Investment A is a one-year investment. It is available in each of the years and may be started at the beginning of any year. At the end of a year it will return £1.15 for every £1 invested.

Investment B is a three-year investment. It may be started at the beginning of year 1, year 2, or year 3. It will return £1.55 for every £1 invested.

Investment C is another one-year investment, but it is not available until the start of year 3. It will return £1.20 per annum for every £1 invested.

The investor has £50 000 to invest.

(i) Define appropriate variables and formulate the investor's problem as an LP. [10]

(ii) Solve your LP using your LP package, and interpret your solution.

You should enclose printouts of your formulation and your output with your solution. [4]

(iii) You should have found that it is not worth investing in B. By experimenting with your LP, or otherwise, find what the return on B would have to be to make it worthwhile investing in it. [3]

2 The Fibonacci recurrence relation is $u_{n+2} = u_{n+1} + u_n$, with $u_0 = 1$ and $u_1 = 1$.

(i) Build a spreadsheet with two columns, the first giving the numbers 0, 1, 2, …, and the second giving the corresponding Fibonacci numbers.

Print out the first 20 Fibonacci numbers. [3]

(ii) Write down and solve the auxiliary equation for the Fibonacci recurrence relation. Hence find an expression for the n^{th} Fibonacci number, and show that it can be expressed in the form

$$u_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right).$$ [9]

(iii) Verify that the formula is correct by coding the formula into the third column of your spreadsheet.

Print out your spreadsheet formula and print out your spreadsheet. [2]

(iv) In the fourth column of your spreadsheet compute the Fibonacci ratios R_n, where R_{n+1} is the $(n+1)^{\text{th}}$ Fibonacci number divided by the n^{th} Fibonacci number.

Describe what happens.

Find the exact value of the limit (which is known as the Golden Ratio). [5]

3 Four shops, S1, S2, S3 and S4 are to be supplied with crates of material from three warehouses, W1, W2 and W3. The requirements at the shops are 10 crates at S1, 15 at S2, 12 at S3 and 20 at S4. There are 20 crates available at each warehouse.

The costs of delivering a single case from each warehouse to each shop are shown in Table 3.1.

cost (£)	S1	S2	S3	S4
W1	2	2	1	5
W2	3	2	2	4
W3	5	5	1	2

Table 3.1

(i) Formulate an LP to solve the problem of moving crates from warehouses to shops at minimum total cost. Produce a printout of your formulation. [7]

(ii) Use your LP package to solve your LP. Produce a printout of your solution.

Interpret your solution. [4]

Two customers, C1 and C2, require 30 and 27 crates respectively. The costs per crate of supplying each of them from each of the shops is shown in Table 3.2.

cost (£)	S1	S2	S3	S4
C1	4	6	3	2
C2	1	4	2	5

Table 3.2

(iii) Formulate, solve and interpret an LP to find the cheapest way of supplying the two customers from the warehouses via the shops. There are still 20 crates available at each warehouse, but the shop requirements no longer apply. [7]

4 The weather in Brighting is either wet, showery or dry. On the day following a wet day there is a 20% chance that it will be wet and a 30% chance that it will be showery. On the day following a showery day there is a 40% chance that it will be wet and a 15% chance that it will be showery. On the day following a dry day there is a 15% chance that it will be wet and a 25% chance that it will be showery.

Today the weather in Brighting is dry.

(i) Find the probabilities of it being wet, showery or dry in Brighting on the day after tomorrow. [4]

(ii) Build a spreadsheet to simulate the weather in Brighting tomorrow, and the day after tomorrow.

(You may wish to set up lookup tables to model the probabilities, and to use "= IF(… ,… ,…)" statements to branch to the appropriate lookup columns.)

Print out the formulae which you use in your spreadsheet. [7]

(iii) Run your simulation 10 times, putting your results into a table.

Estimate the probabilities of it being wet, showery or dry the day after tomorrow. [3]

(iv) Extend your spreadsheet to investigate what happens after 20 days.

Print out your spreadsheet.

Run your simulation 10 times, putting your results into a table.

Estimate the probabilities of it being wet, showery or dry after 20 days. [4]

Mark Scheme 4773
June 2006

Qu. 1

(i)	Variables	M1	
	ai = amount invested in A in year i, i = 1, 2, 3, 4, 5	A1	a's
	bi = amount invested in B in year i, i = 1, 2, 3	A1	b's
	ci = amount invested in C in year i, i = 3, 4, 5	A1	c's
	Maximise 1.15a5+1.55b3+1.20c5	B1	
	st a1+b1 = 50000	B1	
	a2+b2 = 1.15a1	B1	
	a3+b3+c3 = 1.15a2	B1	
	a4+c4 = 1.15a3+1.55b1+1.20c3	B1	
	a5+c5 = 1.15a4+1.55b2+1.20c4	B1	
(ii)	OBJECTIVE FUNCTION VALUE		
	1) 114264.0		

VARIABLE	VALUE	REDUCED COST	
A5	0.000000	0.050000	
B3	0.000000	0.178000	M1
C5	95220.000000	0.000000	A1
A1	50000.000000	0.000000	
B1	0.000000	0.053280	
A2	57500.000000	0.000000	
B2	0.000000	0.127200	
A3	0.000000	0.072000	
C3	66125.000000	0.000000	
A4	0.000000	0.060000	
C4	79350.000000	0.000000	

	Invest all in A in year 1. Put all into A in year 2	B1
	Thence all into C in years 3, 4 and 5.	
	Gives £114264 at the end of 5 years.	B1
(iii)	£1.59	M1 A1 (£1.57 to £1.61)
		A1

Qu. 2

(i) See below – first two columns of s/sheet	M1 A1 A1
(ii) $\mathbf{x^2 - x - 1 = 0}$	M1
$x = \frac{1 \pm \sqrt{5}}{2}$	A1
$x = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$	B1
A + B = 1 and $A\left(\frac{1+\sqrt{5}}{2}\right) + B\left(\frac{1-\sqrt{5}}{2}\right) = 1$	B1 B1
giving $u_n = \frac{1}{\sqrt{5}}\frac{\sqrt{5}+1}{2}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{1}{\sqrt{5}}\frac{\sqrt{5}-1}{2}\left(\frac{1-\sqrt{5}}{2}\right)^n$	M1 solving A1 A1
$= \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{n+1}$	B1
(iii) =(1/SQRT(5))*(((1+SQRT(5))/2)^(A2+1)-((1-SQRT(5))/2)^(A2+1)) plus printout	M1 A1 M1 A1
(iv) See s/sheet below.	B1
Converges to 1.61803…	
$\left(\frac{1+\sqrt{5}}{2}\right)$	M1 A1

n	F(n)	Formula	Ratios	n	F(n)	Formula	Ratios
0	1	1		10	89	89	1.61818
1	1	1	1	11	144	144	1.61798
2	2	2	2	12	233	233	1.61806
3	3	3	1.5	13	377	377	1.61803
4	5	5	1.66667	14	610	610	1.61804
5	8	8	1.6	15	987	987	1.61803
6	13	13	1.625	16	1597	1597	1.61803
7	21	21	1.61538	17	2584	2584	1.61803
8	34	34	1.61905	18	4181	4181	1.61803
9	55	55	1.61765	19	6765	6765	1.61803

Qu. 3

(i) Min 2W1S1+2W1S2+W1S3+5W1S4+3W2S1+2W2S2	B1 variables
	M1 objective
+2W2S3+4W2S4+5W3S1+5W3S2+W3S3+2W3S4	A1
	M1 w/house
st W1S1+W1S2+W1S3+W1S4<20	A1 availabilities
W2S1+W2S2+W2S3+W2S4<20	
W3S1+W3S2+W3S3+W3S4<20	M1 shop
W1S1+W2S1+W3S1>10	A1 requirements
W1S2+W2S2+W3S2>15	
W1S3+W2S3+W3S3>12	
W1S4+W2S4+W3S4>20	
(ii)	
OBJECTIVE FUNCTION VALUE	
1) 104.0000	
VARIABLE VALUE REDUCED COST	
W1S1 8.000000 0.000000	
W1S2 0.000000 1.000000	
W1S3 12.000000 0.000000	
W1S4 0.000000 3.000000	
W2S1 2.000000 0.000000	
W2S2 15.000000 0.000000	B1
W2S3 0.000000 0.000000	
W2S4 0.000000 1.000000	
W3S1 0.000000 3.000000	
W3S2 0.000000 4.000000	
W3S3 0.000000 0.000000	
W3S4 20.000000 0.000000	
	M1
	A1
Supply shop 1 with 8 from warehouse 1 and 2 from 2	
Supply shop 2 from warehouse 2	
Supply shop 3 from warehouse 1	B1
Supply shop 4 from warehouse 3	
Cost = £104	

Qu. 3 (cont)

(iii)	Min	2W1S1+2W1S2+W1S3+5W1S4+3W2S1+2W2S2	B1	new variables
		+2W2S3+4W2S4+5W3S1+5W3S2+W3S3+2W3S4	B1	new objective
		+4S1C1+6S2C1+3S3C1+2S4C1		
		+S1C2+4S2C2+2S3C2+5S4C2	B1	supply constraints
	st	W1S1+W1S2+W1S3+W1S4<20		
		W2S1+W2S2+W2S3+W2S4<20		
		W3S1+W3S2+W3S3+W3S4<20		
		S1C1+S2C1+S3C1+S4C1=30	B1	receipt constraints
		S1C2+S2C2+S3C2+S4C2=27		
		S1C1+S1C2-W1S1-W2S1-W3S1=0		
		S2C1+S2C2-W1S2-W2S2-W3S2=0		
		S3C1+S3C2-W1S3-W2S3-W3S3=0	B1	in/out constraints
		S4C1+S4C2-W1S4-W2S4-W3S4=0		
	A solution is:			
	W1 to S3 20			
	W2 to S3 17		B1	
	W3 to S4 20			
	S3 to C1 10			
	S4 to C1 20		B1	
	S3 to C2 27			

Qu. 4

(i) 0.22, 0.2325, 0.5475	M1 A1 A1 A1
(ii) e.g. look-up tables | | wet(1) | showery(2) | dry(3) | | wet | 0 | 0 | 0 | | showery | 0.2 | 0.4 | 0.15 | | dry | 0.5 | 0.55 | 0.4 |	M1 probability A1 distributions
simulation run | day | 0 | 1 | 2 | | rand | | 0.14227 | 0.43734 | | weather | dry | wet | dry | =IF(B8="wet",LOOKUP(C7,$B$2:$B$4,$A$2:$A$4),(IF(B8=" showery", LOOKUP(C7,$C$2:$C$4,$A$2:$A$4),	M1 selecting dist. A1 by weather M1 sampling from A1 distribution B1 two days handled
(iii) repeating and tabulating calculating experimental probabilities	B1 M1 A1
(iv) 20 transitions handling $s repeating and tabulating experimental probabilities (theoretical = 0.22, 0.24 and 0.54)	B1 B1 B1 B1

4773 - Decision Mathematics Computation

General Comments

Candidates should ensure that all necessary printed computer outputs are supplied and appropriately labelled. In a number of cases candidates were not awarded marks because they had failed to show sufficient evidence of methods used. In a few cases candidates referred to computer output in their written work, but the relevant sheets were not attached.

Comments on Individual Questions

Q 1 **LP modelling**

This question was generally poorly done.
In part (i) the majority of candidates did not appreciate the need for a variable for each investment type for each potential start year and therefore were not able to formulate the LP correctly.
Some candidates did not appreciate the "compound interest" nature of the annual investments and hence did not realise that both A and C were higher yield than B over a 3-year period. However, those that did were often able to give a reasoned argument as to what yield B needed to succeed in part (iii).

Q 2 **Recurrence relations**

Other than a few candidates who used u(0)=0 and started the Fibonacci sequence at n=1, part (i) was straightforward and done well.
Good candidates were able to complete the proof in part (ii), whilst others struggled to solve the simultaneous equations.
Part (iii) was occasionally lacking in evidence of method.
In part (iv) a significant number of candidates failed to generate/demonstrate the ratios in the 21 rows of their spreadsheet.

Q 3 **Networks**

Some candidates complicated this question by using a separate letter of the alphabet for each Warehouse/Shop pairing, making it difficult for them to cross check their LP against the data in the question.
Candidates with a working LP generally gave clear interpretation of the solution in part (ii).
In part (iii) candidates either needed to add a new set of variables for the Transportation from Shop to Customer, or "double up" their original variables, one set of 12 for each Customer. In reading that the shop requirements no longer applied, many candidates lost sight of the fact that a shop cannot supply more crates than it receives and therefore did not include the relevant transhipment constraints in their formulation. Consequently their results did not correspond to a practical solution.

Q 4 **Simulation**

Most candidates achieved full marks in part (i).
Whilst some excellent solutions were seen from some candidates in the rest of the question, others failed to show appropriate evidence of their work. Often spreadsheets were poorly laid out, printed across multiple pages, and with little annotation.
A significant proportion of candidates appeared to be unaware that Excel can cope with testing text cells, and so coded the 3 weather types numerically, necessitating additional processing, and making their work less readable.