ADVANCED GCE

MATHEMATICS (MEI)

4773

Decision Mathematics Computation

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 19 June 2009
Afternoon

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

1 At one-minute intervals the automatic steering on a boat finds the bearing which gives the boat's actual direction of motion. This bearing is compared with the desired bearing. A correction equal to the difference between the two bearings is applied. However, this correction takes two minutes to take effect.

(i) Letting the actual bearing at time n (minutes) be B_n, explain why $\mathrm{B}_{n+2} - \mathrm{B}_{n+1} + \mathrm{B}_n = 0$ when the boat's desired bearing is 0° (due North). **[2]**

(ii) Given that $\mathrm{B}_0 = 2$ and $\mathrm{B}_1 = 4$, build a spreadsheet model to show B_n for $0 \leqslant n \leqslant 20$.
Describe what happens. **[3]**

The performance of the automatic steering is improved by making the correction equal in magnitude to half the difference between the actual bearing and the desired bearing.

(iii) Given that the desired bearing is 0°, give the recurrence relation for B_n with this improvement.
Build a spreadsheet model to show B_n for $0 \leqslant n \leqslant 20$ given that $\mathrm{B}_0 = 2$ and $\mathrm{B}_1 = 4$.
Describe what happens. **[3]**

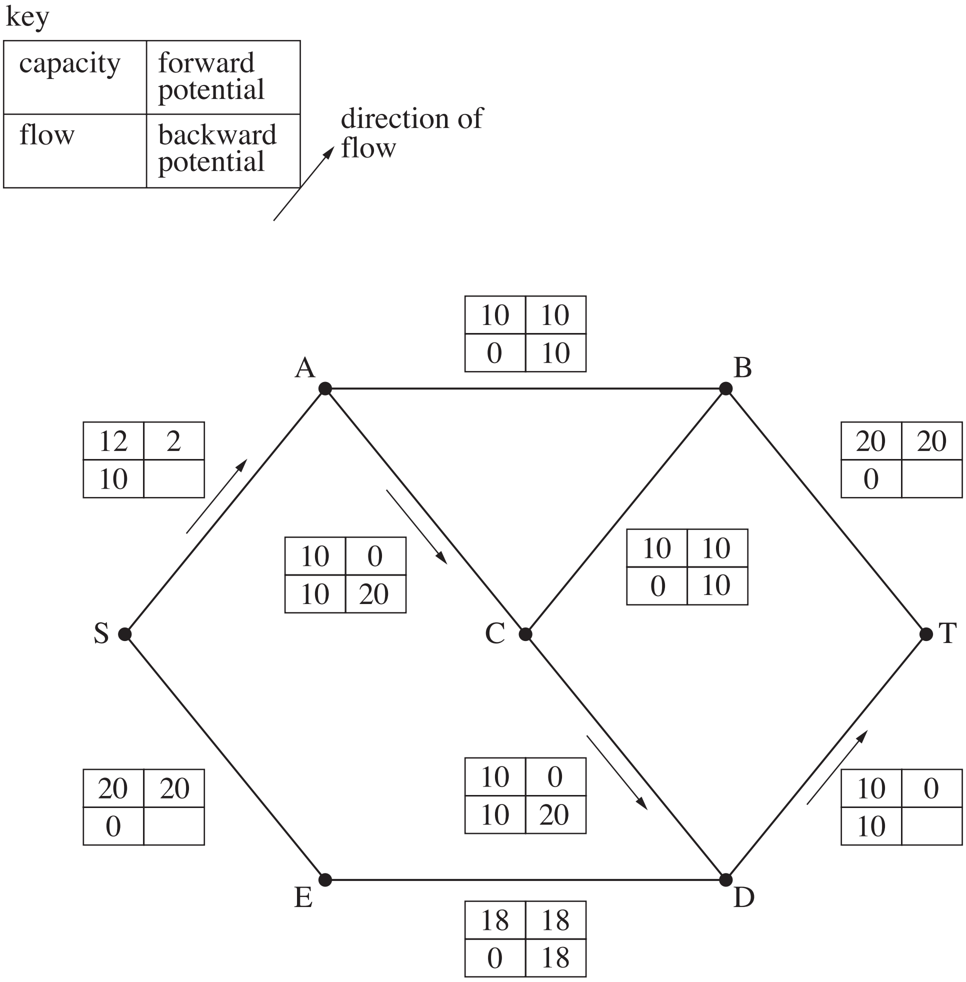
The performance is further improved by making the correction equal in magnitude to a quarter of the difference between the actual bearing and the desired bearing.

(iv) The desired bearing is still 0°.
Build a spreadsheet model incorporating this improvement to show B_n for $0 \leqslant n \leqslant 20$ given that $\mathrm{B}_0 = 2$ and $\mathrm{B}_1 = 4$.
Describe what happens. **[3]**

(v) Solve the recurrence relation $\mathrm{B}_{n+2} - \mathrm{B}_{n+1} + \frac{1}{4}\mathrm{B}_n = 0$, with $\mathrm{B}_0 = 2$ and $\mathrm{B}_1 = 4$.
Compare the values for $\mathrm{B}_2, \mathrm{B}_3, \ldots, \mathrm{B}_{20}$ given by your solution to those you found in part **(iv)**. **[7]**

Print out your spreadsheets.

2 The diagram shows a pipe network with a flow of 10 units established through it. Except for pipes leaving the source (S), or entering the sink (T), flows can be in either direction.



(i) Use flow augmentation to establish a flow of 30 through the network. Redraw the network after each augmentation, showing updated labels. **[7]**

(ii) Give a cut to prove that the flow of 30 units is maximal. **[2]**

(iii) Formulate an LP to find the maximum flow through the network. **[7]**

(iv) Run your LP and interpret the results. **[2]**

3 Bill and Fred are playing each other at cards. Bill is a better player than Fred and has a probability of 0.55 of winning each hand. The loser of a hand pays £1 to the winner.
Bill currently has £5 and Fred has £5.

(i) Build a spreadsheet to simulate what happens. **[3]**

(ii) Run your simulation ten times, each time continuing until one of Bill and Fred is ruined (i.e. runs out of money). Hence estimate the probability that Bill is ruined. **[4]**

Bill plays the same game in a casino, again starting with £5. His probability of winning a hand is now 0.45, and this time his opponent, the casino, has a limitless amount of money and cannot be ruined. When the probability of winning a hand is less than 0.5, Bill will eventually be ruined.

(iii) Use simulation with ten repetitions to estimate how many hands Bill can expect to play before he is ruined. **[5]**

Tim is playing a singles tennis game, and the score is “deuce”. This means that the score is level, and that the first player to be two points ahead wins the game. The probability of Tim winning each point is 0.55.

(iv) Build a spreadsheet to simulate the remainder of the game.
Use ten repetitions to produce an estimate of the probability of Tim winning the game. **[4]**

Minal has a large pot into which she sometimes puts a £1 coin. She also sometimes takes a £1 coin out. Deposits and withdrawals are in the ratio 55:45.

(v) Build a spreadsheet to simulate this situation, given that there are currently five £1 coins in the pot.
What difficulty would you encounter in trying to use simulation to estimate the probability of the pot being emptied? **[2]**

4 J. R. Wing has £4 million to invest in three oil well sites. The revenue from each site will depend on the investment in that site, as shown in the table. The amount invested must be an exact multiple of £1 million.

Amount invested (£ million)	Revenue (£ million)		
	Site 1	Site 2	Site 3
0	4	3	3
1	7	6	7
2	8	10	8
3	9	12	13
4	11	14	15

(i) Let s*i*m*j* be an indicator variable which takes the value 1 if £*j* million is invested in site *i*, and 0 otherwise ($i = 1, 2$ or 3 and $j = 0, 1, 2, 3$ or 4). Thus, for instance, s2m3 takes the value 1 if £3 million is invested in site 2.
Formulate the investment problem as an LP involving 15 such indicator variables.
(The instruction "INT 15" after "END" will make LINDO regard all 15 variables as indicator variables.) **[8]**

(ii) Run your LP and interpret the results. **[6]**

(iii) Adapt your LP and solve the problem if the amount that Mr Wing has to invest is

(*A*) £3 million,

(*B*) £8 million.

Interpret your solution in each case. **[4]**

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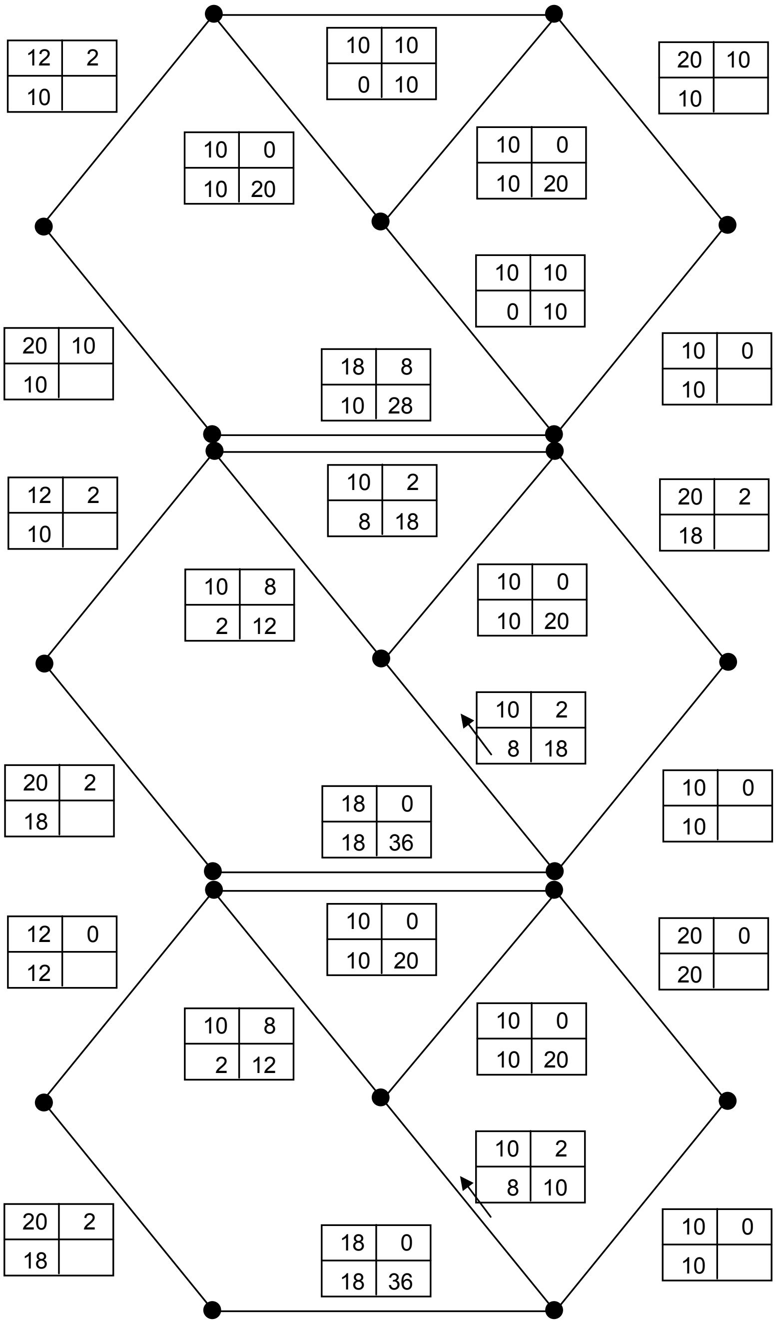
Question 1.

(i)	$B_{n+2} = B_{n+1} + (0 - B_n)$	M1 A1
(ii)	Oscillation: 2, 4, 2, –2, –4, –2, 2, 4, …	M1 A1 B1
(iii)	$B_{n+2} - B_{n+1} + \frac{1}{2}B_n = 0$	B1
	2, 4, 3, 1, –0.5, –1, …, 0.00391, –0.00195	B1
	Oscillatory convergence	B1
(iv)	2, 4, 3.5, 2.5, 1.625, 1, …, 0.00022, 0.00012	B1
	Faster and uniform convergence	B1 B1
(v)	Auxiliary eqn: $x^2 - x + \frac{1}{4} = 0$	B1
	$x = \frac{1}{2}$	B1
	$B_n = A\left(\frac{1}{2}\right)^n + Bn\left(\frac{1}{2}\right)^n$	B1
	2 = A	B1
	$4 = 1 + \frac{1}{2}B$ giving B = 6	B1
	$B_n = (2+6n)\left(\frac{1}{2}\right)^n$ or $(1+3n)\left(\frac{1}{2}\right)^{n-1}$	B1
	"the same"	B1

Question 2.

(i) e.g. 	M1 A1 M1 reversal A1 B1 rest M1 A1

(ii)	{S, A, C, D, E} / {B, T}	M1 A1
(iii)	e.g.	M1 variables A1 objective M1 balancing A1 M1 capacities A1 forwards A1 backwards

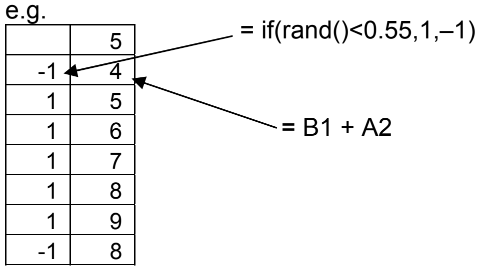
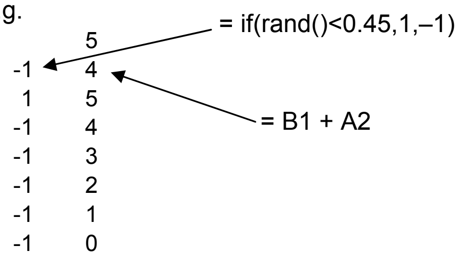
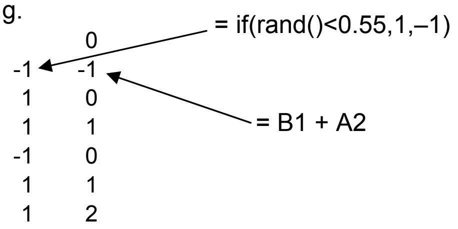
```
Max  SA + SE
st   SA + BA + CA – AB – AC = 0
     AB + CB – BA – BC – BT = 0
     AC + DC + BC – CA – CB – CD = 0
     SE + DE – ED = 0
     CD + ED – DC – DE – DT = 0
     SA < 12
     SE < 20
     AB < 10
     BA < 10
     AC < 10
     CA < 10
     BC < 10
     CB < 10
     CD < 10
     DC < 10
     ED < 18
     DE < 18
     BT < 20
     DT < 10
end
```

(iv)		B1 running

```
OBJECTIVE FUNCTION VALUE
1)     30.00000
VARIABLE        VALUE          REDUCED COST
SA              12.000000      0.000000
SE              18.000000      0.000000
BA        0.000000       0.000000
CA        0.000000       0.000000
AB              10.000000      0.000000
AC        2.000000       0.000000
CB              10.000000      0.000000
BC              0.000000       0.000000
BT              20.000000      0.000000
DC        8.000000       0.000000
CD        0.000000       0.000000
DE              0.000000       1.000000
ED              18.000000      0.000000
DT              10.000000      0.000000
```

	Solution as per part (i)	B1

Question 3.

(i)	e.g.  etc.	M1 A1 "if" or equivalent A1 accumulation
(ii)	repeating until a player is ruined repeating 10 times estimating the probability (theoretical value is 0.2683)	M1 A1 M1 A1
(iii)	e.g.  etc. estimating the run length The theoretical value is 50, so there should be some long runs seen.	B1 change of parameter M1 count to ruin A1 repetitions M1 A1
(iv)	e.g.  etc repetitions + probability estimate (theoretical answer = 0.599)	M1 A1 termination condition B1 B1 B1
(v)	As above How can one tell when a simulation is not emptying the pot?	B1

Question 4.

(i) max 4s1m0+7s1m1+8s1m2+9s1m3+11s1m4+3s2m0	M1
+6s2m1+10s2m2+12s2m3+14s2m4+3s3m0	A1
+7s3m1+8s3m2+13s3m3+15s3m4	
st s1m0+s1m1+s1m2+s1m3+s1m4=1	B1
s2m0+s2m1+s2m2+s2m3+s2m4=1	B1
s3m0+s3m1+s3m2+s3m3+s3m4=1	B1
s1m1+2s1m2+3s1m3+4s1m4+s2m1+2s2m2+3s2m3	M1
+4s2m4 +s3m1+2s3m2+3s3m3+4s3m4=4	A1
end	
int 15	B1

(ii)

```
LP OPTIMUM FOUND AT STEP     14
OBJECTIVE VALUE =   24.0000000

NEW INTEGER SOLUTION OF 24.0000000 AT BRANCH 0 PIVOT 14
RE-INSTALLING BEST SOLUTION...

OBJECTIVE FUNCTION VALUE
1)     24.00000

VARIABLE     VALUE     REDUCED COST
     S1M0        0.000000        -4.000000
     S1M1        1.000000        -7.000000
     S1M2        0.000000        -8.000000                M1
     S1M3        0.000000        -9.000000                A1
     S1M4        0.000000       -11.000000
     S2M0        0.000000        -3.000000
     S2M1        0.000000        -6.000000
     S2M2        1.000000       -10.000000
     S2M3        0.000000       -12.000000
     S2M4        0.000000       -14.000000
     S3M0        0.000000        -3.000000
     S3M1        1.000000        -7.000000
     S3M2        0.000000        -8.000000
     S3M3        0.000000       -13.000000
     S3M4        0.000000       -15.000000

ROW   SLACK OR SURPLUS     DUAL PRICES
     2)        0.000000              0.000000
     3)        0.000000              0.000000
     4)        0.000000              0.000000
     5)        0.000000              0.000000                B3
                                                            B1
```

Invest £1 million at site 1, £2 million at site 2 and £1 million at site 3.	M1
Revenue = £24 million.	A1
(iii) (a) £2 million at site 2 and £1 million at site 3.	M1
Revenue = £21 million.	A1
(b) £1 million at site 1, £3 million at site 2 and £4 million at site 3 (or £1m, £4m and £3m).	
Revenue = £34 million.	

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General Comments

There were fewer entrants for this paper than has been the case since it was first set. This is a shame. Responses to the paper over the years have been excellent. They have demonstrated that the concept is viable. The mathematics, and its development of mathematical modelling skills, has been exciting.

Comments on Individual Questions

Overall, performances were good. The comments below focus on problem issues.

1) **Recurrence Relations**
The Excel modelling in this question was disappointing. Many candidates contrived to get it completely wrong, despite the prompts within the question.
(Note that the question was identical in structure to the 2001 Chancellor of the Exchequer in Mathland question in paper 2622 in 2001.)

   (i) This was quite difficult.
   The markscheme says it all – "$B_{n+2} = B_{n+1} + (0 – B_n)$" – but that is a rather sophisticated response. Candidates needed to identify that the correction is – B_n, to be added on to B_{n+1}, and to take effect at B_{n+2}.

   (ii) There was very little excuse for not getting this right. All that was required was given in the question.

   (iii) One can understand that some candidates might get this wrong initially, and many did. They applied the factor of 0.5 to the whole of the RHS. However, one might have expected this to be corrected once the question for part (v) had been read.

2) **Network Flows**
Not all candidates seemed to be prepared for the question about flow labelling in part (i). Many were not able to "augment", and many attempts were seen in which flows were increased along several routes at the same time.
Attempts at the LP formulation were generally much better, although some candidates failed to capture all possible flow directions in the vertex constraints.

3) **Simulation**
This question turned into a test of candidates' organisational abilities. Most were able, to a greater or lesser extent, to handle the mathematics and the associated Excel work. Only the best candidates were able to keep a grip on all that was required. Typically, part (iv), the tennis game, had a clear need for the identification of a stopping criterion. Whether or not one was provided by the candidate was clearly discriminating.

4) **LP Modelling**
It was anticipated that less good candidates would be confused by the s*i*m*j* notation for the variables, and this proved to be the case. Fewer good solutions were seen than might have been expected, given student responses in the past.