RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

MONDAY 2 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (36 marks)

1 **(i)** Write down the matrix for reflection in the y-axis. [1]

(ii) Write down the matrix for enlargement, scale factor 3, centred on the origin. [1]

(iii) Find the matrix for reflection in the y-axis, followed by enlargement, scale factor 3, centred on the origin. [2]

2 Indicate on a single Argand diagram

(i) the set of points for which $|z-(-3+2\text{j})|=2$, [3]

(ii) the set of points for which $\arg(z-2\text{j})=\pi$, [3]

(iii) the two points for which $|z-(-3+2\text{j})|=2$ and $\arg(z-2\text{j})=\pi$. [1]

3 Find the equation of the line of invariant points under the transformation given by the matrix $\mathbf{M}=\begin{pmatrix}-1 & -1\\ 2 & 2\end{pmatrix}$. [3]

4 Find the values of A, B, C and D in the identity $3x^3-x^2+2\equiv A(x-1)^3+(x^3+Bx^2+Cx+D)$. [5]

5 You are given that $\mathbf{A}=\begin{pmatrix}1 & 2 & 4\\ 3 & 2 & 5\\ 4 & 1 & 2\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}-1 & 0 & 2\\ 14 & -14 & 7\\ -5 & 7 & -4\end{pmatrix}$.

(i) Calculate $\mathbf{AB}$. [3]

(ii) Write down $\mathbf{A}^{-1}$. [2]

6 The roots of the cubic equation $2x^3+x^2-3x+1=0$ are α, β and γ. Find the cubic equation whose roots are 2α, 2β and 2γ, expressing your answer in a form with integer coefficients. [5]

7 **(i)** Show that $\dfrac{1}{3r-1}-\dfrac{1}{3r+2}\equiv\dfrac{3}{(3r-1)(3r+2)}$ for all integers r. [2]

(ii) Hence use the method of differences to find $\displaystyle\sum_{r=1}^{n}\frac{1}{(3r-1)(3r+2)}$. [5]

Section B (36 marks)

8 A curve has equation $y = \dfrac{2x^2}{(x-3)(x+2)}$.

(i) Write down the equations of the three asymptotes. [3]

(ii) Determine whether the curve approaches the horizontal asymptote from above or below for

(A) large positive values of x,

(B) large negative values of x. [3]

(iii) Sketch the curve. [3]

(iv) Solve the inequality $\dfrac{2x^2}{(x-3)(x+2)} < 0$. [3]

9 Two complex numbers, α and β, are given by $\alpha = 2 - 2\mathrm{j}$ and $\beta = -1 + \mathrm{j}$.

α and β are both roots of a quartic equation $x^4 + Ax^3 + Bx^2 + Cx + D = 0$, where A, B, C and D are real numbers.

(i) Write down the other two roots. [2]

(ii) Represent these four roots on an Argand diagram. [2]

(iii) Find the values of A, B, C and D. [7]

10 **(i)** Using the standard formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, prove that

$$\sum_{r=1}^{n} r^2(r+1) = \tfrac{1}{12}n(n+1)(n+2)(3n+1).$$ [5]

(ii) Prove the same result by mathematical induction. [8]

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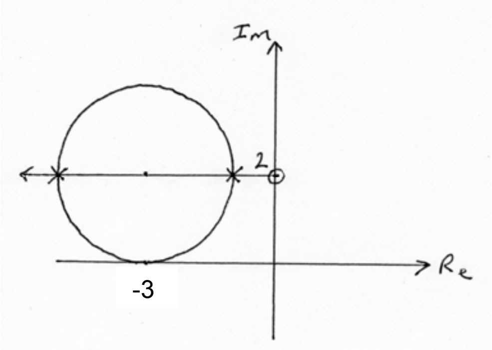
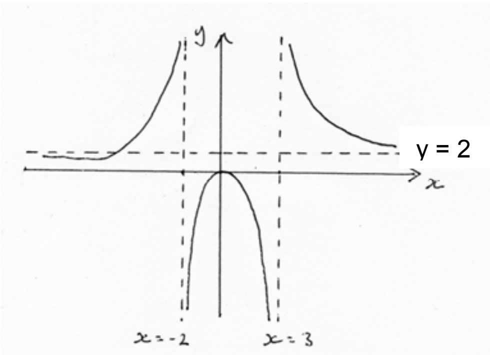
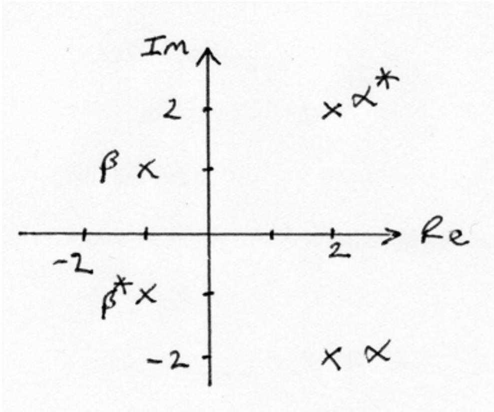
Qu	Answer	Mark	Comment
Section A			
1(i)	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	B1	
1(ii)	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	B1	
1(iii)	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}$	M1 A1 **[4]**	Multiplication, or other valid method (may be implied) c.a.o.
2	 	B3 B3 B1 **[7]**	Circle, B1; centre $-3+2\text{j}$, B1; radius = 2, B1 Line parallel to real axis, B1; through (0, 2), B1; correct half line, B1 Points $-1+2\text{j}$ and $-5+2\text{j}$ indicated c.a.o.
3	$\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix}$ $\Rightarrow -x-y=x,\ 2x+2y=y$ $\Rightarrow y=-2x$	M1 M1 B1 **[3]**	For $\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix}$
4	$3x^3-x^2+2\equiv A(x-1)^3+(x^3+Bx^2+Cx+D)$ $\equiv Ax^3-3Ax^2+3Ax-A+x^3+Bx^2+Cx+D$ $\equiv(A+1)x^3+(B-3A)x^2+(3A+C)x+(D-A)$ $\Rightarrow A=2,\ B=5,\ C=-6,\ D=4$	M1 B4 **[5]**	Attempt to compare coefficients One for each correct value

5(i)	$\mathbf{AB} = \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$	B3 **[3]**	Minus 1 each error to minimum of 0
5(ii)	$\mathbf{A}^{-1} = \frac{1}{7}\begin{pmatrix} -1 & 0 & 2 \\ 14 & -14 & 7 \\ -5 & 7 & -4 \end{pmatrix}$	M1 A1 **[2]**	Use of B c.a.o.
6	$w = 2x \Rightarrow x = \frac{w}{2}$	B1	Substitution. For substitution $x = 2w$ give B0 but then follow through for a maximum of 3 marks
	$\Rightarrow 2\left(\frac{w}{2}\right)^3 + \left(\frac{w}{2}\right)^2 - 3\left(\frac{w}{2}\right) + 1 = 0$	M1 A1	Substitute into cubic Correct substitution
	$\Rightarrow w^3 + w^2 - 6w + 4 = 0$	A2 **[5]**	Minus 1 for each error (including '= 0' missing), to a minimum of 0 Give full credit for integer multiple of equation
6	**OR**		
	$\alpha + \beta + \gamma = -\frac{1}{2}$ $\alpha\beta + \alpha\gamma + \beta\gamma = -\frac{3}{2}$ $\alpha\beta\gamma = -\frac{1}{2}$	B1	All three
	Let new roots be *k*, *l*, *m* then $k + l + m = 2(\alpha + \beta + \gamma) = -1 = \frac{-B}{A}$ $kl + km + lm = 4(\alpha\beta + \alpha\gamma + \beta\gamma) = -6 = \frac{C}{A}$ $klm = 8\alpha\beta\gamma = -4 = \frac{-D}{A}$	M1 A1	Attempt to use sums and products of roots of original equation to find sums and products of roots in related equation Sums and products all correct
	$\Rightarrow \omega^3 + \omega^2 - 6\omega + 4 = 0$	A2 **[5]**	ft their coefficients; minus one for each error (including '= 0' missing), to minimum of 0 Give full credit for integer multiple of equation

7(i)	$\frac{1}{3r-1}-\frac{1}{3r+2}\equiv\frac{3r+2-(3r-1)}{(3r-1)(3r+2)}$	M1	Attempt at correct method
	$\equiv\frac{3}{(3r-1)(3r+2)}$	A1	Correct, without fudging
		[2]	
7(ii)	$\sum_{r=1}^{n}\frac{1}{(3r-1)(3r+2)}=\frac{1}{3}\sum_{r=1}^{n}\left[\frac{1}{3r-1}-\frac{1}{3r+2}\right]$	M1	Attempt to use identity
	$=\frac{1}{3}\left[\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{8}\right)+.......+\left(\frac{1}{3n-1}-\frac{1}{3n+2}\right)\right]$	A1 M1	Terms in full (at least two) Attempt at cancelling
	$=\frac{1}{3}\left[\frac{1}{2}-\frac{1}{3n+2}\right]$	A2	A1 if factor of $\frac{1}{3}$ missing,
		[5]	A1 max if answer not in terms of *n*
			Section A Total: 36

Section B			
8(i)	$x=3$, $x=-2$, $y=2$	B1 B1 B1 **[3]**	
8(ii)	Large positive *x*, $y \to 2^+$ (e.g. consider $x=100$) Large negative *x*, $y \to 2^-$ (e.g. consider $x=-100$)	M1 B1 B1 **[3]**	Evidence of method required
8(iii)	Curve Central and RH branches correct Asymptotes correct and labelled LH branch correct, with clear minimum  	B1 B1 B1 **[3]**	
8(iv)	$-2 < x < 3$ $x \neq 0$	B2 B1 **[3]**	B2 max if any inclusive inequalities appear B3 for $-2 < x < 0$ and $0 < x < 3$,

9(i)	$2+2\text{j}$ and $-1-\text{j}$	B2 **[2]**	1 mark for each
9(ii)		B2 **[2]**	1 mark for each correct pair
9(iii)	$(x-2-2\text{j})(x-2+2\text{j})(x+1+\text{j})(x+1-\text{j})$	M1 B2	Attempt to use factor theorem Correct factors, minus 1 each error B1 if only errors are sign errors
	$=(x^2-4x+8)(x^2+2x+2)$	A1	One correct quadratic with real coefficients (may be implied)
	$=x^4+2x^3+2x^2-4x^3-8x^2-8x+8x^2+16x+16$ $=x^4-2x^3+2x^2+8x+16$	M1	Expanding
	$\Rightarrow A=-2,\ B=2,\ C=8,\ D=16$	A2 **[7]**	Minus 1 each error, A1 if only errors are sign errors
	OR		
	$\sum\alpha=2$	B1	
	$\alpha\beta\gamma\delta=16$	B1	
	$\sum\alpha\beta=\alpha\alpha^*+\alpha\beta+\alpha\beta^*+\beta\beta^*+\beta\alpha^*+\beta^*\alpha^*$	M1	
	$\sum\alpha\beta\gamma=\alpha\alpha^*\beta+\alpha\alpha^*\beta^*+\alpha\beta\beta^*+\alpha^*\beta\beta^*$	M1	
	$\sum\alpha\beta=2$, $\sum\alpha\beta\gamma=-8$	A1	Both correct
	$A=-2,\ B=2,\ C=8,\ D=16$	A2 **[7]**	Minus 1 each error, A1 if only errors are sign errors
	OR		
	Attempt to substitute in one root Attempt to substitute in a second root	M1 M1 A1	Both correct
	Equating real and imaginary parts to 0 Attempt to solve simultaneous equations	M1 M1	
	$A=-2,\ B=2,\ C=8,\ D=16$	A2 **[7]**	Minus 1 each error, A1 if only errors are sign errors

Qu	Answer	Mark	Comment
Section B (continued)			
10(i)	$\sum_{r=1}^{n} r^2(r+1) = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r^2$	M1	Separation of sums (may be implied)
	$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{6}n(n+1)(2n+1)$	B1	One mark for both parts
	$= \frac{1}{12}n(n+1)\left[3n(n+1)+2(2n+1)\right]$	M1	Attempt to factorise (at least two linear algebraic factors)
	$= \frac{1}{12}n(n+1)(3n^2+7n+2)$	A1	Correct
	$= \frac{1}{12}n(n+1)(n+2)(3n+1)$	E1	Complete, convincing argument
		[5]	
10(ii)	$\sum_{r=1}^{n} r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(3n+1)$		
	n = 1, LHS = RHS = 2	B1	2 must be seen
	Assume true for *n* = *k*	E1	Assuming true for *k*
	$\sum_{r=1}^{k} r^2(r+1) = \frac{1}{12}k(k+1)(k+2)(3k+1)$		
	$\sum_{r=1}^{k+1} r^2(r+1)$		
	$= \frac{1}{12}k(k+1)(k+2)(3k+1) + (k+1)^2(k+2)$	B1	(*k* + 1)th term
	$= \frac{1}{12}(k+1)(k+2)\left[k(3k+1)+12(k+1)\right]$	M1	Attempt to factorise
	$= \frac{1}{12}(k+1)(k+2)(3k^2+13k+12)$	A1	Correct
	$= \frac{1}{12}(k+1)(k+2)(k+3)(3k+4)$	A1	Complete convincing argument
	$= \frac{1}{12}(k+1)((k+1)+1)((k+1)+2)(3(k+1)+1)$		
	But this is the given result with *k* + 1 replacing *k*. Therefore if it is true for *k* it is true for *k* + 1. Since it is true for *k* = 1, it is true for *k* = 1, 2, 3 and so true for all positive integers.	E1 E1 **[8]**	Dependent on previous A1 and previous E1 Dependent on first B1 and previous E1
			Section B Total: 36
			Total: 72

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General comments
Candidates generally performed well, demonstrating a good knowledge of the syllabus. The paper enabled the candidates to demonstrate their knowledge, but also differentiated well between them.

Some candidates dropped marks through careless algebraic manipulation, and a smaller number by failing to label diagrams and graphs clearly.

Comments on Individual Questions

1) There were many good answers, but some surprising errors, including vectors given as answers. In (iii) candidates must show the multiplication of the two matrices to avoid loss of a method mark if either of their earlier answers was incorrect.

2) (i) A circle of radius 2 was usually clearly shown. An incorrect centre was the most common error.

(ii) A full line was often shown, instead of a half line, and many lines went through -2j instead of 2j. Most candidates correctly drew lines parallel to the real axis.

(iii) If (i) and (ii) were correct, (iii) was usually correct too, though not all candidates showed the points clearly on their Argand diagrams.

3) This was very well done by about half the candidates. Others either omitted it or made errors such as failing to use an invariant vector, or setting the transformed point (-x – y, 2x + 2y) to (0, 0).

4) There were many good answers, but also many careless mistakes. Often $(1-x)^3$ was expanded without multiplying all terms by *A* (especially the -1). There was also evidence of careless solving of the equation leading to *B*.

5) (i) This was well done by almost all candidates.

(ii) This was often badly done, indicating that many students did not properly understand inverse matrices. Many tried to use an algorithm to invert the matrix; some did this correctly, but wasted a lot of time in doing so; others tried to apply "rules" for inverting a 2×2 matrix to the 3×3 case.

6) The most popular method involved using sums and products of roots, but there were quite a few careless errors involving signs. Those who substituted $x=\frac{w}{2}$ were usually successful. Several candidates omitted the '= 0' from their equation and so lost the final mark.

7) (i) This was well done by almost all candidates.

(ii) The factor of $\frac{1}{3}$ was very often lost following successful expansion and cancellation of terms. A few candidates multiplied by 3 instead of dividing. A few left answers in terms of *r* not *n*.

8) (i) This was well answered, but *y* = 1 and *y* = 0 were quite frequent errors for the asymptote parallel to the *x*-axis.

(ii) Most candidates showed their method, usually by substituting large positive and negative values of *x*. The results were not always put to correct use in (iii).

(iii) The left-hand branch was often incorrect. Many candidates seemed to hold the misconception that the graph cannot cross an asymptote. Even where graphs were shown crossing the asymptote, they often did not show a clear minimum.

(iv) Many correct answers, but $x \neq 0$ was often omitted. A fairly common error was $x < -2, x > 3$, presumably from a misinterpretation of the "< 0" in the question. Several candidates used algebraic methods when the answer could be most easily found directly from the graph.

9) (i) and (ii) were both well done by almost all candidates.

(iii) Use of the sums and products of roots was the most common method, but candidates found difficulty in correctly finding $\sum \alpha\beta$ and $\sum \alpha\beta\gamma$. They also had problems in assigning the correct signs to four coefficients, and got muddled by *A B C D* and *a b c d* and *e*.

(iv) Multiplying factors with conjugate pairs was a less popular but more successful method, apart from occasional sign errors.

A few candidates tried to substitute roots, equating the real and imaginary parts to 0 and then solving simultaneous equations. This was the least successful method.

10) (i) This was usually started well, but a surprising number did not take out the common factors to ease the working. As a result there were algebraic errors and sometimes fudging of the final factorisation of the quartic.

(ii) This was well done in many cases, but there were signs that some candidates were under time pressure. The basic structure was well done in many cases, but the presentation and notation were often poor, with r, k and n often used in the wrong places. Summation signs were often omitted so that statements effectively meaning "last term = sum of k terms + last term" frequently appeared.

(iii) Candidates often failed to take out common factors in the algebraic manipulation, and their proofs faltered as a result. The final words of explanation were not always convincing.

(iv) However, it was encouraging that many perfect solutions were seen, showing that candidates understood the proof and were able to communicate their arguments clearly.