OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Two complex numbers are given by $\alpha = -3 + \mathrm{j}$ and $\beta = 5 - 2\mathrm{j}$.

Find $\alpha\beta$ and $\dfrac{\alpha}{\beta}$, giving your answers in the form $a + b\mathrm{j}$, showing your working. **[5]**

2 You are given that $\mathbf{A} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 & 1 \\ 2 & -3 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 5 & 1 & 8 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} -2 & 0 \\ 4 & 1 \end{pmatrix}$.

(i) Calculate, where they exist, **AB**, **CA**, **B** + **D** and **AC** and indicate any that do not exist. **[5]**

(ii) Matrices **B** and **D** represent transformations B and D respectively. Find the single matrix that represents transformation B followed by transformation D. **[2]**

3 The roots of the cubic equation $4x^3 - 12x^2 + kx - 3 = 0$ may be written $a - d$, a and $a + d$. Find the roots and the value of k. **[6]**

4 You are given that if $\mathbf{M} = \begin{pmatrix} 4 & 0 & 1 \\ -6 & 1 & 1 \\ 5 & 2 & 5 \end{pmatrix}$ then $\mathbf{M}^{-1} = \dfrac{1}{k}\begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}$.

Find the value of k. Hence solve the following simultaneous equations. **[6]**

$$\begin{aligned} 4x \quad\;\; + \; z &= 9 \\ -6x + \; y + \; z &= 32 \\ 5x + 2y + 5z &= 81 \end{aligned}$$

5 Use standard series formulae to show that $\sum_{r=1}^{n} (r+2)(r-3) = \frac{1}{3}n(n^2 - 19)$. **[6]**

6 Prove by induction that $1 \times 2 + 2 \times 3 + \ldots + n(n+1) = \dfrac{n(n+1)(n+2)}{3}$ for all positive integers n. **[6]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{5x - 9}{(2x - 3)(2x + 7)}$.

(i) Write down the equations of the two vertical asymptotes and the one horizontal asymptote. **[3]**

(ii) Describe the behaviour of the curve for large positive and large negative values of x, justifying your answers. **[3]**

(iii) Sketch the curve. **[3]**

(iv) Solve the inequality $\dfrac{5x - 9}{(2x - 3)(2x + 7)} \leqslant 0$. **[3]**

8 **(a)** Fig. 8 shows an Argand diagram.

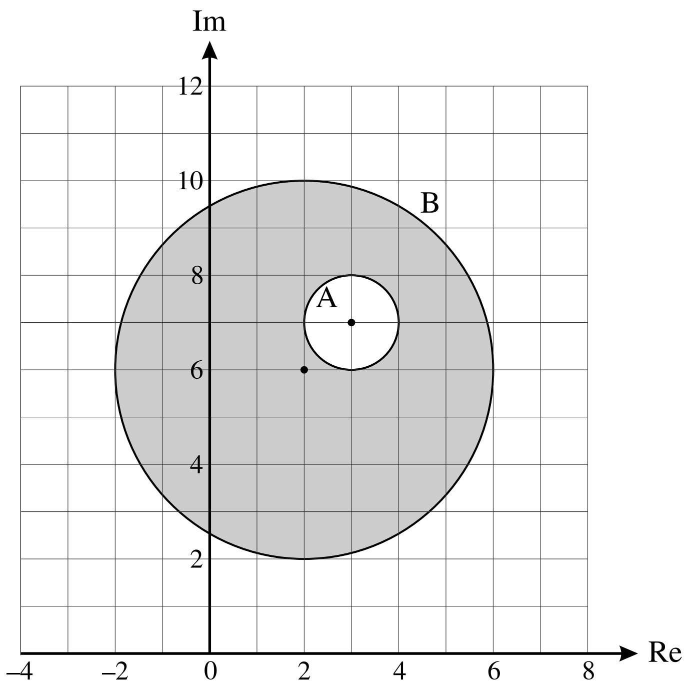


Fig. 8

(i) Write down the equation of the locus represented by the circumference of circle B. **[3]**

(ii) Write down the two inequalities that define the shaded region between, but not including, circles A and B. **[3]**

(b) **(i)** Draw an Argand diagram to show the region where

$$\frac{\pi}{4} < \arg\big(z - (2 + \mathrm{j})\big) < \frac{3\pi}{4}.$$ **[3]**

(ii) Determine whether the point $43 + 47\mathrm{j}$ lies within this region. **[3]**

9 **(i)** Verify that $\dfrac{4+r}{r(r+1)(r+2)} = \dfrac{2}{r} - \dfrac{3}{r+1} + \dfrac{1}{r+2}$. **[2]**

(ii) Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{4+r}{r(r+1)(r+2)} = \frac{3}{2} - \frac{2}{n+1} + \frac{1}{n+2}.$$ **[6]**

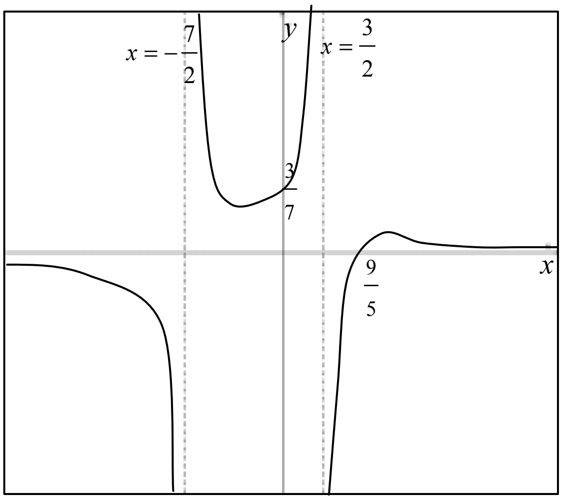
(iii) Write down the limit to which $\displaystyle\sum_{r=1}^{n} \frac{4+r}{r(r+1)(r+2)}$ converges as n tends to infinity. **[1]**

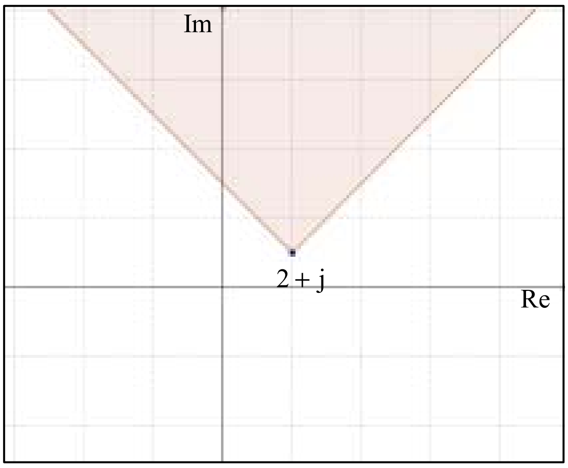
(iv) Find $\displaystyle\sum_{r=50}^{100} \frac{4+r}{r(r+1)(r+2)}$, giving your answer to 3 significant figures. **[3]**

4755 (FP1) Further Concepts for Advanced Mathematics

1	$\alpha\beta = (-3+\mathrm{j})(5-2\mathrm{j}) = -13+11\mathrm{j}$	M1 A1 **[2]**	Use of $\mathrm{j}^2 = -1$
	$\dfrac{\alpha}{\beta} = \dfrac{-3+\mathrm{j}}{5-2\mathrm{j}} = \dfrac{(-3+\mathrm{j})(5+2\mathrm{j})}{29} = \dfrac{-17}{29} - \dfrac{1}{29}\mathrm{j}$	M1 A1 A1 **[3]**	Use of conjugate 29 in denominator All correct
2 **(i)**	**AB** is impossible $\mathbf{CA} = (50)$ $\mathbf{B}+\mathbf{D} = \begin{pmatrix} 3 & 1 \\ 6 & -2 \end{pmatrix}$ $\mathbf{AC} = \begin{pmatrix} 20 & 4 & 32 \\ -10 & -2 & -16 \\ 20 & 4 & 32 \end{pmatrix}$	B1 B1 B1 B2 **[5]**	 –1 each error
(ii)	$\mathbf{DB} = \begin{pmatrix} -2 & 0 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} -10 & -2 \\ 22 & 1 \end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply in correct order c.a.o.
3	$\alpha+\beta+\gamma = a-d+a+a+d = \dfrac{12}{4} \Rightarrow a = 1$	M1 A1	Valid attempt to use sum of roots $a = 1$, c.a.o.
	$(a-d)a(a+d) = \dfrac{3}{4} \Rightarrow d = \pm\dfrac{1}{2}$	M1	Valid attempt to use product of roots
	So the roots are $\dfrac{1}{2}$, 1 and $\dfrac{3}{2}$	A1	All three roots
	$\alpha\beta+\alpha\gamma+\beta\gamma = \dfrac{k}{4} = \dfrac{11}{4} \Rightarrow k = 11$	M1 A1 **[6]**	Valid attempt to use $\alpha\beta+\alpha\gamma+\beta\gamma$, or to multiply out factors, or to substitute a root $k = 11$ c.a.o.

4	$\mathbf{MM}^{-1} = \frac{1}{k}\begin{pmatrix} 4 & 0 & 1 \\ -6 & 1 & 1 \\ 5 & 2 & 5 \end{pmatrix}\begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}$	M1	Attempt to consider $\mathbf{MM}^{-1}$ or $\mathbf{M}^{-1}\mathbf{M}$ (may be implied)
	$= \frac{1}{k}\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} \Rightarrow k = 5$	A1 **[2]**	c.a.o.
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}\begin{pmatrix} 9 \\ 32 \\ 81 \end{pmatrix}$	M1	Attempt to pre-multiply by $\mathbf{M}^{-1}$
		M1	Attempt to multiply matrices
	$\frac{1}{5}\begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}\begin{pmatrix} 9 \\ 32 \\ 81 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -10 \\ 15 \\ 85 \end{pmatrix}$	A1	Correct
	$\Rightarrow x = -2,\ y = 3,\ z = 17$	A1 **[4]**	All 3 correct s.c. B1 if matrices not used
5	$\sum_{r=1}^{n}(r+2)(r-3) = \sum_{r=1}^{n}(r^2 - r - 6)$		
	$= \sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r - 6n$	M1	Separate into 3 sums
	$= \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - 6n$	A2	–1 each error
	$= \frac{1}{6}n[(n+1)(2n+1) - 3(n+1) - 36]$	M1	Valid attempt to factorise (with n as a factor)
	$= \frac{1}{6}n(2n^2 - 38) = \frac{1}{3}n(n^2 - 19)$	A1 A1 **[6]**	Correct expression c.a.o. Complete, convincing argument
6	When $n = 1$, $\frac{n(n+1)(n+2)}{3} = 2$,	B1	
	so true for $n = 1$ Assume true for $n = k$	E1	Assume true for k
	$2 + 6 + ... + k(k+1) = \frac{k(k+1)(k+2)}{3}$		
	$\Rightarrow 2 + 6 + ... + (k+1)(k+2)$	M1	Add $(k+1)$th term to both sides
	$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$		
	$= \frac{1}{3}(k+1)(k+2)(k+3)$	A1	c.a.o. with correct simplification
	$= \frac{(k+1)((k+1)+1)((k+1)+2)}{3}$		
	But this is the given result with $k + 1$ replacing k. Therefore if it is true for $n = k$ it is true for $n = k + 1$.	E1	Dependent on A1 and previous E1
	Since it is true for $n = 1$, it is true for $n = 1, 2, 3$ and so true for all positive integers.	E1 **[6]**	Dependent on B1 and previous E1

7 (i)	$x=\frac{-7}{2},\ x=\frac{3}{2}, y=0$	B1 B1 B1 **[3]**	
(ii)	Large positive x, $y \to 0^+$ (e.g. consider $x = 100$) Large negative x, $y \to 0^-$ (e.g. consider $x = -100$)	B1 B1 M1 **[3]**	 Evidence of method
(iii)		B1 B1 B1 **[3]**	Intercepts correct and labelled LH and central branches correct RH branch correct, with clear maximum
(iv)	$x<-\frac{7}{2}$ or $\frac{3}{2}<x\le\frac{9}{5}$	B1 B2 **[3]**	 Award B1 if only error relates to inclusive/exclusive inequalities

8(a) (i)	$\lvert z-(2+6\mathrm{j})\rvert = 4$	B1 B1 B1 **[3]**	2 + 6j seen (expression in z) = 4 Correct equation
(ii)	$\lvert z-(2+6\mathrm{j})\rvert < 4$ and $\lvert z-(3+7\mathrm{j})\rvert > 1$	B1 B1 B1 **[3]**	$\lvert z-(2+6\mathrm{j})\rvert < 4$ $\lvert z-(3+7\mathrm{j})\rvert > 1$ (allow errors in inequality signs) Both inequalities correct
(b)(i)		B1 B1 B1 **[3]**	Any straight line through 2 + j Both correct half lines Region between their two half lines indicated
(ii)	$43+47\mathrm{j}-(2+\mathrm{j}) = 41+46\mathrm{j}$ $\arg(41+46\mathrm{j}) = \arctan\left(\frac{46}{41}\right) = 0.843$ $\frac{\pi}{4} < 0.843 < \frac{3\pi}{4}$ so $43+47\mathrm{j}$ does fall within the region	M1 A1 E1 **[3]**	Attempt to calculate argument, or other valid method such as comparison with $y = x - 1$ Correct Justified

9	**(i)**	$\frac{2}{r}-\frac{3}{r+1}+\frac{1}{r+2}$ $=\frac{2(r+1)(r+2)-3r(r+2)+r(r+1)}{r(r+1)(r+2)}$ $=\frac{2r^2+6r+4-3r^2-6r+r^2+r}{r(r+1)(r+2)}=\frac{4+r}{r(r+1)(r+2)}$	M1 A1 **[2]**	Attempt a common denominator Convincingly shown
	(ii)	$\sum_{r=1}^{n}\frac{4+r}{r(r+1)(r+2)}=\sum_{r=1}^{n}\left[\frac{2}{r}-\frac{3}{r+1}+\frac{1}{r+2}\right]$ $=\left(\frac{2}{1}-\frac{3}{2}+\frac{1}{3}\right)+\left(\frac{2}{2}-\frac{3}{3}+\frac{1}{4}\right)+\left(\frac{2}{3}-\frac{3}{4}+\frac{1}{5}\right)+..$ $..+\left(\frac{2}{n-1}-\frac{3}{n}+\frac{1}{n+1}\right)+\left(\frac{2}{n}-\frac{3}{n+1}+\frac{1}{n+2}\right)$ $=\frac{2}{1}-\frac{3}{2}+\frac{2}{2}+\frac{1}{n+1}-\frac{3}{n+1}+\frac{1}{n+2}$ $=\frac{3}{2}-\frac{2}{n+1}+\frac{1}{n+2}$ as required	M1 M1 A2 M1 A1 **[6]**	Use of the given result (may be implied) Terms in full (at least first and one other) At least 3 consecutive terms correct, -1 each error Attempt to cancel, including algebraic terms Convincingly shown
	(iii)	$\frac{3}{2}$	B1 **[1]**	
	(iv)	$\sum_{r=50}^{100}\frac{4+r}{r(r+1)(r+2)}$ $=\sum_{r=1}^{100}\frac{4+r}{r(r+1)(r+2)}-\sum_{r=1}^{49}\frac{4+r}{r(r+1)(r+2)}$ $=\left(\frac{3}{2}-\frac{2}{101}+\frac{1}{102}\right)-\left(\frac{3}{2}-\frac{2}{50}+\frac{1}{51}\right)$ $=0.0104$ (3s.f.)	M1 M1 A1 **[3]**	Splitting into two parts Use of result from (ii) c.a.o.

4755 Further Concepts for Advanced Mathematics (FP1)

General comments

Most candidates showed a good mastery of the material and were well prepared for the examination; many were able to score highly.

The standard of algebra was generally high, but some candidates fell down on several questions through lack of algebraic fluency and it appeared that a small proportion of candidates had been entered for the examination before they were ready.

Comments on Individual Questions

1 Complex numbers

This was generally well done, though a few made careless sign errors. In part (ii) almost all could use the complex conjugate to carry out the division, but some incorrectly assumed the numerator of $\frac{\alpha}{\beta}$ would be -13 + 11j, the answer to part (i).

2 Matrices

(i) **AB** not possible and **CA** and **B** + **D** were mostly correctly found. Several errors seen were: **CA** and **AC** not possible; **AC** = **CA**; **CA** found but **AC** thought impossible. Surprisingly, these errors were often made even by the candidates who scored highly on the paper as a whole.

(ii) **DB** was usually chosen and calculated correctly, but a significant minority of candidates opted for **BD**.

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3 Relationships between the roots of a cubic

There were many very good answers, but there were several common errors:

- A surprising number of candidates thought that $3a = 3$ leads to $a = 3$;
- There were errors with signs in using the standard results for $\sum \alpha$, etc;
- Many forgot to take account of the coefficient of x^3 and so did not divide by 4 when using the standard results;
- Some did not give the roots of the equation, despite a specific request to do so.

Finding k was usually done using $\sum \alpha\beta$, but quite a few chose to substitute a root, usually $x = 1$, which is probably quicker and easier.

Some tried to expand the product $(x-a+d)(x-a)(x-a-d)$, but very few did so without error in lengthy algebraic expressions and those that did often did not take account of the coefficient of x^3.

4 Three by three matrix and simultaneous equations

Most candidates chose to multiply the given matrices and thus found $k = 5$ easily. Some attempted to find the determinant of **M** (finding the determinant of a 3×3 matrix is not on the FP1 specification and so is never needed in the examination), but most of those that were able to do so correctly then assumed that $k = \det$ **M**, failing to check the cofactors. A few candidates failed to use the inverse matrix to solve the equations, as required, thus limiting their marks.

5 Summation of a series using standard results

There were many fully correct answers to this question, but a common error was to assume $\sum 6 = 6$, rather than $6n$. The resulting lack of common factor *n* either led to fudged workings or to a reassessment of the expressions, which was not always accurately pursued.

6 Proof by induction

A good answer requires a clearly stated argument with all the logical steps shown explicitly. In many cases this was achieved, perhaps helped by the simplicity of the algebra needed to establish the result for $1\times 2 + 2\times 3 + \ldots + k(k+1) + (k+1)(k+2)$. This manipulation was often carefully expressed, with correctly placed brackets, but there was often careless treatment of the summation, including omitting it altogether, leading to clearly incorrect statements such as $k(k+1) = \frac{k(k+1)(k+2)}{3}$.

Some candidates do not appear to appreciate the conditional part of the argument, "<u>If true for $n = k$ then true for $n = k + 1$...</u>" and try to run together the $n = 1$, $n = k$ and $n = k + 1$ cases as all true. This is made even more illogical if there was no earlier statement about assuming the truth of the statement when $n = k$.

7 Curve sketching

There were many very good answers to this question.

(i) A fairly common incorrect answer was to give $y = \frac{5}{9}$ for the horizontal asymptote.

(ii) Most candidates showed their method, usually by substituting large positive and negative values of *x*. The results were not always used correctly in (iii).

(iii) The right-hand branch was often incorrect, with no maximum shown. Approaches to asymptotes were sometimes sketched carelessly and many candidates did not label the points where the curve crossed the axes. Several candidates' graphs had inverted sections, suggesting they did not check the behaviour of the function near to the vertical asymptotes.

(iv) This was often answered correctly, given a correctly sketched graph. Some candidates appeared to misread ≤ 0, and gave answers for ≥ 0. Some gave inclusive inequalities at the vertical asymptotes $x = 3/2$ and $x = -7/2$. Some became extremely muddled in an attempt to solve the inequality algebraically, rather than using the sketch of the graph.

8 Loci on the Argand diagram

This question was probably the least well answered of the paper.

(a) (i) Modulus lines were often omitted and "≤ 4" was inserted instead of "$= 4$", although an equation was clearly requested. There were a few correct Cartesian equations given, but also some incorrect ones.

(a) (ii) This produced some misunderstanding and curious notations, despite the instruction to write down two inequalities.

(b) (i) It was not uncommon to see $-2 - \mathrm{j}$ used as the apex of the region. Candidates' diagrams often showed lines from the point $2 + \mathrm{j}$ (or other) to the origin and lines parallel to the Real Axis. Many diagrams were spoilt by inadequate annotation (especially failing to show the angles of the half-lines), in the absence of a properly labelled grid.

(b) (ii) There were few correct answers. Most commonly candidates found $\arg(43+47\mathrm{j})$ instead of $\arg(41+46\mathrm{j})$. Occasionally $\tan^{-1}(41/46)$ or $\tan^{-1}(43/47)$ were used. Those candidates who recognised that $y = x - 1$ was relevant often did well, but some only had a vague idea of this and failed to produce a convincing explanation.

9 Series and proof by induction

This question was often done well, but there was evidence that a few candidates were pushed for time at the end of the paper.

(i) This was usually correctly done, but those who chose to use partial fractions (not required for FP1), rather than simply showing the result as requested, probably used up valuable time on two marks.

(ii) Part (ii) was not always clearly presented. With a given result to work towards, many candidates either fudged their algebra, or did not show sufficient working for a convincing argument. It is always good practice to show at least the first three and the last two terms of the series in full, so that cancelling can be properly justified.

(iii) This was usually answered correctly, but some candidates who considered the structure of the general term instead of the total were led to a give a limit of 0.

(iv) There were few correct answers. A large proportion of candidates calculated Sum(100 terms) – Sum(50 terms). A surprising number believed that "0.01" shows three significant figures. Some candidates successfully repeated the method of differences, rather than using the totals for 100 terms and 49 terms.