OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Thursday 27 May 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Find the values of A, B and C in the identity $4x^2 - 16x + C \equiv A(x + B)^2 + 2$. **[4]**

2 You are given that $\mathbf{M} = \begin{pmatrix} 2 & -5 \\ 3 & 7 \end{pmatrix}$.

$\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ -1 \end{pmatrix}$ represents two simultaneous equations.

(i) Write down these two equations. **[2]**

(ii) Find $\mathbf{M}^{-1}$ and use it to solve the equations. **[4]**

3 The cubic equation $2z^3 - z^2 + 4z + k = 0$, where k is real, has a root $z = 1 + 2\text{j}$.

Write down the other complex root. Hence find the real root and the value of k. **[6]**

4 The roots of the cubic equation $x^3 - 2x^2 - 8x + 11 = 0$ are α, β and γ.

Find the cubic equation with roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$. **[6]**

5 Use the result $\frac{1}{5r-1} - \frac{1}{5r+4} \equiv \frac{5}{(5r-1)(5r+4)}$ and the method of differences to find

$$\sum_{r=1}^{n} \frac{1}{(5r-1)(5r+4)},$$

simplifying your answer. **[6]**

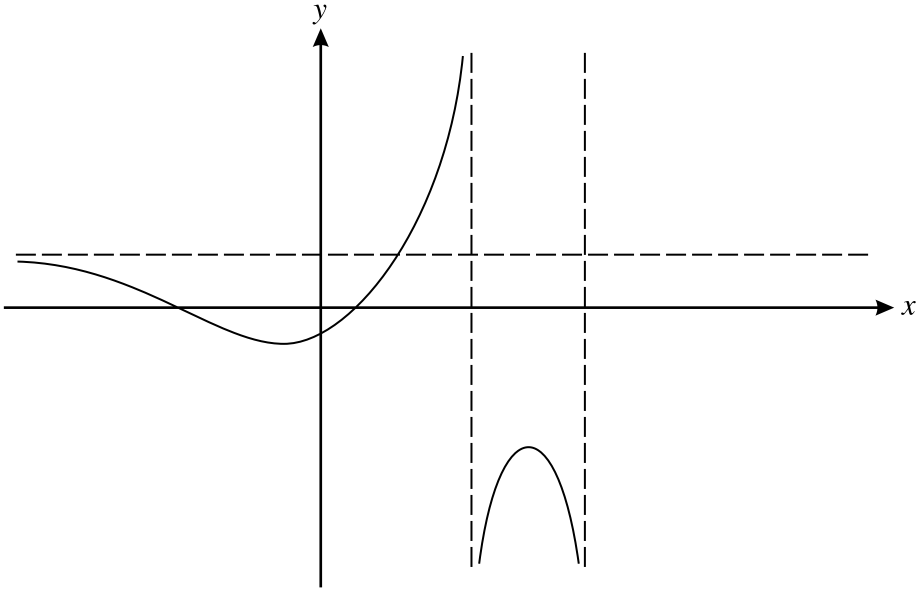
6 A sequence is defined by $u_1 = 2$ and $u_{n+1} = \frac{u_n}{1 + u_n}$.

(i) Calculate u_3. **[2]**

(ii) Prove by induction that $u_n = \frac{2}{2n-1}$. **[6]**

Section B (36 marks)

7 Fig. 7 shows an incomplete sketch of $y = \dfrac{(2x-1)(x+3)}{(x-3)(x-2)}$.



Not to scale

Fig. 7

(i) Find the coordinates of the points where the curve cuts the axes. **[2]**

(ii) Write down the equations of the three asymptotes. **[3]**

(iii) Determine whether the curve approaches the horizontal asymptote from above or below for large positive values of x, justifying your answer. Copy and complete the sketch. **[3]**

(iv) Solve the inequality $\dfrac{(2x-1)(x+3)}{(x-3)(x-2)} < 2$. **[4]**

8 Two complex numbers, α and β, are given by $\alpha = \sqrt{3} + \mathrm{j}$ and $\beta = 3\mathrm{j}$.

(i) Find the modulus and argument of α and β. **[3]**

(ii) Find $\alpha\beta$ and $\dfrac{\beta}{\alpha}$, giving your answers in the form $a + b\mathrm{j}$, showing your working. **[5]**

(iii) Plot α, β, $\alpha\beta$ and $\dfrac{\beta}{\alpha}$ on a single Argand diagram. **[2]**

[Question 9 is printed overleaf.]

9 The matrices $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ represent transformations P and Q respectively.

(i) Describe fully the transformations P and Q. **[4]**

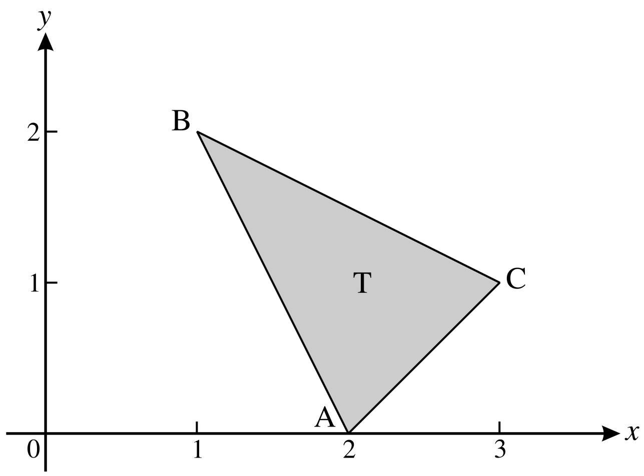


Fig. 9

Fig. 9 shows triangle T with vertices A (2, 0), B (1, 2) and C (3, 1).

Triangle T is transformed first by transformation P, then by transformation Q.

(ii) Find the single matrix that represents this composite transformation. **[2]**

(iii) This composite transformation maps triangle T onto triangle T′, with vertices A′, B′ and C′. Calculate the coordinates of A′, B′ and C′. **[2]**

T′ is reflected in the line $y = -x$ to give a new triangle, T″.

(iv) Find the matrix **R** that represents reflection in the line $y = -x$. **[2]**

(v) A single transformation maps T″ onto the original triangle, T. Find the matrix representing this transformation. **[4]**

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE **4755**

Further Concepts for Advanced Mathematics (FP1)

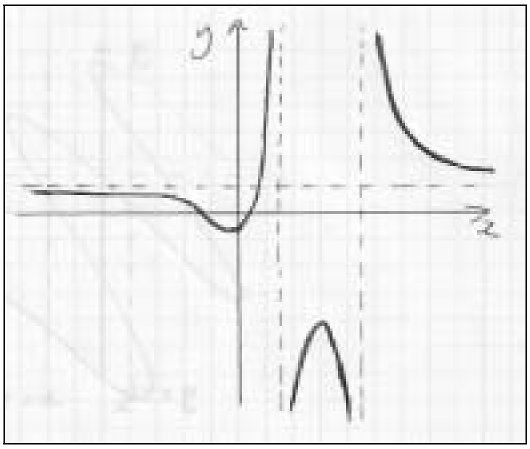
Mark Scheme for June 2010

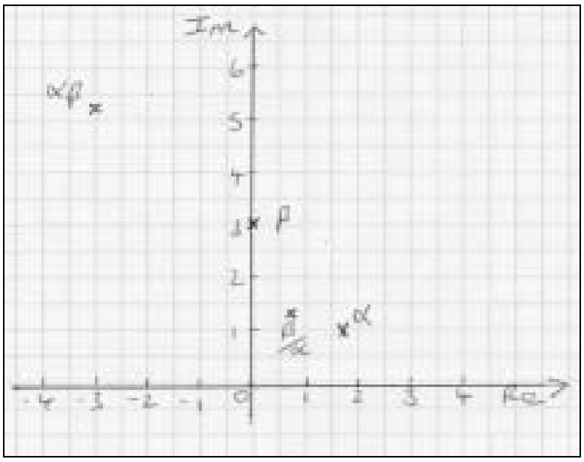
Oxford Cambridge and RSA Examinations

Qu	Answer	Mark	Comment
Section A			
1	$4x^2 - 16x + C \equiv A\left(x^2 + 2Bx + B^2\right) + 2$	B1	$A = 4$
	$\Leftrightarrow 4x^2 - 16x + C \equiv Ax^2 + 2ABx + AB^2 + 2$	M1	Attempt to expand RHS or other valid method (may be implied)
	$\Leftrightarrow A = 4,\ B = -2,\ C = 18$	A2, 1 **[4]**	1 mark each for B and C, c.a.o.
2(i)	$2x - 5y = 9$	B1	
	$3x + 7y = -1$	B1 **[2]**	
2(ii)	$\mathbf{M}^{-1} = \frac{1}{29}\begin{pmatrix} 7 & 5 \\ -3 & 2 \end{pmatrix}$	M1 A1 **[2]**	Divide by determinant c.a.o.
	$\frac{1}{29}\begin{pmatrix} 7 & 5 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 9 \\ -1 \end{pmatrix} = \frac{1}{29}\begin{pmatrix} 58 \\ -29 \end{pmatrix}$ $\Rightarrow x = 2,\ y = -1$	M1 A1(ft) **[2]**	Pre-multiply by their inverse For both
3	$z = 1 - 2\text{j}$	B1	
	$1 + 2\text{j} + 1 - 2\text{j} + \alpha = \frac{1}{2}$	M1	Valid attempt to use sum of roots, or other valid method
	$\Rightarrow \alpha = -\frac{3}{2}$	A1	c.a.o.
	$\frac{-k}{2} = -\frac{3}{2}(1 - 2\text{j})(1 + 2\text{j}) = -\frac{15}{2}$	M1 A1(ft)	Valid attempt to use product of roots, or other valid method Correct equation – can be implied
	$k = 15$	A1 **[6]**	c.a.o.
	OR		
	$\left(z - (1 + 2\text{j})\right)\left(z - (1 - 2\text{j})\right) = z^2 - 2z + 5$	M1 A1	Multiplying correct factors Correct quadratic, c.a.o.
	$2z^3 - z^2 + 4z + k = \left(z^2 - 2z + 5\right)(2z + 3)$	M1	Attempt to find linear factor
	$\alpha = \frac{-3}{2}$	A1(ft)	
	$k = 15$	A1 **[6]**	c.a.o.

4	$w = x+1 \Rightarrow x = w-1$ $x^3 - 2x^2 - 8x + 11 = 0,\ w = x-1$ $\Rightarrow (w-1)^3 - 2(w-1)^2 - 8(w-1) + 11 = 0$ $\Rightarrow w^3 - 5w^2 - w + 16 = 0$	B1 M1 M1 A3 **[6]**	Substitution. For $x = w+1$ give B0 but then follow for a maximum of 3 marks Attempt to substitute into cubic Attempt to expand -1 for each error (including omission of = 0)
	OR $\alpha+\beta+\gamma = 2$ $\alpha\beta+\alpha\gamma+\beta\gamma = -8$ $\alpha\beta\gamma = -11$ Let the new roots be *k*, *l* and *m* then $k+l+m = \alpha+\beta+\gamma+3 = 2+3 = 5$ $kl+km+lm = (\alpha\beta+\alpha\gamma+\beta\gamma)+2(\alpha+\beta+\gamma)+3$ $= -8+4+3 = -1$ $klm = \alpha\beta\gamma+(\alpha\beta+\alpha\gamma+\beta\gamma)+(\alpha+\beta+\gamma)+1$ $= -11-8+2+1 = -16$ $\Rightarrow w^3 - 5w^2 - w + 16 = 0$	B1 M1 M1 A3 **[6]**	All 3 correct Valid attempt to use their sum of roots in original equation to find sum of roots in new equation Valid attempt to use their product of roots in original equation to find one of $\sum\alpha\beta$ or $\alpha\beta\gamma$ -1 each error (including omission of = 0)
5	$\sum_{r=1}^{n}\frac{1}{(5r-1)(5r+4)} = \frac{1}{5}\sum_{r=1}^{n}\left(\frac{1}{5r-1}-\frac{1}{5r+4}\right)$ $= \frac{1}{5}\left(\left(\frac{1}{4}-\frac{1}{9}\right)+\left(\frac{1}{9}-\frac{1}{14}\right)+.....+\left(\frac{1}{5n-1}-\frac{1}{5n+4}\right)\right)$ $= \frac{1}{5}\left(\frac{1}{4}-\frac{1}{5n+4}\right) = \frac{1}{5}\left(\frac{5n+4-4}{4(5n+4)}\right) = \frac{n}{4(5n+4)}$	M1 A1 M1 A1 A1 A1 **[6]**	Attempt to use identity – may be implied Terms in full (at least first and last) Attempt at cancelling $\left(\frac{1}{4}-\frac{1}{5n+4}\right)$ factor of $\frac{1}{5}$ Correct answer as a single algebraic fraction

6(i)	$u_2 = \frac{2}{1+2} = \frac{2}{3}$, $u_3 = \frac{\frac{2}{3}}{1+\frac{2}{3}} = \frac{2}{5}$	M1 A1 **[2]**	Use of inductive definition c.a.o.
6(ii)	When n = 1, $\frac{2}{2\times 1-1} = 2$, so true for n = 1	B1	Showing use of $u_n = \frac{2}{2n-1}$
	Assume $u_k = \frac{2}{2k-1}$	E1	Assuming true for k
	$\Rightarrow u_{k+1} = \frac{\frac{2}{2k-1}}{1+\frac{2}{2k-1}}$	M1	u_{k+1}
	$= \frac{\frac{2}{2k-1}}{\frac{2k-1+2}{2k-1}} = \frac{2}{2k+1}$	A1	Correct simplification
	$= \frac{2}{2(k+1)-1}$		
	But this is the given result with k + 1 replacing k. Therefore if it is true for k it is also true for k + 1.	E1	Dependent on A1 and previous E1
	Since it is true for k = 1, it is true for all positive integers.	E1 **[6]**	Dependent on B1 and previous E1
			Section A Total: 36

Section B			
7(i)	$\left(0,\ -\frac{1}{2}\right)$ $(-3,\ 0),\ \left(\frac{1}{2},\ 0\right)$	B1 B1 **[2]**	 For both
7(ii)	$x=3,\ x=2$ and $y=2$	B1 B1 B1 **[3]**	
7(iii)	Large positive x, $y \to 2^{+}$ (e.g. substitute $x=100$ to give 2.15…, or convincing algebraic argument) 	M1 A1 B1 **[3]**	Must show evidence of method A0 if no valid method Correct RH branch
7(iv)	$\frac{(2x-1)(x+3)}{(x-3)(x-2)}=2$ $\Rightarrow (2x-1)(x+3)=2(x-3)(x-2)$ $\Rightarrow x=1$ From graph $x<1$ or $2<x<3$	M1 A1 B1 B1 **[4]**	Or other valid method to find intersection with horizontal asymptote For $x<1$ For $2<x<3$

8(i)	$\arg\alpha = \frac{\pi}{6},\ \lvert\alpha\rvert = 2$ $\arg\beta = \frac{\pi}{2},\ \lvert\beta\rvert = 3$	B1 B1 B1 **[3]**	Modulus of α Argument of α (allow 30°) Both modulus and argument of β (allow 90°)
8(ii)	$\alpha\beta = \left(\sqrt{3}+\mathrm{j}\right)3\mathrm{j} = -3 + 3\sqrt{3}\mathrm{j}$	M1 A1	Use of $\mathrm{j}^2 = -1$ Correct
	$\frac{\beta}{\alpha} = \frac{3\mathrm{j}}{\sqrt{3}+\mathrm{j}} = \frac{3\mathrm{j}\left(\sqrt{3}-\mathrm{j}\right)}{\left(\sqrt{3}+\mathrm{j}\right)\left(\sqrt{3}-\mathrm{j}\right)}$ $= \frac{3+3\sqrt{3}\mathrm{j}}{4} = \frac{3}{4} + \frac{3\sqrt{3}\mathrm{j}}{4}$	M1 A1 A1 **[5]**	. Correct use of conjugate of denominator Denominator = 4 All correct
8(iii)		M1 A1(ft) **[2]**	Argand diagram with at least one correct point Correct relative positions with appropriate labelling

Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	P is a rotation through 90 degrees about the origin in a clockwise direction.	B1 B1	Rotation about origin 90 degrees clockwise, or equivalent
	Q is a stretch factor 2 parallel to the *x*-axis	B1 B1 **[4]**	Stretch factor 2 Parallel to the *x*-axis
9(ii)	$\mathbf{QP}=\begin{pmatrix}2&0\\0&1\end{pmatrix}\begin{pmatrix}0&1\\-1&0\end{pmatrix}=\begin{pmatrix}0&2\\-1&0\end{pmatrix}$	M1 A1 **[2]**	Correct order c.a.o.
9(iii)	$\begin{pmatrix}0&2\\-1&0\end{pmatrix}\begin{pmatrix}2&1&3\\0&2&1\end{pmatrix}=\begin{pmatrix}0&4&2\\-2&-1&-3\end{pmatrix}$	M1	Pre-multiply by their **QP** - may be implied
	$A'=(0,\ -2),\ B'=(4,\ -1),\ C'=(2,\ -3)$	A1(ft) **[2]**	For all three points
9(iv)	$\mathbf{R}=\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$	B1 B1 **[2]**	One for each correct column
9(v)	$\mathbf{RQP}=\begin{pmatrix}0&-1\\-1&0\end{pmatrix}\begin{pmatrix}0&2\\-1&0\end{pmatrix}=\begin{pmatrix}1&0\\0&-2\end{pmatrix}$	M1 A1(ft)	Multiplication of their matrices in correct order
	$(\mathbf{RQP})^{-1}=\frac{-1}{2}\begin{pmatrix}-2&0\\0&1\end{pmatrix}$	M1 A1 **[4]**	Attempt to calculate inverse of their **RQP** c.a.o.
			Section B Total: 36
			Total: 72