

Friday 18 May 2012 – Morning

AS GCE MATHEMATICS (MEI)

4755 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4755
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

- 1 You are given that the matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a transformation A, and that the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ represents a transformation B.
- (i) Describe the transformations A and B. [2]
- (ii) Find the matrix representing the composite transformation consisting of A followed by B. [2]
- (iii) What single transformation is represented by this matrix? [1]
- 2 You are given that z_1 and z_2 are complex numbers.
 $z_1 = 3 + 3\sqrt{3}j$, and z_2 has modulus 5 and argument $\frac{\pi}{3}$.
- (i) Find the modulus and argument of z_1 , giving your answers exactly. [4]
- (ii) Express z_2 in the form $a + bj$, where a and b are to be given exactly. [2]
- (iii) Explain why, when plotted on an Argand diagram, z_1 , z_2 and the origin lie on a straight line. [1]
- 3 The cubic equation $3x^3 + 8x^2 + px + q = 0$ has roots α , $\frac{\alpha}{6}$ and $\alpha - 7$. Find the values of α , p and q . [6]
- 4 Solve the inequality $\frac{3}{x-4} > 1$. [4]
- 5 (i) Show that $\frac{1}{2r+1} - \frac{1}{2r+3} \equiv \frac{2}{(2r+1)(2r+3)}$. [2]
- (ii) Use the method of differences to find $\sum_{r=1}^{30} \frac{1}{(2r+1)(2r+3)}$, expressing your answer as a fraction. [5]
- 6 A sequence is defined by $a_1 = 1$ and $a_{k+1} = 3(a_k + 1)$.
- (i) Calculate the value of the third term, a_3 . [1]
- (ii) Prove by induction that $a_n = \frac{5 \times 3^{n-1} - 3}{2}$. [6]

Section B (36 marks)

- 7 A curve has equation $y = \frac{x^2 - 25}{(x - 3)(x + 4)(3x + 2)}$.
- (i) Write down the coordinates of the points where the curve crosses the axes. [3]
- (ii) Write down the equations of the asymptotes. [4]
- (iii) Determine how the curve approaches the horizontal asymptote for large positive values of x , and for large negative values of x . [3]
- (iv) Sketch the curve. [4]
- 8 (i) Verify that $1 + 3j$ is a root of the equation $3z^3 - 2z^2 + 22z + 40 = 0$, showing your working. [4]
- (ii) Explain why the equation must have exactly one real root. [1]
- (iii) Find the other roots of the equation. [5]

- 9 You are given that $\mathbf{A} = \begin{pmatrix} -3 & -4 & 1 \\ 2 & 1 & k \\ 7 & -1 & -1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -4 & -5 & 11 \\ -19 & -4 & -7 \\ -9 & -31 & 2 - k \end{pmatrix}$ and

$$\mathbf{AB} = \begin{pmatrix} 79 & 0 & -3 - k \\ -9k - 27 & -31k - 14 & q \\ p & 0 & 82 + k \end{pmatrix} \text{ where } p \text{ and } q \text{ are to be determined.}$$

- (i) Show that $p = 0$ and $q = 15 + 2k - k^2$. [3]

It is now given that $k = -3$.

- (ii) Find \mathbf{AB} and hence write down the inverse matrix \mathbf{A}^{-1} . [5]

- (iii) Use a matrix method to find the values of x , y and z that satisfy the equation $\mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ -23 \\ 9 \end{pmatrix}$. [4]

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE



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AS GCE MATHEMATICS (MEI)

4755 Further Concepts for Advanced Mathematics (FP1)

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:

- Question Paper 4755 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



Candidate forename		Candidate surname	
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Centre number						Candidate number				
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Section A (36 marks)

1 (i)	
1 (ii)	
1 (iii)	

2 (i)	
2 (ii)	
2 (iii)	

3	

Section B (36 marks)

7 (i)	
7 (ii)	

7 (iii)	
7 (iv)	

8 (iii)	(continued)
9 (i)	

9 (iii)	



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Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4755**: Further Concepts for Advanced Mathematics

Mark Scheme for June 2012

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he / she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

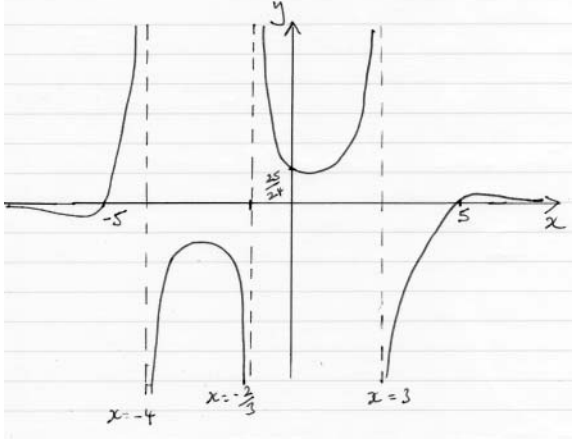
h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1	(i)	Transformation A is a reflection in the y -axis. Transformation B is a rotation through 90° clockwise about the origin.	B1 B1 [2]	
1	(ii)	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	M1 A1 [2]	Attempt to multiply in correct order cao
1	(iii)	Reflection in the line $y = x$	B1 [1]	
2	(i)	$ z_1 = \sqrt{3^2 + (3\sqrt{3})^2} = 6$ $\arg(z_1) = \arctan \frac{3\sqrt{3}}{3} = \frac{\pi}{3}$	M1 A1 M1 A1 [4]	Use of Pythagoras cao cao
2	(ii)	$z_2 = \frac{5}{2} + \frac{5\sqrt{3}}{2}j$	M1 A1 [2]	May be implied cao
2	(iii)	Because z_1 and z_2 have the same argument	E1 [1]	Consistent with (i)
3		$\alpha + \frac{\alpha}{6} + \alpha - 7 = \frac{-8}{3} \Rightarrow \alpha = 2$ Other roots are -5 and $\frac{1}{3}$ Product of roots = $\frac{-q}{3} = \frac{-10}{3} \Rightarrow q = 10$ Sum of products in pairs = $\frac{p}{3} = -11 \Rightarrow p = -33$	M1 A1 M1 A1 M1 A1	Attempt to use sum of roots Value of α (cao) Attempt to use product of roots $q = 10$ c.a.o. Attempt to use sum of products of roots in pairs $p = -33$ cao

Question	Answer	Marks	Guidance
	<p>OR, for final four marks</p> $(x-2)(x+5)(3x-1)$ $= 3x^3 + 8x^2 - 33x + 10$ $\Rightarrow p = -33 \text{ and } q = 10$	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[6]</p>	<p>Express as product of factors</p> <p>Expanding</p> <p>$p = -33$ cao</p> <p>$q = 10$ cao</p>
4	$\frac{3}{x-4} > 1 \Rightarrow 3(x-4) > (x-4)^2$ $\Rightarrow 0 > x^2 - 11x + 28$ $\Rightarrow 0 > (x-4)(x-7)$ $\Rightarrow 4 < x < 7$ <p>OR</p> $\frac{3}{x-4} - 1 > 0 \Rightarrow \frac{7-x}{x-4} > 0$ <p>Consideration of graph sketch or table of values/signs</p> $\Rightarrow 4 < x < 7$ <p>OR</p> <p>$3 = x - 4 \Rightarrow x = 7$ (each side equal)</p> <p>$x = 4$ (asymptote)</p> <p>Critical values at $x = 7$ and $x = 4$</p> <p>Consideration of graph sketch or table of values/signs</p> $4 < x < 7$ <p>OR</p> <p>Consider inequalities arising from both $x < 4$ and $x > 4$</p> <p>Solving appropriate inequalities to their $x > 7$ and $x < 7$</p> $4 < x < 7$	<p>M1*</p> <p>M1dep*</p> <p>B2</p> <p>M1*</p> <p>M1dep*</p> <p>B2</p> <p>M1*</p> <p>M1dep*</p> <p>B2</p> <p>M1*</p> <p>M1dep*</p> <p>B2</p> <p>[4]</p>	<p>Multiply through by $(x-4)^2$</p> <p>Factorise quadratic</p> <p>One each for $4 < x$ and $x < 7$</p> <p>Obtain single fraction > 0</p> <p>One each for $4 < x$ and $x < 7$</p> <p>Identification of critical values at $x = 7$ and $x = 4$</p> <p>One each for $4 < x$ and $x < 7$</p> <p>One for each $4 < x$ and $x < 7$, and no other solutions</p>

Question	Answer	Marks	Guidance
5 (i)	$\frac{1}{2r+1} - \frac{1}{2r+3} = \frac{2r+3 - (2r+1)}{(2r+1)(2r+3)} = \frac{2}{(2r+1)(2r+3)}$	M1 A1 [2]	Attempt at common denominator
5 (ii)	$\sum_{r=1}^{30} \frac{1}{(2r+1)(2r+3)} = \frac{1}{2} \sum_{r=1}^{30} \left[\frac{1}{2r+1} - \frac{1}{2r+3} \right]$ $= \frac{1}{2} \left[\left(\frac{1}{3} - \frac{1}{5} \right) + \left(\frac{1}{5} - \frac{1}{7} \right) + \dots + \left(\frac{1}{59} - \frac{1}{61} \right) + \left(\frac{1}{61} - \frac{1}{63} \right) \right]$ $= \frac{1}{2} \left(\frac{1}{3} - \frac{1}{63} \right) = \frac{10}{63}$	M1 M1 M1 A1 A1 oe cao [5]	Use of (i); do not penalise missing factor of $\frac{1}{2}$ Sufficient terms to show pattern Cancelling terms Factor $\frac{1}{2}$ used oe cao
6 (i)	$a_2 = 3 \times 2 = 6, a_3 = 3 \times 7 = 21$	B1 [1]	cao
6 (ii)	<p>When $n = 1$, $\frac{5 \times 3^0 - 3}{2} = 1$, so true for $n = 1$</p> <p>Assume $a_k = \frac{5 \times 3^{k-1} - 3}{2}$</p> $\Rightarrow a_{k+1} = 3 \left(\frac{5 \times 3^{k-1} - 3}{2} + 1 \right)$ $= \frac{5 \times 3^k - 9}{2} + 3 = \frac{5 \times 3^k - 9 + 6}{2}$ $= \frac{5 \times 3^k - 3}{2} = \frac{5 \times 3^{(k+1)-1} - 3}{2}$ <p>But this is the given result with $k + 1$ replacing k. Therefore if it is true for $n = k$ it is also true for $n = k + 1$. Since it is true for $n = 1$, it is true for all positive integers.</p>	B1 E1 M1 A1 E1 E1 [6]	Showing use of $a_n = \frac{5 \times 3^{n-1} - 3}{2}$ Assuming true for $n = k$ a_{k+1} , using a_k and attempting to simplify Correct simplification to left hand expression. May be identified with a 'target' expression using $n = k + 1$ Dependent on A1 and previous E1 Dependent on B1 and previous E1

Question		Answer	Marks	Guidance
7	(i)	$(-5, 0), (5, 0), \left(0, \frac{25}{24}\right)$	B1 B1 B1 [3]	-1 for each additional point
7	(ii)	$x = 3, x = -4, x = -\frac{2}{3}$ and $y = 0$	B1 B1 B1 B1 [4]	
7	(iii)	Some evidence of method needed e.g. substitute in 'large' values or argument involving signs Large positive $x, y \rightarrow 0^+$ Large negative $x, y \rightarrow 0^-$	M1 B1 B1 [3]	
7	(iv)		B1* B1dep* B1 B1 [4]	4 branches correct Asymptotic approaches clearly shown Vertical asymptotes correct and labelled Intercepts correct and labelled

Question	Answer	Marks	Guidance
8 (i)	$3(1+3j)^3 - 2(1+3j)^2 + 22(1+3j) + 40$ $= 3(-26-18j) - 2(-8+6j) + 22(1+3j) + 40$ $= (-78+16+22+40) + (-54-12+66)j$ $= 0$ So $z = 1+3j$ is a root	M1 A1 A1 A1 [4]	Substitute $z = 1+3j$ into cubic $(1+3j)^2 = -8+6j$, $(1+3j)^3 = -26-18j$ Simplification (correct) to show that this comes to 0 and so $z = 1+3j$ is a root
8 (ii)	All cubics have 3 roots. As the coefficients are real, the complex conjugate is also a root. This leaves the third root, which must therefore be real.	E1 [1]	Convincing explanation
8 (iii)	$1-3j$ must also be a root Sum of roots = $-\frac{-2}{3} = \frac{2}{3}$ OR product of roots = $-\frac{40}{3}$ $\text{OR } \sum \alpha\beta = \frac{22}{3}$ $(1+3j) + (1-3j) + \alpha = \frac{2}{3}$ OR $(1+3j)(1-3j)\alpha = -\frac{40}{3}$ $\text{OR } (1-3j)(1+3j) + (1-3j)\alpha + (1+3j)\alpha = \frac{22}{3}$ $\Rightarrow \alpha = \frac{-4}{3}$ is the real root	B1 M1 A2,1,0 A1	Attempt to use one of $\sum \alpha, \alpha\beta\gamma, \sum \alpha\beta$ Correct equation Cao
	OR $1-3j$ must also be a root $(z-1+3j)(z-1-3j) = z^2 - 2z + 10$ $3z^3 - 2z^2 + 22z + 40 \equiv (z^2 - 2z + 10)(3z + 4) = 0$ $\Rightarrow z = \frac{-4}{3}$ is the real root	B1 M1 A1 A1 A1 [5]	Use of factors Correct quadratic factor Correct linear factor (by inspection or division) Cao

Question		Answer	Marks	Guidance
9	(i)	$p = 7 \times (-4) + (-1) \times (-19) + (-1) \times (-9) = 0$ $q = 2 \times 11 + 1 \times (-7) + k \times (2 - k)$ $\Rightarrow q = 15 + 2k - k^2$	E1 M1 A1 [3]	AG must see correct working AG Correct working
9	(ii)	$\mathbf{AB} = \begin{pmatrix} 79 & 0 & 0 \\ 0 & 79 & 0 \\ 0 & 0 & 79 \end{pmatrix}$ $\mathbf{A}^{-1} = \frac{1}{79} \begin{pmatrix} -4 & -5 & 11 \\ -19 & -4 & -7 \\ -9 & -31 & 5 \end{pmatrix}$	B2 M1 B1 A1 [5]	-1 each error Use of B $\frac{1}{79}$ Correct inverse
9	(iii)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{79} \begin{pmatrix} -4 & -5 & 11 \\ -19 & -4 & -7 \\ -9 & -31 & 5 \end{pmatrix} \begin{pmatrix} 14 \\ -23 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 8 \end{pmatrix}$ $\Rightarrow x = 2, y = -3, z = 8$	M1 A1 A1 A1 [4]	Attempt to pre-multiply by their \mathbf{A}^{-1} SC A2 for x, y, z unspecified sSC B1 for \mathbf{A}^{-1} not used or incorrectly placed.

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4755 Further Concepts for Advanced Mathematics (FP1)

General Comments

This paper was well answered by the majority of candidates. Nearly all candidates were able to attempt all the questions in the time. There were many extremely good scripts, with well-expressed work. Some candidates would benefit from taking more care with the quality of their written communication. It is unfortunately a common practice to misuse the implication sign ' \Rightarrow ', which frequently is seen to replace '=' or words, to the detriment of sense. Graph paper is not needed and can be extremely difficult to read on screen. There appeared to be rather more candidates writing answers in the wrong places in the answer booklets than on previous occasions.

Comments on Individual Questions

- 1) This straightforward question produced a varied response. In (i), one mark was frequently lost through incomplete description of the rotation. Not many candidates felt the need to show any working or diagrams; not essential, but might have helped some.
- In (ii), the product was mostly found correctly, the common error being to multiply in the wrong sequence, which usually led to the wrong transformation in (iii). A visual check on the sequence of transformations could either reinforce or, in some cases, provide a correction following a wrong result. A point of language: reflections are usually *in* a line not *along* it.
- 2) (i) This was well done by many, but the following comments apply. The arithmetic was not always correct. Several candidates made the mistake of using $3\sqrt{3}j$ in calculating the modulus of z_1 . An argument expressed in degrees was not acceptable.
- (ii) Those that sketched the position of z_2 were usually correct. A number of candidates responded to $|z_2| = 5$ by deducing that $a = 3$ and $b = 4$, or vice versa. An attempt to solve the simultaneous equations $\tan \frac{\pi}{3} = \frac{b}{a}$ and $a^2 + b^2 = 25$ was rarely successful. Not all answers gave the exact form of b and it is preferable to give the values as ratios, not decimal fractions. It was quite common to finish with a statement about a and b , rather than to write the full expression for z_2 which was requested.
- (iii) It was sufficient here to state that the two complex numbers had the same argument. Explaining this in terms of an angle was rarely coherent, and sometimes misleading. Angles cannot be made 'with the origin'. 'At the origin' requires a little more explanation. Some realised that one complex number was a real multiple of the other, but did not specify the scale factor, nor mention that the multiplier was real.

- 3) Most often answers to this question were completely correct and well set out. The relationships between roots and coefficients was the most popular route to the solutions, and probably the easiest. The common mistake was to forget the coefficient of x^3 , either altogether or at some point during the solution. Another was to omit the minus sign in finding q , and sometimes in finding α .
- Candidates who began by trying to expand factors using α , $\frac{\alpha}{6}$ and $\alpha - 7$ were often defeated by the algebra, but those who used the factors after finding α managed perfectly well.
- 4) This probably produced the least satisfactory answers. There were many partial solutions with inadequate working where the sign of $x - 4$ was ignored. Most successful were those who multiplied both sides by $(x - 4)^2$, and then solved the resulting quadratic inequality. Candidates who chose a graphical approach were also usually successful, although many made algebraic errors, and the sketches produced were often extremely scruffy. The best solution, very rarely seen, considered the two inequalities $0 < x - 4 < 3$ which immediately supply the solution.
- 5) (i) This result was nearly always correctly shown, unless there was loss of a necessary bracket. It is expected that correct notation is used at this level.
- (ii) This question was also successfully answered by the great majority of candidates. There were those that forgot the factor $\frac{1}{2}$ in the final stages and some who showed a careless disregard of signs.
- 6) (i) Very few errors were seen here, as would be expected.
- (ii) There were numerous satisfactory and well expressed answers, where all the details were included. Many candidates coped well with the algebra but failed to produce the desired argument at the final stages, one place where ' \Rightarrow ' could usefully and correctly be employed. In words, 'if...then...' are those needed, and few others are adequate. There were candidates who made the mistake of trying to add a term, as in a series, and others who were less than attentive to every line of their working in finding the expression for a_{k+1} .
- 7) (i) Mostly correct. Any errors were usually in notation. It would be good to see co-ordinates presented in the conventional manner. Equations of lines were permitted if each point had two.
- (ii) Vertical asymptotes were correctly identified in the majority of scripts. There was some confusion over the equation of the horizontal asymptote; $y = \frac{1}{3x}$ was fairly frequently seen, and also, less appropriately, $y = \frac{1}{3}$.
- (iii) When $y = 0$ had been found in (ii) this was usually correctly answered. Otherwise the marks were only available to those who specified that y approached zero, as their calculations should have demonstrated.
- (iv) There were many clear and carefully drawn graphs. Some diagrams failed to show some of the features. In particular the approach to the horizontal asymptote, following obvious turning points, was wanted, also labelling of all three intercepts on the axes, with no extras. Some graphs failed to show all four branches; a quick numerical check should have revealed that they existed.

- 8) (i) This was in general well answered. Most candidates substituted $z = 1 + 3j$ and reduced the polynomial to zero without mishap. Not many bothered to state that this demonstrated that $1 + 3j$ was indeed a root of the equation. Some evidence of manipulating the required result occurred. Necessary alterations should be traced back to their source if marks for accuracy are to be earned. Some answers took the long route of showing the factorisation of the polynomial, assuming that $1 + 3j$ was a root, which earned the marks provided there was adequate explanation, and of course made short work of part (iv).
- (ii) It was needful to refer to the complex conjugate as another root and to explain that there were only three roots to a cubic equation. This was not always achieved.
- (iii) The root $1 - 3j$ was usually recognised, but not always stated to be a root. The neatest solutions used the root relationships, but some candidates made errors with signs or, in using the coefficients, $a = 3$ was sometimes forgotten. Those candidates who used the complex roots to find a quadratic factor were usually successful in proceeding to the real root, but it was evident that some did not know the difference between a factor and a root.
- 9) (i) This was well done, but the answers were given and as a result there was a penalty for carelessly written expressions.
- (ii) Nearly all realised that the inverse of **A** involved **B**, and only a few forgot the factor $\frac{1}{79}$.
- (iii) Again, well answered by nearly all candidates. Some were unable to show that a matrix method was used to solve the equations.

GCE Mathematics (MEI)		Max Mark	a	b	c	d	e	u
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	57	50	44	38	32	0
	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	54	48	42	36	31	0
	UMS	100	80	70	60	50	40	0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	60	53	47	41	34	0
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	65	57	50	43	36	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	56	49	42	35	0
	UMS	100	80	70	60	50	40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	54	47	40	34	28	0
	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	39	0
	Raw	18	15	13	11	9	8	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	58	50	42	34	27	0
	UMS	100	80	70	60	50	40	0
4762/01 (M2) MEI Mechanics 2	Raw	72	58	51	44	38	32	0
	UMS	100	80	70	60	50	40	0
4763/01 (M3) MEI Mechanics 3	Raw	72	63	56	50	44	38	0
	UMS	100	80	70	60	50	40	0
4764/01 (M4) MEI Mechanics 4	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4766/01 (S1) MEI Statistics 1	Raw	72	54	46	38	30	23	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw	72	61	55	49	43	38	0
	UMS	100	80	70	60	50	40	0
4768/01 (S3) MEI Statistics 3	Raw	72	58	51	44	38	32	0
	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	53	47	42	37	32	0
	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	56	50	44	39	34	0
	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
	UMS	100	80	70	60	50	40	0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	50	44	38	33	27	0
	Raw	18	14	12	10	8	7	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0