ADVANCED GCE

4757/01

MATHEMATICS (MEI)

Further Applications of Advanced Mathematics (FP3)

FRIDAY 6 JUNE 2008

Afternoon

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Option 1: Vectors

1 A tetrahedron ABCD has vertices A $(-3, 5, 2)$, B $(3, 13, 7)$, C $(7, 0, 3)$ and D $(5, 4, 8)$.

(i) Find the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$, and hence find the equation of the plane ABC. [4]

(ii) Find the shortest distance from D to the plane ABC. [3]

(iii) Find the shortest distance between the lines AB and CD. [4]

(iv) Find the volume of the tetrahedron ABCD. [4]

The plane P with equation $3x - 2z + 5 = 0$ contains the point B, and meets the lines AC and AD at E and F respectively.

(v) Find λ and μ such that $\overrightarrow{AE} = \lambda\overrightarrow{AC}$ and $\overrightarrow{AF} = \mu\overrightarrow{AD}$. Deduce that E is between A and C, and that F is between A and D. [5]

(vi) Hence, or otherwise, show that P divides the tetrahedron ABCD into two parts having volumes in the ratio 4 to 17. [4]

Option 2: Multi-variable calculus

2 You are given $g(x, y, z) = 6xz - (x + 2y + 3z)^2$.

(i) Find $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$ and $\frac{\partial g}{\partial z}$. [4]

A surface S has equation $g(x, y, z) = 125$.

(ii) Find the equation of the normal line to S at the point P $(7, -7.5, 3)$. [3]

(iii) The point Q is on this normal line and is close to P. At Q, $g(x, y, z) = 125 + h$, where h is small. Find the vector $\mathbf{n}$ such that $\overrightarrow{PQ} = h\mathbf{n}$ approximately. [5]

(iv) Show that there is no point on S at which the normal line is parallel to the z-axis. [4]

(v) Find the two points on S at which the tangent plane is parallel to $x + 5y = 0$. [8]

Option 3: Differential geometry

3 The curve C has parametric equations $x = 8t^3$, $y = 9t^2 - 2t^4$, for $t \geqslant 0$.

(i) Show that $\dot{x}^2 + \dot{y}^2 = (18t + 8t^3)^2$. Find the length of the arc of C for which $0 \leqslant t \leqslant 2$. [6]

(ii) Find the area of the surface generated when the arc of C for which $0 \leqslant t \leqslant 2$ is rotated through 2π radians about the x-axis. [6]

(iii) Show that the curvature at a general point on C is $\frac{-6}{t(4t^2 + 9)^2}$. [5]

(iv) Find the coordinates of the centre of curvature corresponding to the point on C where $t = 1$. [7]

Option 4: Groups

4 A binary operation $*$ is defined on real numbers x and y by

$$x * y = 2xy + x + y.$$

You may assume that the operation $*$ is commutative and associative.

(i) Explain briefly the meanings of the terms 'commutative' and 'associative'. [3]

(ii) Show that $x * y = 2\left(x + \frac{1}{2}\right)\left(y + \frac{1}{2}\right) - \frac{1}{2}$. [1]

The set S consists of all real numbers greater than $-\frac{1}{2}$.

(iii) (A) Use the result in part **(ii)** to show that S is closed under the operation $*$.

(B) Show that S, with the operation $*$, is a group. [9]

(iv) Show that S contains no element of order 2. [3]

The group $G = \{0, 1, 2, 4, 5, 6\}$ has binary operation $\circ$ defined by

$x \circ y$ is the remainder when $x * y$ is divided by 7.

(v) Show that $4 \circ 6 = 2$. [2]

The composition table for G is as follows.

$\circ$	0	1	2	4	5	6
0	0	1	2	4	5	6
1	1	4	0	6	2	5
2	2	0	5	1	6	4
4	4	6	1	5	0	2
5	5	2	6	0	4	1
6	6	5	4	2	1	0

(vi) Find the order of each element of G. [3]

(vii) List all the subgroups of G. [3]

[Question 5 is printed overleaf.]

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 Every day, a security firm transports a large sum of money from one bank to another. There are three possible routes A, B and C. The route to be used is decided just before the journey begins, by a computer programmed as follows.

On the first day, each of the three routes is equally likely to be used.

If route A was used on the previous day, route A, B or C will be used, with probabilities 0.1, 0.4, 0.5 respectively.

If route B was used on the previous day, route A, B or C will be used, with probabilities 0.7, 0.2, 0.1 respectively.

If route C was used on the previous day, route A, B or C will be used, with probabilities 0.1, 0.6, 0.3 respectively.

The situation is modelled as a Markov chain with three states.

(i) Write down the transition matrix $\mathbf{P}$. [2]

(ii) Find the probability that route B is used on the 7th day. [4]

(iii) Find the probability that the same route is used on the 7th and 8th days. [3]

(iv) Find the probability that the route used on the 10th day is the same as that used on the 7th day. [4]

(v) Given that $\mathbf{P}^n \to \mathbf{Q}$ as $n \to \infty$, find the matrix $\mathbf{Q}$ (give the elements to 4 decimal places). Interpret the probabilities which occur in the matrix $\mathbf{Q}$. [4]

The computer program is now to be changed, so that the long-run probabilities for routes A, B and C will become 0.4, 0.2 and 0.4 respectively. The transition probabilities after routes A and B remain the same as before.

(vi) Find the new transition probabilities after route C. [4]

(vii) A long time after the change of program, a day is chosen at random. Find the probability that the route used on that day is the same as on the previous day. [3]

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1 (i)	$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}6\\8\\5\end{pmatrix}\times\begin{pmatrix}10\\-5\\1\end{pmatrix}=\begin{pmatrix}33\\44\\-110\end{pmatrix}$ ABC is $3x+4y-10z=-9+20-20$ $3x+4y-10z+9=0$	B2 M1 A1 **4**	*Ignore subsequent working* Give B1 for one element correct SC1 for minus the correct vector For $3x+4y-10z$ Accept $33x+44y-110z=-99$ etc
(ii)	Distance is $\dfrac{3\times5+4\times4-10\times8+9}{\sqrt{3^2+4^2+10^2}}$ $=(-)\ \dfrac{40}{\sqrt{125}}\quad\left(=\dfrac{8}{\sqrt{5}}\right)$	M1 A1 ft A1 **3**	Using distance formula (or other complete method) *Condone negative answer* *Accept a.r.t. 3.58*
(iii)	$\overrightarrow{AB}\times\overrightarrow{CD}=\begin{pmatrix}6\\8\\5\end{pmatrix}\times\begin{pmatrix}-2\\4\\5\end{pmatrix}=\begin{pmatrix}20\\-40\\40\end{pmatrix}\quad\left[=20\begin{pmatrix}1\\-2\\2\end{pmatrix}\right]$ Distance is $\overrightarrow{AC}\,.\,\hat{\mathbf{n}}=\dfrac{\begin{pmatrix}10\\-5\\1\end{pmatrix}.\begin{pmatrix}1\\-2\\2\end{pmatrix}}{\sqrt{1^2+2^2+2^2}}$ $=\dfrac{22}{3}$	M1 A1 M1 A1 **4**	Evaluating $\overrightarrow{AB}\times\overrightarrow{CD}$ *or* method for finding end-points of common perp PQ *or* P $(\tfrac{3}{2},11,\tfrac{23}{4})$ & Q $(\tfrac{71}{18},\tfrac{55}{9},\tfrac{383}{36})$ *or* $\overrightarrow{PQ}=(\tfrac{22}{9},-\tfrac{44}{9},\tfrac{44}{9})$
(iv)	Volume is $\tfrac{1}{6}(\overrightarrow{AB}\times\overrightarrow{AC})\,.\,\overrightarrow{AD}$ $=\dfrac{1}{6}\begin{pmatrix}33\\44\\-110\end{pmatrix}.\begin{pmatrix}8\\-1\\6\end{pmatrix}$ $=(-)\ \dfrac{220}{3}$	M1 A1 M1 A1 **4**	Scalar triple product *Accept a.r.t. 73.3*
(v)	E is $(-3+10\lambda,\ 5-5\lambda,\ 2+\lambda)$ $3(-3+10\lambda)-2(2+\lambda)+5=0$ $\lambda=\dfrac{2}{7}$ F is $(-3+8\mu,\ 5-\mu,\ 2+6\mu)$ $3(-3+8\mu)-2(2+6\mu)+5=0$ $\mu=\dfrac{2}{3}$ Since $0<\lambda<1$, E is between A and C Since $0<\mu<1$, F is between A and D	M1 A1 M1 A1 B1 **5**	

(vi)	$V_{\mathrm{ABEF}} = \frac{1}{6}(\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AE}})\,.\,\overrightarrow{\mathrm{AF}}$ $= \frac{1}{6}\lambda\mu(\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AC}})\,.\,\overrightarrow{\mathrm{AD}}$ $= \lambda\mu V_{\mathrm{ABCD}}$ $= \frac{4}{21}V_{\mathrm{ABCD}}$ Ratio of volumes is $\frac{4}{21} : \frac{17}{21}$ $= 4 : 17$	M1 A1 M1 A1 ag **4**	($13\frac{61}{63}$) *ft if numerical* Finding ratio of volumes of two parts SC1 for 4 : 17 deduced from $\frac{4}{21}$ without working
2 (i)	$\frac{\partial g}{\partial x} = 6z - 2(x+2y+3z) = -2x-4y$ $\frac{\partial g}{\partial y} = -4(x+2y+3z)$ $\frac{\partial g}{\partial z} = 6x - 6(x+2y+3z) = -12y-18z$	M1 A1 A1 A1 **4**	Partial differentiation *Any correct form, ISW*
(ii)	At P, $\frac{\partial g}{\partial x} = 16,\ \frac{\partial g}{\partial y} = -4,\ \frac{\partial g}{\partial z} = 36$ Normal line is $\mathbf{r} = \begin{pmatrix} 7 \\ -7.5 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} 4 \\ -1 \\ 9 \end{pmatrix}$	M1 A1 A1 ft **3**	Evaluating partial derivatives at P All correct *Condone omission of* ' **r =** '
(iii)	$\delta g \approx 16\,\delta x - 4\,\delta y + 36\,\delta z$ If $\overrightarrow{\mathrm{PQ}} = \lambda\begin{pmatrix} 4 \\ -1 \\ 9 \end{pmatrix}$, $\delta g \approx 16(4\lambda) - 4(-\lambda) + 36(9\lambda) \quad (= 392\lambda)$ $h = \delta g$, so $h \approx 392\lambda$ $\overrightarrow{\mathrm{PQ}} \approx \frac{h}{392}\begin{pmatrix} 4 \\ -1 \\ 9 \end{pmatrix}$, so $\mathbf{n} = \frac{1}{392}\begin{pmatrix} 4 \\ -1 \\ 9 \end{pmatrix}$	M1 M1 A1 ft M1 A1 **5**	*Alternative:* *M3 for substituting* $x = 7 + 4\lambda$ *, …* *into* $g = 125 + h$ *and neglecting* λ^2 *A1 ft for linear equation in* λ *and* h *A1 for* **n** *correct*
(iv)	Require $\frac{\partial g}{\partial x} = \frac{\partial g}{\partial y} = 0$ $-2x-4y=0$ and $x+2y+3z=0$ $x+2y=0$ and $z=0$ $g(x,y,z) = 0 - 0^2 = 0 \neq 125$ Hence there is no such point on *S*	M1 M1 M1 A1 **4**	Useful manipulation using both eqns Showing there is no such point on *S* Fully correct proof
(v)	Require $\frac{\partial g}{\partial z} = 0$ and $\frac{\partial g}{\partial y} = 5\frac{\partial g}{\partial x}$ $-4x-8y-12z = 5(-2x-4y)$	M1 M1 M1	Implied by $\frac{\partial g}{\partial x} = \lambda,\ \frac{\partial g}{\partial y} = 5\lambda$ *This M1 can be awarded for* $-2x-4y=1$ and $-4x-8y-12z=5$

	$y=-\frac{3}{2}z$ and $x=5z$ $6(5z)z-(5z)^2=125$ $z=\pm 5$ Points are $(25,\ -7.5,\ 5)$ and $(-25,\ 7.5,\ -5)$	A1 M1 M1 A1 A1 ft **8**	*or* $z=-\frac{2}{3}y$ and $x=-\frac{10}{3}y$ *or* $y=-\frac{3}{10}x$ and $z=\frac{1}{5}x$ *or* $x=-\frac{5}{4}\lambda,\ y=\frac{3}{8}\lambda,\ z=-\frac{1}{4}\lambda$ *or* $x:y:z=10:-3:2$ Substituting into $g(x,y,z)=125$ Obtaining one value of *x, y, z or* λ *Dependent on previous M1* *ft is minus the other point, provided all M marks have been earned*
3 (i)	$\dot{x}^2+\dot{y}^2=(24t^2)^2+(18t-8t^3)^2$ $=576t^4+324t^2-288t^4+64t^6$ $=324t^2+288t^4+64t^6$ $=(18t+8t^3)^2$ Arc length is $\int_0^2(18t+8t^3)\,dt$ $=\left[\ 9t^2+2t^4\ \right]_0^2$ $=68$	B1 M1 A1 ag M1 A1 A1 **6**	*Note* $\int_0^2(18+8t^3)\,dt\ =\left[\ 18t+2t^4\ \right]_0^2=68$ *earns M1A0A0*
(ii)	Curved surface area is $\int 2\pi y\,ds$ $=\int_0^2 2\pi(9t^2-2t^4)(18t+8t^3)\,dt$ $=\int_0^2 \pi(324t^3+72t^5-32t^7)\,dt$ $=\pi\left[\ 81t^4+12t^6-4t^8\ \right]_0^2$ $=1040\pi\quad(\approx 3267)$	M1 M1 A1 M1 M1 A1 **6**	Using $ds=(18t+8t^3)\,dt$ Correct integral expression including limits *(may be implied by later work)*
(iii)	$\kappa=\dfrac{\dot{x}\ddot{y}-\ddot{x}\dot{y}}{(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}}}=\dfrac{(24t^2)(18-24t^2)-(48t)(18t-8t^3)}{(18t+8t^3)^3}$ $=\dfrac{48t^2(9-12t^2-18+8t^2)}{8t^3(9+4t^2)^3}=\dfrac{-48t^2(9+4t^2)}{8t^3(9+4t^2)^3}$ $=\dfrac{-6}{t(4t^2+9)^2}$	M1 A1A1 M1 A1 ag **5**	Using formula for κ (or ρ) For numerator and denominator Simplifying the numerator

(iv)	When $t=1,\ x=8,\ y=7,\ \kappa=-\frac{6}{169}$ $\rho=(-)\,\dfrac{169}{6}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\dot{y}}{\dot{x}}=\dfrac{18t-8t^3}{24t^2}=\dfrac{10}{24}$ $\hat{\mathbf{n}}=\begin{pmatrix}5/13\\-12/13\end{pmatrix}$ $\mathbf{c}=\begin{pmatrix}8\\7\end{pmatrix}+\frac{169}{6}\begin{pmatrix}5/13\\-12/13\end{pmatrix}$ Centre of curvature is $(\,18\frac{5}{6},\ -19\,)$	B1 M1 M1 A1 M1 A1A1 **7**	Finding gradient (or tangent vector) Finding direction of the normal Correct unit normal (either direction)
4 (i)	*Commutative:* $x*y=y*x$ (for all *x, y*) *Associative:* $(x*y)*z=x*(y*z)$ (for all *x, y, z*)	B1 B2 **3**	Accept e.g. 'Order does not matter' Give B1 for a partial explanation, e.g. 'Position of brackets does not matter'
(ii)	$2(x+\frac{1}{2})(y+\frac{1}{2})-\frac{1}{2}=2xy+x+y+\frac{1}{2}-\frac{1}{2}$ $=2xy+x+y=x*y$	B1 ag **1**	*Intermediate step required*
(iii)(*A*)	If $x,y\in S$ then $x>-\frac{1}{2}$ and $y>-\frac{1}{2}$ $x+\frac{1}{2}>0$ and $y+\frac{1}{2}>0$, so $2(x+\frac{1}{2})(y+\frac{1}{2})>0$ $2(x+\frac{1}{2})(y+\frac{1}{2})-\frac{1}{2}>-\frac{1}{2}$, so $x*y\in S$	M1 A1 A1 **3**	
(*B*)	0 is the identity since $0*x=0+x+0=x$ If $x\in S$ and $x*y=0$ then $2xy+x+y=0$ $y=\dfrac{-x}{2x+1}$ $y+\frac{1}{2}=\dfrac{1}{2(2x+1)}>0$ (since $x>-\frac{1}{2}$) so $y\in S$ *S* is closed and associative; there is an identity; and every element of *S* has an inverse in *S*	B1 B1 M1 A1 M1 A1 **6**	or $2(x+\frac{1}{2})(y+\frac{1}{2})-\frac{1}{2}=0$ or $y+\frac{1}{2}=\dfrac{1}{4(x+\frac{1}{2})}$ *Dependent on M1A1M1*
(iv)	If $x*x=0,\ 2x^2+x+x=0$ $x=0$ or -1 0 is the identity (and has order 1) -1 is not in *S*	M1 A1 A1 **3**	

<table>
<tr><td>(v)</td><td>$4*6=48+4+6=58$
$\quad =56+2=7\times 8+2$
So $4\circ 6=2$</td><td>B1
B1 ag
2</td><td></td></tr>
<tr><td>(vi)</td><td><table><tr><td>Element</td><td>0</td><td>1</td><td>2</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Order</td><td>1</td><td>6</td><td>6</td><td>3</td><td>3</td><td>2</td></tr></table></td><td>B3
3</td><td>Give B2 for 4 correct
B1 for 2 correct</td></tr>
<tr><td>(vii)</td><td>$\{0\},\ G$
$\{0,\ 6\}$
$\{0,\ 4,\ 5\}$</td><td>B1
B1
B1
3</td><td><i>Condone omission of G</i>
If more than 2 non-trivial subgroups are given, deduct 1 mark (from final B1B1) for each non-trivial subgroup in excess of 2</td></tr>
</table>

Pre-multiplication by transition matrix

5 (i)	$\mathbf{P} = \begin{pmatrix} 0.1 & 0.7 & 0.1 \\ 0.4 & 0.2 & 0.6 \\ 0.5 & 0.1 & 0.3 \end{pmatrix}$	B2 **2**	Give B1 for two columns correct
(ii)	$\mathbf{P}^6 \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 0.328864 \\ 0.381536 \\ 0.2896 \end{pmatrix}$ P(*B* used on 7th day) $= 0.3815$	M1 M1 M1 A1 **4**	Using $\mathbf{P}^6$ (or $\mathbf{P}^7$) For matrix of initial probabilities For evaluating matrix product *Accept 0.381 to 0.382*
(iii)	$0.328864 \times 0.1 + 0.381536 \times 0.2 + 0.2896 \times 0.3$ $= 0.1961$	M1 M1 A1 **3**	Using diagonal elements from **P** Correct method *Accept a.r.t. 0.196*
(iv)	$\mathbf{P}^3 = \begin{pmatrix} 0.352 & 0.328 & 0.304 \\ 0.364 & 0.404 & 0.372 \\ 0.284 & 0.268 & 0.324 \end{pmatrix}$ $0.328864 \times 0.352 + 0.381536 \times 0.404 + 0.2896 \times 0.324$ $= 0.3637$	M1 M1 M1 A1 **4**	For evaluating $\mathbf{P}^3$ Using diagonal elements from $\mathbf{P}^3$ Correct method *Accept a.r.t. 0.364*
(v)	$\mathbf{Q} = \begin{pmatrix} 0.3289 & 0.3289 & 0.3289 \\ 0.3816 & 0.3816 & 0.3816 \\ 0.2895 & 0.2895 & 0.2895 \end{pmatrix}$ 0.3289, 0.3816, 0.2895 are the long-run probabilities for the routes *A, B, C*	B1 B1 B1 B1 **4**	*Deduct 1 if not given as a (3×3) matrix* *Deduct 1 if not 4 dp* *Accept 'equilibrium probabilities'*
(vi)	$\begin{pmatrix} 0.1 & 0.7 & a \\ 0.4 & 0.2 & b \\ 0.5 & 0.1 & c \end{pmatrix} \begin{pmatrix} 0.4 \\ 0.2 \\ 0.4 \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.2 \\ 0.4 \end{pmatrix}$ $0.04 + 0.14 + 0.4a = 0.4$, so $a = 0.55$ $0.16 + 0.04 + 0.4b = 0.2$, so $b = 0$ $0.2 + 0.02 + 0.4c = 0.4$, so $c = 0.45$ After *C*, routes *A, B, C* will be used with probabilities 0.55, 0, 0.45	M1 M1 A2 **4**	Obtaining a value for *a, b* or *c* Give A1 for one correct
(vii)	$0.4 \times 0.1 + 0.2 \times 0.2 + 0.4 \times 0.45$ $= 0.26$	M1 M1 A1 **3**	Using long-run probs 0.4, 0.2, 0.4 Using diag elements from new matrix

Post-multiplication by transition matrix

5 (i)	$\mathbf{P} = \begin{pmatrix} 0.1 & 0.4 & 0.5 \\ 0.7 & 0.2 & 0.1 \\ 0.1 & 0.6 & 0.3 \end{pmatrix}$	B2 **2**	Give B1 for two rows correct
(ii)	$\left(\tfrac{1}{3} \quad \tfrac{1}{3} \quad \tfrac{1}{3}\right)\mathbf{P}^6 = \left(0.328864 \quad 0.381536 \quad 0.2896\right)$ P(*B* used on 7th day) $= 0.3815$	M1 M1 M1 A1 **4**	Using $\mathbf{P}^6$ (or $\mathbf{P}^7$) For matrix of initial probabilities For evaluating matrix product *Accept 0.381 to 0.382*
(iii)	$0.328864 \times 0.1 + 0.381536 \times 0.2 + 0.2896 \times 0.3$ $= 0.1961$	M1 M1 A1 **3**	Using diagonal elements from **P** Correct method *Accept a.r.t. 0.196*
(iv)	$\mathbf{P}^3 = \begin{pmatrix} 0.352 & 0.364 & 0.284 \\ 0.328 & 0.404 & 0.268 \\ 0.304 & 0.372 & 0.324 \end{pmatrix}$ $0.328864 \times 0.352 + 0.381536 \times 0.404 + 0.2896 \times 0.324$ $= 0.3637$	M1 M1 M1 A1 **4**	For evaluating $\mathbf{P}^3$ Using diagonal elements from $\mathbf{P}^3$ Correct method *Accept a.r.t. 0.364*
(v)	$\mathbf{Q} = \begin{pmatrix} 0.3289 & 0.3816 & 0.2895 \\ 0.3289 & 0.3816 & 0.2895 \\ 0.3289 & 0.3816 & 0.2895 \end{pmatrix}$ 0.3289, 0.3816, 0.2895 are the long-run probabilities for the routes *A, B, C*	B1B1B1 B1 **4**	*Deduct 1 if not given as a (3×3) matrix* *Deduct 1 if not 4 dp* *Accept 'equilibrium probabilities'*
(vi)	$\left(0.4 \quad 0.2 \quad 0.4\right)\begin{pmatrix} 0.1 & 0.4 & 0.5 \\ 0.7 & 0.2 & 0.1 \\ a & b & c \end{pmatrix} = \left(0.4 \quad 0.2 \quad 0.4\right)$ $0.04 + 0.14 + 0.4a = 0.4, \text{ so } a = 0.55$ $0.16 + 0.04 + 0.4b = 0.2, \text{ so } b = 0$ $0.2 + 0.02 + 0.4c = 0.4, \text{ so } c = 0.45$ After *C*, routes *A, B, C* will be used with probabilities 0.55, 0, 0.45	M1 M1 A2 **4**	Obtaining a value for *a, b* or *c* Give A1 for one correct
(vii)	$0.4 \times 0.1 + 0.2 \times 0.2 + 0.4 \times 0.45$ $= 0.26$	M1 M1 A1 **3**	Using long-run probs 0.4, 0.2, 0.4 Using diag elements from new matrix

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General Comments

Most of the candidates for this paper were well prepared and demonstrated a sound understanding of their chosen three topics. Candidates appeared to have sufficient time to do all they could, and their presentation was generally very good. However, there were parts which proved difficult for all but the most able candidates, and overall the marks were again disappointingly low. About 15% of candidates scored 60 marks or more (out of 72) and about 10% scored fewer than 30 marks. Questions 1 and 2 were about twice as popular as the other three questions, and the most frequent selection this year was questions 1, 2 and 4.

Comments on Individual Questions

1) *(Vectors)*

This was the most popular question, attempted by about 90% of the candidates, and the average mark was about 16 (out of 24). Appropriate vector products were evaluated confidently and accurately by most candidates, although sign errors were fairly common. Another error which occurred quite frequently was misinterpreting a scalar product as a vector; after quoting a formula involving, say, $|\mathbf{p}.\mathbf{q}|$, this was evaluated as $\sqrt{(p_1q_1)^2+(p_2q_2)^2+(p_3q_3)^2}$ instead of $|\, p_1q_1+p_2q_2+p_3q_3\,|$.

Parts (i) to (iv) were very well understood and often answered correctly. Parts (v) and (vi) were quite often omitted, but a good number of candidates did find the values of λ and μ. When explaining why E is between A and C, most mentioned $\lambda<1$ bur failed to mention $\lambda>0$. Candidates who attempted part (vi) usually made good progress with it, although some wasted time by calculating the volume of the tetrahedron ABEF from the coordinates of E and F, instead of using the values of λ and μ.

2) *(Multi-variable calculus)*

This was the worst answered question, with an average mark of about 11; parts (iii) and (v) posed significant difficulties for a large number of candidates.

Most candidates found the partial derivatives correctly in part (i), although quite a number made algebraic slips when simplifying their expressions; this was not penalised here, but it led to problems later on. Most also knew how to find the equation of the normal line in part (ii), although some just gave the normal vector, and some gave the equation of the tangent plane.

In part (iii) most candidates tried to use $h\approx(\partial g/\partial x)\delta x+(\partial g/\partial y)\delta y+(\partial g/\partial z)\delta z$, but few could combine this with PQ being parallel to the normal vector they had found in part (ii).

Part (iv) was done quite well. After manipulating $\partial g/\partial x=\partial g/\partial y=0$ to obtain $z=0$, a good proportion of the candidates could use the equation of *S* to explain why this is impossible. Many also incorrectly stated that $\partial g/\partial z=1$, but provided this was not used in the subsequent argument, they were not penalised.

In part (v), those who started with $\partial g/\partial y=5\partial g/\partial x$ and $\partial g/\partial z=0$ were often successful. However, a large number of candidates assumed that $\partial g/\partial x=1$ and $\partial g/\partial y=5$; they could then find (incorrect) values of *x, y* and *z* without substituting into the equation of *S*, and scored only 2 marks out of the 8.

3) *(Differential geometry)*
This was by far the best answered question, with an average mark of about 19. The techniques required in all four parts were very well known. The differentiations and integrations were almost always carried out accurately, but some candidates had difficulties with the algebra when deriving the given results in parts (i) and (iii).

4) *(Groups)*
The average mark on this question was about 15. Candidates attempting it generally demonstrated a good basic knowledge of groups, and parts (i), (ii), (v), (vi) and (vii) were answered very well.
In part (iii), closure was often not shown satisfactorily, with many candidates showing in effect that $x * y \neq -½$ rather than $x * y > -½$. The identity and inverse were usually found correctly, but many omitted to show that $x^{-1} > -½$.
Part (iv) was often omitted, but there were also many correct answers.

5) *(Markov chains)*
This was the least popular question, attempted by about 30% of the candidates, and the average mark was about 16. Most of those who attempted the question demonstrated a sound knowledge of the techniques and used their calculator effectively to evaluate powers and products of matrices. Parts (i), (ii), (v) and (vi) were answered very well.
In part (iii), a very large number of candidates started by finding the probabilities for the 8th day, then proceeded as if the 7th and 8th days were independent. Similar errors were made in parts (iv) and (vii).