

ADVANCED SUBSIDIARY GCE
MATHEMATICS (MEI)
Mechanics 1

4761

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Wednesday 21 January 2009
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Section A (36 marks)

- 1 A particle is travelling in a straight line. Its velocity $v \text{ m s}^{-1}$ at time t seconds is given by

$$v = 6 + 4t \quad \text{for } 0 \leq t \leq 5.$$

- (i) Write down the initial velocity of the particle and find the acceleration for $0 \leq t \leq 5$. [2]

- (ii) Write down the velocity of the particle when $t = 5$. Find the distance travelled in the first 5 seconds. [3]

For $5 \leq t \leq 15$, the acceleration of the particle is 3 m s^{-2} .

- (iii) Find the total distance travelled by the particle during the 15 seconds. [3]

- 2 Fig. 2 shows an acceleration-time graph modelling the motion of a particle.

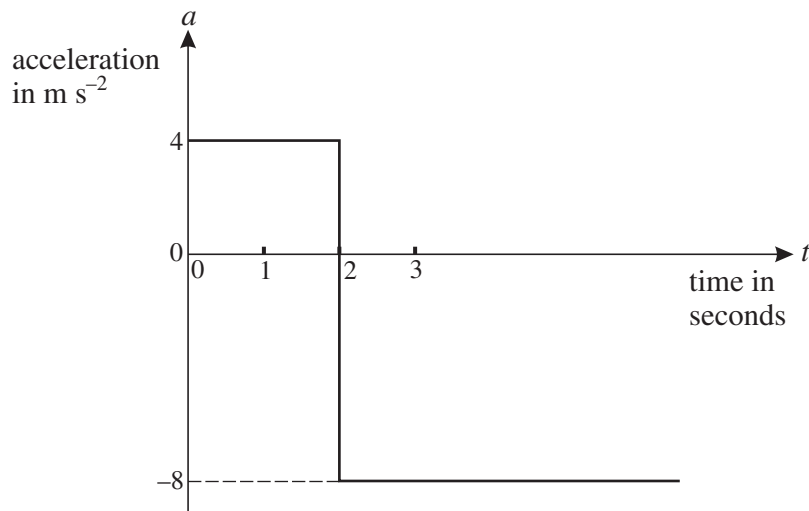


Fig. 2

At $t = 0$ the particle has a velocity of 6 m s^{-1} in the positive direction.

- (i) Find the velocity of the particle when $t = 2$. [2]

- (ii) At what time is the particle travelling in the negative direction with a speed of 6 m s^{-1} ? [2]

- 3 The resultant of the force $\begin{pmatrix} -4 \\ 8 \end{pmatrix} \text{ N}$ and the force \mathbf{F} gives an object of mass 6 kg an acceleration of $\begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ m s}^{-2}$.

- (i) Calculate \mathbf{F} . [4]

- (ii) Calculate the angle between \mathbf{F} and the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. [2]

- 4 Sandy is throwing a stone at a plum tree. The stone is thrown from a point O at a speed of 35 m s^{-1} at an angle of α to the horizontal, where $\cos \alpha = 0.96$. You are *given* that, t seconds after being thrown, the stone is $(9.8t - 4.9t^2)$ m higher than O.

When descending, the stone hits a plum which is 3.675 m higher than O. Air resistance should be neglected.

Calculate the horizontal distance of the plum from O. [6]

- 5 A man of mass 75 kg is standing in a lift. He is holding a parcel of mass 5 kg by means of a light inextensible string, as shown in Fig. 5. The tension in the string is 55 N.

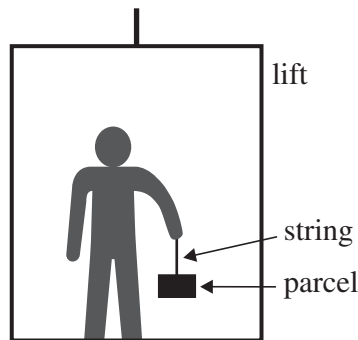


Fig. 5

- (i) Find the upward acceleration. [3]
- (ii) Find the reaction on the man of the lift floor. [2]
- 6 Small stones A and B are initially in the positions shown in Fig. 6 with B a height H m directly above A.

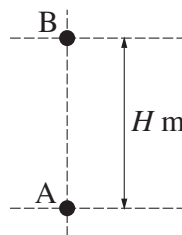


Fig. 6

At the instant when B is released from rest, A is projected vertically upwards with a speed of 29.4 m s^{-1} . Air resistance may be neglected.

The stones collide T seconds after they begin to move. At this instant they have the same speed, $V \text{ m s}^{-1}$, and A is still rising.

By considering when the speed of A upwards is the same as the speed of B downwards, or otherwise, show that $T = 1.5$ and find the values of V and H . [7]

Section B (36 marks)

- 7 An explorer is trying to pull a loaded sledge of total mass 100 kg along horizontal ground using a light rope. The only resistance to motion of the sledge is from friction between it and the ground.

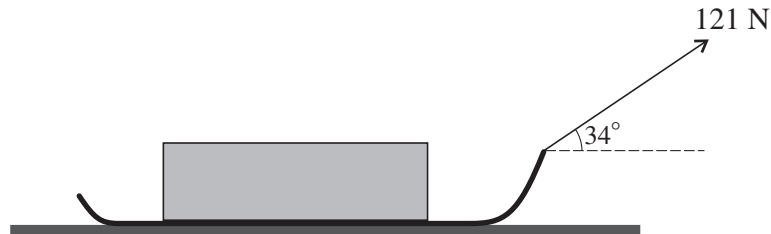


Fig. 7

Initially she pulls with a force of 121 N on the rope inclined at 34° to the horizontal, as shown in Fig. 7, but the sledge does not move.

- (i) Draw a diagram showing all the forces acting on the sledge.

Show that the frictional force between the ground and the sledge is 100 N, correct to 3 significant figures.

Calculate the normal reaction of the ground on the sledge. [7]

The sledge is given a small push to set it moving at 0.5 m s^{-1} . The explorer continues to pull on the rope with the same force and the same angle as before. The frictional force is also unchanged.

- (ii) Describe the subsequent motion of the sledge. [2]

The explorer now pulls the rope, still at an angle of 34° to the horizontal, so that the tension in it is 155 N. The frictional force is now 95 N.

- (iii) Calculate the acceleration of the sledge. [3]

In a new situation, there is no rope and the sledge slides down a uniformly rough slope inclined at 26° to the horizontal. The sledge starts from rest and reaches a speed of 5 m s^{-1} in 2 seconds.

- (iv) Calculate the frictional force between the slope and the sledge. [5]

- 8** A toy boat moves in a horizontal plane with position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$, where \mathbf{i} and \mathbf{j} are the standard unit vectors east and north respectively. The origin of the position vectors is at O. The displacements x and y are in metres.

First consider only the motion of the boat parallel to the x -axis. For this motion

$$x = 8t - 2t^2.$$

The velocity of the boat in the x -direction is $v_x \text{ m s}^{-1}$.

- (i)** Find an expression in terms of t for v_x and determine when the boat instantaneously has zero speed in the x -direction. **[3]**

Now consider only the motion of the boat parallel to the y -axis. For this motion

$$v_y = (t - 2)(3t - 2),$$

where $v_y \text{ m s}^{-1}$ is the velocity of the boat in the y -direction at time t seconds.

- (ii)** Given that $y = 3$ when $t = 1$, use integration to show that $y = t^3 - 4t^2 + 4t + 2$. **[4]**

The position vector of the boat is given in terms of t by $\mathbf{r} = (8t - 2t^2)\mathbf{i} + (t^3 - 4t^2 + 4t + 2)\mathbf{j}$.

- (iii)** Find the time(s) when the boat is due north of O and also the distance of the boat from O at any such times. **[4]**
- (iv)** Find the time(s) when the boat is instantaneously at rest. Find the distance of the boat from O at any such times. **[5]**
- (v)** Plot a graph of the path of the boat for $0 \leq t \leq 2$. **[3]**

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Q 1		Mark	Comment	Sub
(i)	6 m s^{-1} 4 m s^{-2}	B1 B1	Neglect units. Neglect units.	2
(ii)	$v(5) = 6 + 4 \times 5 = 26$ $s(5) = 6 \times 5 + 0.5 \times 4 \times 25 = 80$ so 80 m	B1 M1 A1	Or equiv. FT (i) and their $v(5)$ where necessary. cao	3
(iii)	distance is $80 +$ $26 \times (15 - 5) + 0.5 \times 3 \times (15 - 5)^2$ $= 490 \text{ m}$	M1 M1 A1	Their $80 +$ attempt at distance with $a = 3$ Appropriate <i>uvast</i> . Allow $t = 15$. FT their $v(5)$. cao	3
		8		

Q 2		Mark	Comment	Sub
(i)	When $t = 2$, velocity is $6 + 4 \times 2 = 14$	M1 A1	Recognising that areas under graph represent changes in velocity in (i) or (ii) or equivalent <i>uvast</i> .	2
(ii)	Require velocity of -6 so must inc by -20 $-8 \times (t - 2) = -20$ so $t = 4.5$	M1 F1	FT $\pm(6 + \text{their } 14)$ used in any attempt at area/ <i>uvast</i> FT their 14 [Award SC2 for 4.5 WW and SC1 for 2.5 WW]	2
		4		

Q 3		Mark	Comment	Sub
(i)	$\mathbf{F} + \begin{pmatrix} -4 \\ 8 \end{pmatrix} = 6 \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\mathbf{F} = \begin{pmatrix} 16 \\ 10 \end{pmatrix}$	M1 B1 B1 A1	N2L. $F = ma$. All forces present Addition to get resultant. May be implied. For $\mathbf{F} \pm \begin{pmatrix} -4 \\ 8 \end{pmatrix} = 6 \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. SC4 for $\mathbf{F} = \begin{pmatrix} 16 \\ 10 \end{pmatrix}$ WW. If magnitude is given, final mark is lost unless vector answer is clearly intended.	4
(ii)	$\arctan\left(\frac{16}{10}\right)$ $57.994\dots$ so 58.0° (3 s. f.)	M1 A1	Accept equivalent and FT their \mathbf{F} only. Do not accept wrong angle. Accept $360 - \arctan\left(\frac{16}{10}\right)$ cao. Accept 302° (3 s.f.)	2
		6		

Q4	Mark	Comment	Sub
<p>either We need $3.675 = 9.8t - 4.9t^2$</p> <p>Solving $4t^2 - 8t + 3 = 0$ gives $t = 0.5$ or $t = 1.5$</p> <p>or</p> <p>Time to greatest height $0 = 35 \times 0.28 - 9.8t$ so $t = 1$ Time to drop is 0.5 total is 1.5 s</p> <p>then Horiz distance is $35 \times 0.96t$ So distance is $35 \times 0.96 \times 1.5 = 50.4$ m</p>	<p>*M1</p> <p>M1*</p> <p>A1</p> <p>F1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>F1</p>	<p>Equating given expression or their attempt at y to ± 3.675. If they attempt y, allow sign errors, $g = 9.81$ etc. and $u = 35$.</p> <p>Dependent. Any method of solution of a 3 term quadratic.</p> <p>cao. Accept only the larger root given</p> <p>Both roots shown and larger chosen provided both +ve. Dependent on 1st M1. [Award M1 M1 A1 for 1.5 seen WW]</p> <p>Complete method for total time from motion in separate parts. Allow sign errors, $g = 9.81$ etc. Allow $u = 35$ initially only.</p> <p>Time for 1st part</p> <p>Time for 2nd part</p> <p>cao</p> <p>Use of $x = u \cos at$. May be implied.</p> <p>FT their quoted t provided it is positive.</p>	6
	6		

Q5	Mark	Comment	Sub
(i)	M1	Applying N2L to the parcel. Correct mass. Allow $F = mga$. Condone missing force but do not allow spurious forces.	3
	A1	Allow only sign error(s).	
	A1	Allow -1.2 only if sign convention is clear.	
(ii)	M1	N2L. Must have correct mass. Allow only sign errors.	2
	A1	FT their a cao [NB beware spurious methods giving 880 N]	
	5		

Q6	Mark	Comment	Sub
<p>Method 1</p> <p>$\uparrow v_A = 29.4 - 9.8T \quad \downarrow v_B = 9.8T$</p> <p>For same speed $29.4 - 9.8T = 9.8T$</p> <p>so $T = 1.5$ and $V = 14.7$</p> $H = 29.4 \times 1.5 - 0.5 \times 9.8 \times 1.5^2 + 0.5 \times 9.8 \times 1.5^2 = 44.1$ <p>Method 2</p> $V^2 = 29.4^2 - 2 \times 9.8 \times x = 2 \times 9.8 \times (H - x)$ <p>$29.4^2 = 19.6H$ so $H = 44.1$</p> <p>Relative velocity is 29.4 so</p> $T = \frac{44.1}{29.4}$ <p>Using $v = u + at$</p> $V = 0 + 9.8 \times 1.5 = 14.7$	<p>M1</p> <p>A1</p> <p>M1</p> <p>E1</p> <p>F1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>M1</p> <p>E1</p> <p>M1</p> <p>F1</p>	<p>Either attempted. Allow sign errors and $g = 9.81$ etc</p> <p>Both correct</p> <p>Attempt to equate. Accept sign errors and $T = 1.5$ substituted in both.</p> <p>If 2 subs there must be a statement about equality</p> <p>FT T or V, whichever is found second</p> <p>Sum of the distance travelled by each attempted</p> <p>cao</p> <p>Attempts at V^2 for each particle equated. Allow sign errors, 9.81 etc</p> <p>Allow h_1, h_2 without $h_1 = H - h_2$</p> <p>Both correct. Require $h_1 = H - h_2$ but not an equation.</p> <p>cao</p> <p>Any method that leads to T or V</p> <p>Any method leading to the other variable</p> <p>Other approaches possible. If 'clever' ways seen, reward according to weighting above.</p>	<p>7</p>
	7		

Q7		Mark	Comment	Sub
(i)	Diagram Resolve \rightarrow $121 \cos 34 - F = 0$ $F = 100.313 \dots$ so 100 N (3 s. f.) Resolve \uparrow $R + 121 \sin 34 - 980 = 0$ $R = 912.337 \dots$ so 912 N (3 s. f.)	B1 B1 M1 E1 M1 B1 A1	Weight, friction and 121 N present with arrows. All forces present with suitable labels. Accept W , mg , $100g$ and 980 . No extra forces. Resolving horiz. Accept $s \leftrightarrow c$. Some evidence required for the <i>show</i> , e.g. at least 4 figures. Accept \pm . Resolve vert. Accept $s \leftrightarrow c$ and sign errors. All correct	7
(ii)	It will continue to move at a constant speed of 0.5 m s^{-1} .	E1 E1	Accept no reference to direction Accept no reference to direction [Do not isw: conflicting statements get zero]	2
(iii)	Using N2L horizontally $155 \cos 34 - 95 = 100a$ $a = 0.335008 \dots$ so 0.335 m s^{-2} (3 s. f.)	M1 A1 A1	Use of N2L. Allow $F = mga$, F omitted and 155 not resolved. Use of $F = ma$ with resistance and T resolved. Allow $s \leftrightarrow c$ and signs as the only errors.	3
(iv)	$a = 5 \div 2 = 2.5$ N2L down the slope $100g \sin 26 - F = 100 \times 2.5$ $F = 179.603 \dots$ so 180 N (3 s. f.)	M1 A1 M1 B1 A1	Attempt to find a from information $F = ma$ using their "new" a . All forces present. No extras. Require attempt at wt cpt. Allow $s \leftrightarrow c$ and sign errors. Weight term resolved correctly, seen in an equn or on a diagram. cao. Accept -180 N if consistent with direction of F on their diagram	5
		17		

Q8	Mark	Comment	Sub
(i)	M1 A1 F1	either Differentiating or Finding 'u' and 'a' from x and use of $v = u + at$ FT their $v_x = 0$	3
(ii)	M1 A1 M1 E1	Integrating v_y with at least one correct integrated term. All correct. Accept no arbitrary constant. Clear evidence Clearly shown and stated	4
(iii)	M1 A1 A1 A1	May be implied. Must have both Condone 2j Condone 18j	4
(iv)	M1 M1 A1 B1 B1	either Recognises $v_x = 0$ when $t = 2$ or Finds time(s) when $v_y = 0$ or States or implies $v_x = v_y = 0$ Considers $v_x = 0$ and $v_y = 0$ with their time(s) $t = 2$ recognised as only value (accept as evidence only $t = 2$ used below). For the last 2 marks, no credit lost for reference to $t = \frac{2}{3}$. May be implied FT from their position. Accept one position followed through correctly.	5
(v)	B1 B1 B1	At least one value $0 \leq t < 2$ correctly calc. This need not be plotted Must be x-y curve. Accept sketch. Ignore curve outside interval for t. Accept unlabelled axes. Condone use of line segments. At least three correct points used in x-y graph or sketch. General shape correct. Do not condone use of line segments.	3
	19		

4761 Mechanics 1

General Comments

There were many perfect solutions to every question and it was pleasing to see most of the candidates making good progress with several questions. There was little evidence that time was a problem to any candidate who tackled most of the questions reasonably efficiently.

Q2, involving the interpretation and use of kinematics graphs, presented great problems to many candidates. As has been the case for questions on this topic in recent sessions, it seems that many candidates do not have sufficient experience with answering this type of question.

Q4 and Q6 were unstructured and some of the weaker candidates failed to organise their ideas sufficiently well to make much progress.

Many of the scripts were well presented and showed a good knowledge of the course and the ability to select appropriate techniques.

Comments on Individual Questions

Section A

- 1) A constant acceleration problem in two stages
 - (i) Most candidates obtained the correct answers but many did not seem to recognise that they were dealing with an expression of the form $v = u + at$. These evaluated v at some time (usually 5 s), assumed constant acceleration and then used $v = u + at$ to deduce a .
 - (ii) $v(5)$ was usually found correctly but quite a few candidates made substitution or evaluation errors.
 - (iii) There were many correct answers but some candidates failed properly to deal with the second part of the motion being linked to the first but with different acceleration. The most common errors were: to forget to add the 80 m from part (ii); to take the initial velocity to be 6 m s^{-1} from part (i) instead of 26 m s^{-1} from part (ii); to use the acceleration of the first part of the motion; to think that the second part of the motion lasted for 15 s instead of 10 s; to use integration but omit the arbitrary constants.

2) Use of an acceleration-time graph

Many candidates made little progress with this question as they did not seem properly to understand what the graph was telling them. Others wrote down the correct answers.

- (i) Many candidates did not know what to do. Some were clearly not considering the time interval $0 \leq t \leq 2$ but the time $t = 2$ and tried to take account of the acceleration changing at that time. The most common mistake was to neglect the initial velocity and give the *change* in velocity of 2×4 as if it were the velocity when $t = 2$.
- (ii) Again many candidates did not know what to do. The most common mistakes were in identifying the change of velocity as being from the answer to part (i) to -6 m s^{-1} . Commonly the change was given as being from $+6$ to -6 or from their answer to part (i) to $+6$. Many candidates who obtained 2.5 s forgot that they had to add 2 s .

3) The resultant of two vectors, Newton's second law applied in 2 dimensions and the direction of a vector

- (i) Apart from the few candidates who did not understand the vector notation, this part was usually done accurately with the most common error being subtracting the forces instead of adding them.
- (ii) Many candidates failed to recognise the direction given and gave the complementary angle to the one required. Quite a few thought they had to subtract $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ from their force before using the standard technique for finding the bearing of a vector.

- 4) The horizontal distance travelled by a projectile before it is descending at a given height above the point of projection

This question was unstructured but most candidates were familiar with problems of this type and knew what to do.

A surprising number of candidates threw away the given expression for the height of the stone and started again.

Most candidates used the obvious method of equating the height of the stone at time t to 3.675, solving the quadratic equation and selecting the larger root as the stone is descending. Very few candidates simplified the expression and tried to factorize; most used the quadratic formula. The chief problems were with signs, either because the sign of the 3.675 term became wrong during reorganisation of the quadratic expression or because there was a mistake during substitution into the formula (especially when the expression was left as $-4.9t^2 + 9.8t - 3.675 = 0$). Most candidates chose the greater root (and quite a few said why) but some gave two answers to the question.

Some candidates took on the longer method of finding the time to the top of the motion and then the time to fall to the required height; many gave up or made a mistake.

Finding the horizontal range from the time taken was usually done accurately.

- 5) Connected particles accelerating upwards in a lift

Quite a few candidates scored no marks on this question; more made progress with part (i) than with part (ii). Of course, many candidates wrote down the answers with accomplished ease.

- (i) Many candidates did not realize that they had to consider the equation of motion of the parcel not that of the man or man with parcel. The most common mistake from those who did consider only the parcel was to neglect its weight.
- (ii) Those who considered the man with the parcel often managed to get the right answer if they remembered the weight; those who considered only the man usually forgot the 55 N.

- 6) The collision of two particles moving in the same vertical line.

This unstructured question was less familiar than Q4 and fewer candidates managed to find a satisfactory method. Although many candidates struggled to make much progress, many others did well and produced efficient complete answers.

The most commonly used successful method was to use the information given directly in the question and equate the speeds at time t , thereby establishing that the collision took place after 1.5 s; the speed at collision followed as did the initial separation. The most common error made by candidates who adopted this approach was to equate velocities instead of speeds and then find that their term in t disappeared. Most candidates who used the given answer $T = 1.5$ failed to substitute it in expressions for the speeds of *each* of the particles and/or failed to make a statement explaining how they had established the result.

There were many false assumptions made, a common one being that the two particles travelled equal distances before collision; many candidates based their whole argument on this – others correctly found the distance travelled by one particle and incorrectly doubled it to find H . Other candidates based their attempts on assertions that were *true* but gave no reason *why* they must be true; for instance, the collision takes place after half the time it would have taken particle A to reach its greatest height.

A surprising number of candidates gave the value of H as the distance travelled by just one of the particles.

- 7) Static equilibrium and the application of Newton's second law to motion on horizontal and inclined planes.

Many candidates answered most of this question very well. It was especially pleasing to see many correct solutions to part (v) which involves motion on a slope and many correctly finding the normal reaction in part (i).

- (i) Most candidates produced an accurate diagram. The common mistakes were to omit an arrow (usually on the force in the string – perhaps they were drawing the string not the force) or the normal reaction or a label.

With the answer given, most candidates used the correct trigonometric ratio to find the frictional force. Many merely asserted that $121\cos 34$ is 100 (correct to 3 significant figures) without showing any evidence.

Many candidates accurately found the normal reaction. As always, many candidates wrongly believed that the normal reaction is the component of the weight perpendicular to the tangent of contact and so in the question ignored the component of the force in the string.

- (ii) Many candidates correctly stated that the sledge continued to move at the constant speed of 0.5 m s^{-1} but a few argued that it slowed down or even that it speeded up.
- (iii) Many candidates did this well. The common errors were to forget to resolve the tension and/or to omit the frictional force.

- (iv) There were many completely correct solutions to this part. Most candidates found the new acceleration but not all of these found the frictional force. The common errors were to use the wrong trigonometric ratio or use 'false' resolution obtained from the 'wrong' triangle to give the weight component as $\frac{980}{\sin 26}$.

- 8) The kinematics requiring calculus of a toy boat moving in 2 dimensions; the direction of the boat as seen from the origin, the direction of motion of the boat and the path of the boat

Most of the candidates used calculus appropriately in the question. Compared with previous sessions, more candidates correctly recognised when to use the direction of the position vector and when to use the direction of the velocity vector.

Some of the notation used was poor with candidates writing, for example, $\mathbf{i} = 0$ when they meant (and used) $v_x = 0$.

- (i) Most candidates used calculus to find the velocity of the boat in the x -direction and did so accurately. Quite a few then went on to find v_x when $t = 0$ instead of t when $v_x = 0$ but quoted their answer as a time.
- (ii) It was pleasing to see how well this was generally done. Most candidates convincingly obtained the expression $y = t^2 - 4t^2 + 4t + c$ but a few then simply stated that $c = 2$ without any attempt to demonstrate this is true.
- (iii) Many candidates did this completely correctly. Most realized that they needed the direction of the position vector and needed a zero \mathbf{i} component. It was disappointing that many omitted the $t = 0$ solution.
- (iv) This part presented many difficulties. Many candidates realized that they required the direction of the velocity vector but not all of them considered both $v_x = 0$ and $v_y = 0$ and others did not make it clear exactly what they were doing. A common mistake was to go on from the condition that both $v_x = 0$ and $v_y = 0$ to solve one of the equations $v_x \pm v_y = 0$. It was also clear that some candidates thought that they needed *either* $v_x = 0$ or $v_y = 0$.

Quite a few candidates correctly found the position vector when $t = 2$ but did not, as requested, find the distance of the boat from the origin.

There were many complete answers beautifully displayed and clearly argued.

- (v) There were a few correct answers. A common error was to plot a graph of x against t or y against t or distance against t .