

**ADVANCED SUBSIDIARY GCE
MATHEMATICS (MEI)**
Mechanics 1

4761

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

**Wednesday 27 January 2010
Afternoon**

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Section A (36 marks)

- 1 A ring is moving up and down a vertical pole. The displacement, s m, of the ring above a mark on the pole is modelled by the displacement-time graph shown in Fig. 1. The three sections of the graph are straight lines.

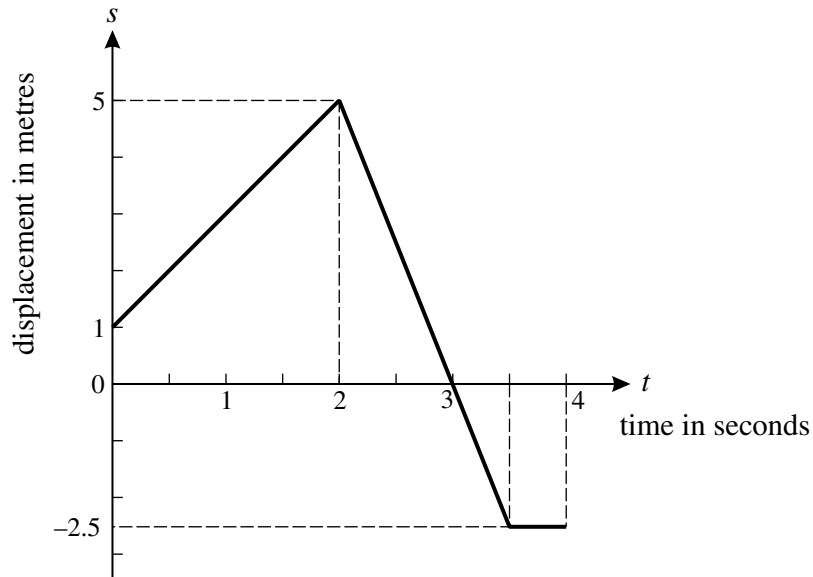


Fig. 1

- (i) Calculate the velocity of the ring in the interval $0 < t < 2$ and in the interval $2 < t < 3.5$. [2]
- (ii) Sketch a velocity-time graph for the motion of the ring during the 4 seconds. [2]
- (iii) State the direction of motion of the ring when
- (A) $t = 1$,
- (B) $t = 2.75$,
- (C) $t = 3.25$. [1]
- 2 A particle of mass 5 kg has constant acceleration. Initially, the particle is at $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ m with velocity $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ m s⁻¹; after 4 seconds the particle has velocity $\begin{pmatrix} 12 \\ 9 \end{pmatrix}$ m s⁻¹.
- (i) Calculate the acceleration of the particle. [2]
- (ii) Calculate the position of the particle at the end of the 4 seconds. [3]
- (iii) Calculate the force acting on the particle. [2]

- 3 In this question, \mathbf{i} is a horizontal unit vector and \mathbf{j} is a unit vector pointing vertically upwards.

A force \mathbf{F} is $-\mathbf{i} + 5\mathbf{j}$.

- (i) Calculate the magnitude of \mathbf{F} .

Calculate also the angle between \mathbf{F} and the upward vertical.

[4]

Force \mathbf{G} is $2a\mathbf{i} + a\mathbf{j}$ and force \mathbf{H} is $-2\mathbf{i} + 3b\mathbf{j}$, where a and b are constants. The force \mathbf{H} is the resultant of forces $4\mathbf{F}$ and \mathbf{G} .

- (ii) Find \mathbf{G} and \mathbf{H} .

[4]

- 4 A box of mass 2.5 kg is on a smooth horizontal table, as shown in Fig. 4. A light string AB is attached to the table at A and the box at B. AB is at an angle of 50° to the vertical. Another light string is attached to the box at C; this string is inclined at 15° above the horizontal and the tension in it is 20 N. The box is in equilibrium.

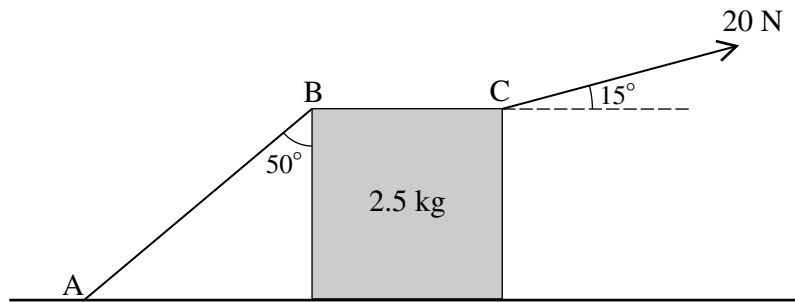


Fig. 4

- (i) Calculate the horizontal component of the force exerted on the box by the string at C. [1]

- (ii) Calculate the tension in the string AB. [2]

- (iii) Calculate the normal reaction of the table on the box. [4]

The string at C is replaced by one inclined at 15° below the horizontal with the same tension of 20 N.

- (iv) Explain why this has no effect on the tension in string AB. [1]

- 5 The velocity, $v \text{ m s}^{-1}$, of a particle moving along a straight line is given by

$$v = 3t^2 - 12t + 14,$$

where t is the time in seconds.

- (i) Find an expression for the acceleration of the particle at time t . [2]

- (ii) Find the displacement of the particle from its position when $t = 1$ to its position when $t = 3$. [4]

- (iii) You are given that v is always positive. Explain how this tells you that the distance travelled by the particle between $t = 1$ and $t = 3$ has the same value as the displacement between these times. [2]

Section B (36 marks)

6

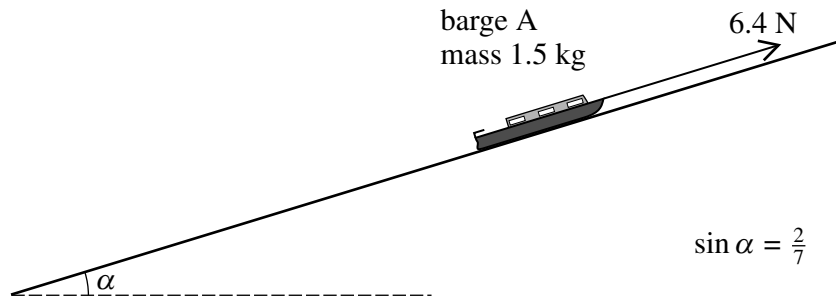


Fig. 6.1

Fig. 6.1 shows a toy barge A of mass 1.5 kg on a rough plane. The plane is at an angle α to the horizontal where $\sin \alpha = \frac{2}{7}$.

- (i) Show that the component of the weight of the barge down the slope is 4.2 N. [2]

The barge is held in equilibrium by a force of 6.4 N acting up and parallel to the plane.

- (ii) Determine the frictional force on the barge and state whether it acts up or down the plane. [2]

The force of 6.4 N is removed and the barge now slides down the plane with acceleration 1.2 m s^{-2} .

- (iii) Calculate the new frictional force on the barge. [4]

- (iv) Determine how far the barge travels while its speed increases from 0.8 m s^{-1} to 2 m s^{-1} . [3]

Fig. 6.2 shows barge A on the same slope with a second barge B of mass 2 kg attached to it by means of a light rigid coupling parallel to the plane. The frictional force on barge B is 0.7 N and the frictional force on barge A is now 2.3 N. At one stage of the motion the two barges are being pulled up the plane by a force of 10 N parallel to the plane.

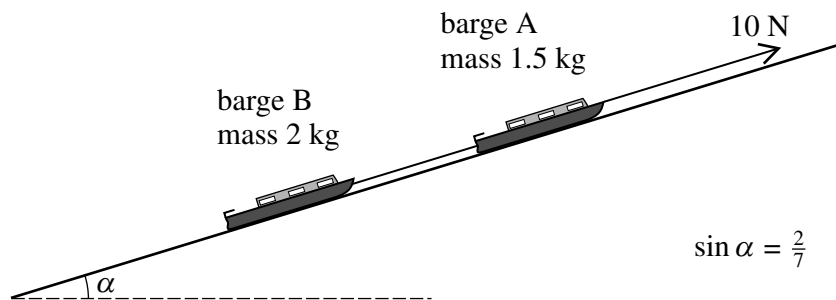


Fig. 6.2

- (v) Draw diagrams showing the forces acting on each barge.

Calculate the acceleration of the barges and clearly indicate its direction.

- Find the force in the coupling, stating whether this is a tension or a thrust (compression). [7]

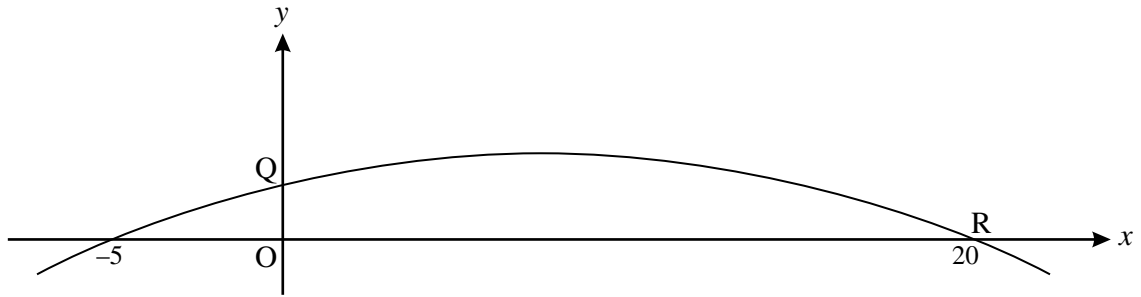


Fig. 7

Fig. 7 shows the graph of $y = \frac{1}{100}(100 + 15x - x^2)$.

For $0 \leq x \leq 20$, this graph shows the trajectory of a small stone projected from the point Q where y m is the height of the stone above horizontal ground and x m is the horizontal displacement of the stone from O. The stone hits the ground at the point R.

- (i) Write down the height of Q above the ground. [1]
- (ii) Find the horizontal distance from O of the highest point of the trajectory and show that this point is 1.5625 m above the ground. [5]
- (iii) Show that the time taken for the stone to fall from its highest point to the ground is 0.565 seconds, correct to 3 significant figures. [3]
- (iv) Show that the horizontal component of the velocity of the stone is 22.1 m s^{-1} , correct to 3 significant figures. Deduce the time of flight from Q to R. [5]
- (v) Calculate the speed at which the stone hits the ground. [4]

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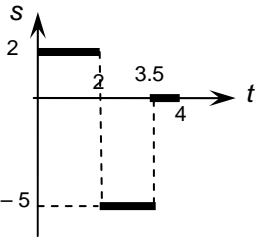
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1 (i)	$0 < t < 2, v = 2$ $2 < t < 3.5, v = -5$	B1 B1	Condone '5 downwards' and '-5 downwards'	2
(ii)		B1 B1	<p>Condone intent – e.g. straight lines free-hand and scales not labelled; accept non-vertical sections at $t = 2$ & 3.5.</p> <p>Only horizontal lines used and 1st two parts present. BOD t-axis section. One of 1st 2 sections correct. FT (i) and allow if answer correct with (i) wrong All correct. Accept correct answer with (i) wrong. FT (i) only if 2nd section –ve in (i)</p>	2
(iii)	(A) upwards; (B) and (C) downwards	E1	All correct. Accept +/- ve but not towards/away from O Accept forwards/backwards. Condone additional wrong statements about position.	1
5				
2 (i)	$\begin{pmatrix} 12 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + 4\mathbf{a}$ so $\mathbf{a} = \begin{pmatrix} 2.5 \\ 3 \end{pmatrix}$	M1 A1	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ If vector \mathbf{a} seen, isw.	2
(ii)	either $\mathbf{r} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} \times 4 + \frac{1}{2} \mathbf{a} \times 4^2$ $\mathbf{r} = \begin{pmatrix} 27 \\ 14 \end{pmatrix} \text{ so } \begin{pmatrix} 27 \\ 14 \end{pmatrix} \text{ m}$ or	M1 A1 A1 M1 A1 A1	For use of $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with their a . Initial position may be omitted. FT their a . Initial position may be omitted. cao. Do not condone magnitude as final answer. Use of $\mathbf{s} = 0.5t(\mathbf{u} + \mathbf{v})$ Initial position may be omitted. Correct substitution. Initial position may be omitted. cao Do not condone mag as final answer. SC2 for $\begin{pmatrix} 28 \\ 12 \end{pmatrix}$	3

(iii)	Using N2L $\mathbf{F} = 5\mathbf{a} = \begin{pmatrix} 12.5 \\ 15 \end{pmatrix}$ so $\begin{pmatrix} 12.5 \\ 15 \end{pmatrix}$ N	M1 F1	Use of $\mathbf{F} = m\mathbf{a}$ or $\mathbf{F} = m\mathbf{g}\mathbf{a}$. FT their a only. Do not accept magnitude as final ans.	2
				7
3 (i)	$ \mathbf{F} = \sqrt{(-1)^2 + 5^2}$ $= \sqrt{26} = 5.0990\dots = 5.10$ (3 s. f.) Angle with \mathbf{j} is $\arctan(0.2)$ so $11.309\dots$ so 11.3° (3 s. f.)	M1 A1 M1 A1	Accept $\sqrt{-1^2 + 5^2}$ even if taken to be $\sqrt{24}$ accept $\arctan(p)$ where $p = \pm 0.2$ or ± 5 o.e. cao	4
(ii)	$\begin{pmatrix} -2 \\ 3b \end{pmatrix} = 4\begin{pmatrix} -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 2a \\ a \end{pmatrix}$ $a = 1, b = 7$ so $\mathbf{G} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{H} = \begin{pmatrix} -2 \\ 21 \end{pmatrix}$ or $\mathbf{G} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{H} = -2\mathbf{i} + 21\mathbf{j}$	M1 M1 A1 A1	$\mathbf{H} = 4\mathbf{F} + \mathbf{G}$ soi Formulating at least 1 scalar equation from their vector equation soi a correct or G follows from their wrong a \mathbf{H} cao	4
				8
4(i)	$20\cos 15 = 19.3185\dots$ so 19.3 N (3 s. f.) in direction BC	B1	Accept no direction. Must be evaluated	1
(ii)	Let the tension be T $T \sin 50 = 19.3185\dots$ so $T = 25.2185\dots$ so 25.2 N (3 s. f.)	M1 F1	Accept $\sin \leftrightarrow \cos$ but not $(i) \times \sin 50$ FT their $19.3\dots$ only. cwo	2
(iii)	$R + 20 \sin 15 - 2.5g - 25.2185\dots \times \cos 50 = 0$ $R = 35.5337\dots$ so 35.5 N (3 s. f.)	M1 B1 A1 A1	Allow 1 force missing or 1 tension not resolved. FT T . No extra forces. Accept mass used. Accept $\sin \leftrightarrow \cos$. Weight correct All correct except sign errors. FT their T cao. Accept 35 or 36 for 2. s.f.	4
(iv)	The horizontal resolved part of the 20 N force is not changed.	E1	Accept no reference to vertical component but do not accept 'no change' to both components. No need to be explicit that value of tension in AB depends only on horizontal component of force at C	1
				8

5(i)	$a = 6t - 12$	M1 A1	Differentiating cao	2
(ii)	We need $\int_1^3 (3t^2 - 12t + 14) dt$ $= [t^3 - 6t^2 + 14t]_1^3$ either $= (27 - 54 + 42) - (1 - 6 + 14)$ $= 15 - 9 = 6$ so 6 m or $s = t^3 - 6t^2 + 14t + C$ $s = 0$ when $t = 1$ gives $0 = 1 - 6 + 14 + C$ so $C = -9$ Put $t = 3$ to give $s = 27 - 54 + 42 - 9 = 6$ so 6 m.	M1 A1 M1 A1 M1 A1	Integrating. Neglect limits. At least two terms correct. Neglect limits. Dep on 1 st M1. Use of limits with attempt at subtraction seen. cao Dep on 1 st M1. An attempt to find C using $s(1) = 0$ and then evaluating $s(3)$. cao	4
(iii)	$v > 0$ so the particle always travels in the same (+ve) direction As the particle never changes direction, the final distance from the starting point is the displacement.	E1 E1	Only award if explicit Complete argument	
				2
				8
6 (i)	Component of weight down the plane is $1.5 \times 9.8 \times \frac{2}{7} = 4.2$ N	M1 E1	Use of mgk where k involves an attempt at resolution Accept $1.5 \times 9.8 \times \frac{2}{7} = 4.2$ or $14.7 \times \frac{2}{7} = 4.2$ seen	2
(ii)	Down the plane. Take F down the plane. $4.2 - 6.4 + F = 0$ so $F = 2.2$. Friction is 2.2 N down the plane	M1 A1	Allow sign errors. All forces present. No extra forces. Must have direction. [Award 1 for 2.2 N seen and 2 for 2.2 N down plane seen]	2
(iii)	F up the plane N2L down the plane $4.2 - F = 1.5 \times 1.2$ so $F = 4.2 - 1.8 = 2.4$ Friction is 2.4 N up the plane	M1 A1 A1 A1	N2L. $F = ma$. No extra forces. Allow weight term missing or wrong Allow only sign errors ± 2.4 cao. Accept no reference to direction if $F = 2.4$.	4
(iv)	$2^2 = 0.8^2 + 2 \times 1.2 \times s$ $s = 1.4$ so 1.4 m	M1 A1 A1	Use of $v^2 = u^2 + 2as$ or sequence All correct in 1 or 2-step method	3

(iii)	$4.9t^2 = \frac{25}{16}$ (1.5625) $t^2 = 0.31887\dots$ so $t = \pm 0.56469\dots$ Hence 0.565 s (3 s. f.)	M1 A1 E1	Use of $s = ut + 0.5at^2$ with $u = 0$. Condone use of ± 10 , ± 9.8 , ± 9.81 . If sequence of <i>suvat</i> used, complete method required. In any method only error accepted is sign error AG. Condone no reference to -ve value. www. 0.565 must be justified as answer to 3 s. f.	3
(iv)	$\dot{x} = \frac{12.5}{0.56469\dots} = 22.1359\dots$ so 22.1 m s ⁻¹ (3 s. f.) Either Time is $\frac{20}{12.5} \times 0.56469\dots$ s so 0.904 s (3 s. f.) or Time is $\frac{20}{22.1359\dots}$ s = 0.903507... so 0.904 s (3 s. f.) or (iii) + $\frac{7.5}{\text{their } \dot{x}}$ so 0.904 s (3 s. f.)	M1 B1 E1 M1 A1 M1 A1 M1 A1	or 25 / (2×0.56469..) Use of 12.5 or equivalent 22.1 must be justified as answer to 3 s. f. Don't penalise if penalty already given in (iii). cao Accept 0.91 (2 s. f.) cao Accept 0.91 (2 s. f.) cao Accept 0.91 (2 s. f.)	5
(v)	$v = \sqrt{\dot{x}^2 + \dot{y}^2}$ $\dot{y}^2 = 0^2 + 2 \times 9.8 \times \frac{25}{16}$ or $\dot{y} = 0 + 9.8 \times 0.5646\dots$ = $\frac{245}{8}$ (30.625) or $\dot{y} = \pm 5.539\dots$ so $v = \sqrt{490 + 30.625} = 22.8172\dots$ m s ⁻¹ so 22.8 m s ⁻¹ (3 s. f.)	M1 M1 A1 A1	Must have attempts at both components Or equiv. $u = 0$. Condone use of ± 10 , ± 9.8 , ± 9.81 . Accept wrong s (or t in alternative method) Or equivalent. May be implied. Could come from (iii) if $v^2 = u^2 + 2as$ used there. Award marks again. cao. www	4
				18

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General Comments

There were many candidates who submitted good, well-presented scripts which showed their sound grasp of the principles and their ability to apply them accurately. Most of the candidates were able to make progress with all of the questions and many produced good answers to all of them; the mistakes mentioned in the detailed analysis below of the candidates' responses should be viewed in this context.

Two matters that were not done well by a surprisingly large number of the candidates were the interpretation and construction of kinematics diagrams in Q1 and the production of the force diagram in Q6 (v); there are more details about these matters below.

It was very pleasing to read the many excellent explanations given in answer to Q4 (iii) and to Q5 (iii) from candidates who clearly exactly understood the points they were trying to make.

Comments on Individual Questions

Section A (36 marks)

1 Construction and use of kinematics graphs

Many candidates failed to score full marks. The meaning of the language of kinematics is not completely understood by many candidates and quite a few do not understand at all well how to interpret a kinematics graph.

- (i) This was done well by most but by no means all of the candidates. Some candidates did not realise they only had to find gradients; few of those who tried to use the *suvat* results instead made much progress. A common error was to give the speed instead of the velocity for $2 < t < 3.5$.
- (ii) Most candidates did this part quite well but some did not use horizontal lines or plotted speeds instead of velocities for $2 < t < 3.5$, even when their answer in part (i) was correct.
- (iii) Most candidates understood what was required but a few answered in terms of moving towards or away from O. Perhaps because of misreading the graph, some said that the ring was not moving when $t = 3.25$.

2 Kinematics and the use of Newton's second law in vector form

Quite a lot of the candidates did this question very well, working accurately throughout in vector forms. Quite a common error was to substitute the initial position where the initial velocity was required throughout the question or just in part (ii).

- (i) Most candidates did this accurately but a few gave the magnitude as their final answer.

(ii) This part was not done so well. Most candidates worked throughout in vector form and gave a vector answer but many failed to take account of the initial position.

(iii) Most candidates did this part well.

3 The magnitude and direction of a vector; solving a vector equation.

(i) This was done well by most of the candidates with the common error being to find the wrong angle.

(ii) It was pleasing to see so many candidates working efficiently at this problem. Although many candidates knew exactly what to do, others failed to get $\mathbf{H} = 4\mathbf{F} + \mathbf{G}$ or, having found it, didn't know how to extract scalar equations from it. A surprisingly large number found a and b but did not then state \mathbf{G} and \mathbf{H} (or got them wrong).

4 A box in equilibrium with two strings attached

This static equilibrium problem received sensible analysis from many of the candidates. It seemed that fewer candidates than in recent series confused sine and cosine. As always, finding a normal reaction presented too much complexity to candidates who lacked a systematic approach.

(i) This part was done well by most of the candidates.

(ii) Although quite a few candidates unnecessarily 'started again', there were many correct answers. The most common error was to resolve the tension in the string at C (or the value found in part (i)) in the direction of AB instead of considering the horizontal components of the two strings.

(iii) Although there were many correct answers, many more were spoiled by one or more of the following: confusing sine and cosine; omission of a tension; omission of resolution of one or more of the tensions; sign errors.

(iv) Very many good answers were seen from candidates who had clearly understood that the size of the force in AB depended only on the horizontal component of the force acting at C and that this latter value had not changed. Common errors were simply to claim that it was because the system is still in equilibrium or instead (or as well) to list (some of) the modelling terms 'light' and 'smooth' used in the question.

5 The kinematics of a particle with non-constant acceleration

It was a pleasure to see so many completely correct answers to this question, including the explanation required in part (iii).

(i) Almost all of the candidates answered this part correctly, even those who used constant acceleration results in part (ii).

- (ii) Some candidates tried to use the constant acceleration results but a large majority used the correct method of integration. A surprisingly common error was for the '14' to become '4' in a miscopy at some stage of the working. A quite common mistake was to find the values of the integral for $t = 3$ and $t = 1$ but not subtract them.
- (iii) A very pleasing number of candidates clearly understood that the key point is that $v > 0$ at all times means that the particle does not change direction. A few candidates seemed to have the wrong idea but rather more did not express themselves clearly enough to receive full marks – often their difficulty seemed to come from not clearly distinguishing between distance and distance travelled.

Section B (36 marks)

6 Situations on an inclined plane involving friction. After considering static equilibrium this is followed by application of Newton's second law to one body and then two connected bodies.

There were many good answers to all parts of this question except (v). It was expected that the complete analysis of this situation would be challenging to all but the strongest candidates but it was surprising how few candidates could produce diagrams showing the forces acting on each barge. Many candidates did not apply the rule that forces should have same label if and only if they are known to have the same magnitude. Many candidates missed out one force due to the coupling or did not get the weights or the normal reactions in the correct directions.

- (i) Most candidates did this part correctly but some failed fully to show the given result.
- (ii) Most candidates answered this well but some did not state the direction of the frictional force.
- (iii) Many candidates knew what to do but quite a few of these made a mistake with signs. Common errors were to continue to involve the 6.4 N force or to start again and this time confuse sine and cosine.
- (iv) Most of the candidates obtained full marks for this part. Most calculated the displacement directly but quite a few unnecessarily used a two-step method and found the time first.
- (v) Most of the diagrams given were poor for the reasons given above and few candidates obtained both marks. Whether the equations of motion of each barge or the equation of motion of the pair of barges was attempted, the most common error was to omit either the weight components or, less commonly, the friction terms. A surprisingly large number of those candidates who considered the barges separately omitted the tension terms. Many candidates produced complete and accurate numerical solutions without full (or any) marks for the diagram.

7 Various questions about the motion of a projectile with the trajectory equation as the initial given information

No question about a projectile set in an earlier series has started with the equation of its trajectory and so this question was structured to lead the candidates through a strategy to establish the required results. After parts (i) and (ii) (which were managed by most candidates), the rest of the question, as usual, required increasing levels of understanding of projectile motion. Most candidates worked parts (iii) and (iv) in the order given and many of these made a lot of progress and scored full or nearly full marks; many of the candidates who tried a different order failed to make much progress. A few candidates adopted approaches to the solutions that involved their knowledge of a general trajectory equation and some of these were successful. A mistake seen more than once was differentiating to find $\frac{dy}{dx}$ and then treating it as if it were $\frac{dy}{dt}$.

- (i) Almost all of the candidates scored this mark.
- (ii) The most common method for finding the horizontal position of the highest point was to argue from symmetry but many candidates used calculus. Most candidates who used either argument clearly obtained the correct value. Almost all candidates found the required height by substituting the x value they had just found in the trajectory equation.
- (iii) Many candidates knew how to do this but quite a few made sign errors in their substitution of $s = ut + \frac{1}{2}at^2$. Candidates were expected properly to show that their answer agreed to 0.565 to 3 significant figures and many did not do so.
- (iv) Most candidates who attempted this could see how to obtain the given horizontal component of velocity. As in part (iii), candidates were expected properly to show that their answer agreed to the given one to 3 significant figures and many did not do so – they were not penalised for this omission in both parts (iii) and (iv).

Although there are many ways to find the time of flight from Q to R, a few candidates who attempted this part were unable to find any of them. This request also seems to have been overlooked by quite a few candidates.

- (v) A pleasingly large proportion of the candidates knew what to do and did it accurately. By far the most common error was to give the vertical component of the velocity at R instead of the speed.