

ADVANCED GCE
MATHEMATICS (MEI)
Mechanics 2

4762

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Friday 9 January 2009
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

- 1 (i) What constant force is required to accelerate a particle of mass m kg from rest to $2u$ m s⁻¹ in 5 seconds? [3]

Two discs P and Q are moving in the same straight line over a smooth, horizontal surface. Fig. 1 shows the masses (in kg) and the velocities (in m s⁻¹) of the discs before and after they collide directly. The collision is perfectly elastic.

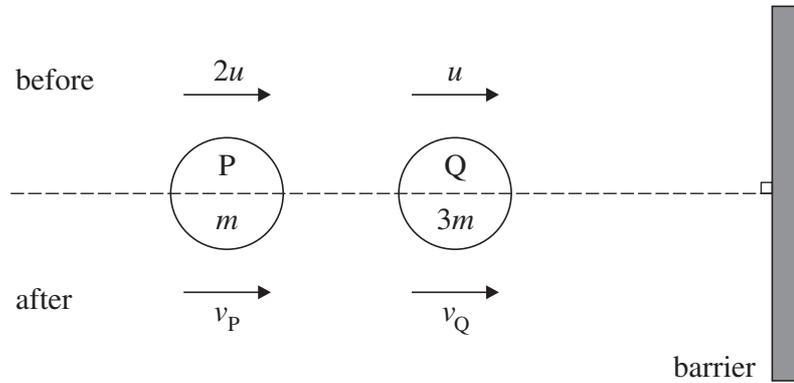


Fig. 1

- (ii) Show that $v_Q = \frac{3}{2}u$ and find the value of v_P . [6]

After the collision, Q hits the barrier shown in Fig. 1 which is perpendicular to its path and bounces back directly. The coefficient of restitution in this collision is e . Q collides again with P and on this occasion they stick together (coalesce) to form a single object, R, that has a speed of $\frac{1}{4}u$ m s⁻¹ away from the barrier.

- (iii) Write down an expression in terms of e and u for the velocity of Q after collision with the barrier. Find, in either order, the value of e and the impulse on the barrier. [9]

- 2 One way to load a box into a van is to push the box so that it slides up a ramp. Some removal men are experimenting with the use of different ramps to load a box of mass 80 kg.

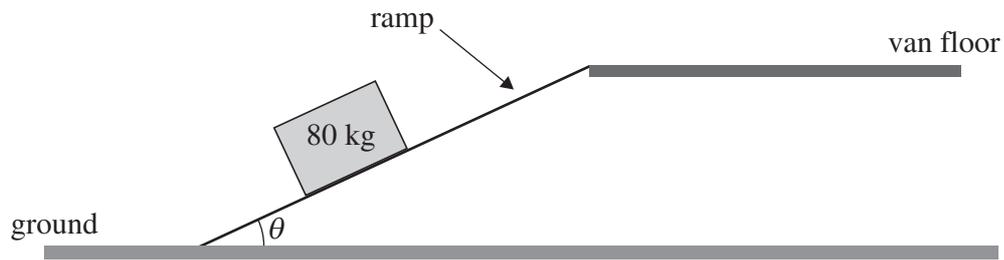


Fig. 2

Fig. 2 shows the general situation. The ramps are all uniformly rough with coefficient of friction 0.4 between the ramp and the box. The men push parallel to the ramp. As the box moves from one end of the ramp to the other it travels a vertical distance of 1.25 m.

- (i) Find the limiting frictional force between the ramp and the box in terms of θ . [3]
- (ii) From rest at the bottom, the box is pushed up the ramp and left at rest at the top. Show that the work done against friction is $\frac{392}{\tan \theta}$ J. [3]
- (iii) Calculate the gain in the gravitational potential energy of the box when it is raised from the ground to the floor of the van. [2]

For the rest of the question take $\theta = 35^\circ$.

- (iv) Calculate the power required to slide the box up the ramp at a steady speed of 1.5 m s^{-1} . [4]
- (v) The box is given an initial speed of 0.5 m s^{-1} at the top of the ramp and then slides down without anyone pushing it. Determine whether it reaches a speed of 3 m s^{-1} while it is on the ramp. [5]

- 3 A fish slice consists of a blade and a handle as shown in Fig. 3.1. The rectangular blade ABCD is of mass 250 g and modelled as a lamina; this is 24 cm by 8 cm and is shown in the Oxy plane. The handle EF is of mass 125 g and is modelled as a thin rod; this is 30 cm long and E is attached to the mid-point of CD. EF is at right angles to CD and inclined at α to the plane containing ABCD, where $\sin \alpha = 0.6$ (and $\cos \alpha = 0.8$). Coordinates refer to the axes shown in Fig. 3.1. Lengths are in centimetres. The y- and z-coordinates of the centre of mass of the fish slice are \bar{y} and \bar{z} respectively.

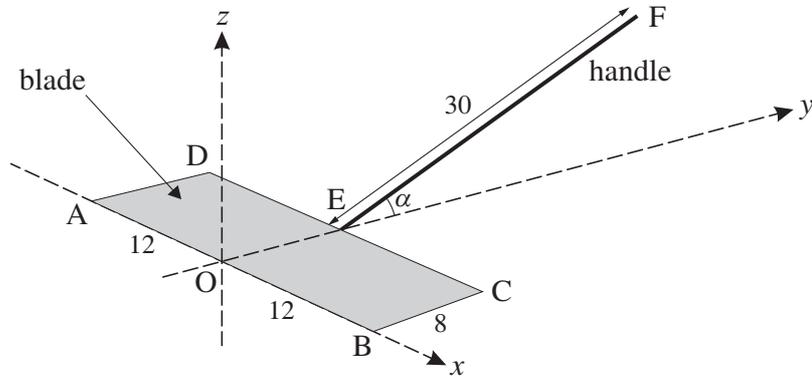


Fig. 3.1

- (i) Show that $\bar{y} = 9\frac{1}{3}$ and $\bar{z} = 3$. [8]
- (ii) Suppose that the plane Oxy in Fig. 3.1 is horizontal and represents a table top and that the fish slice is placed on it as shown. Determine whether the fish slice topples. [2]

The ‘superior’ version of the fish slice has an extra mass of 125 g uniformly distributed over the existing handle for 10 cm from F towards E, as shown in Fig. 3.2. This section of the handle may still be modelled as a thin rod.

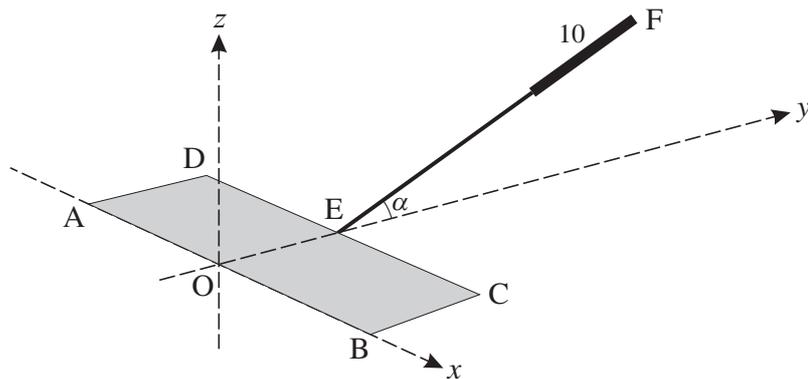


Fig. 3.2

- (iii) In this new situation show that $\bar{y} = 14$ and $\bar{z} = 6$. [4]

A sales feature of the ‘superior’ version is the ability to suspend it using a very small hole in the blade. This situation is modelled as the fish slice hanging in equilibrium when suspended freely about an axis through O.

- (iv) Indicate the position of the centre of mass on a diagram and calculate the angle of the line OE with the vertical. [4]

- 4 (a) A uniform, rigid beam, AB, has a weight of 600 N. It is horizontal and in equilibrium resting on two small smooth pegs at P and Q. Fig. 4.1 shows the positions of the pegs; lengths are in metres.

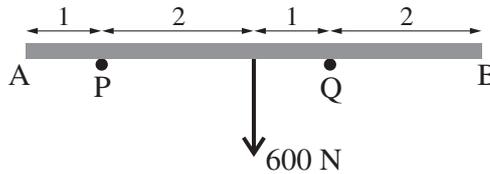


Fig. 4.1

- (i) Calculate the forces exerted by the pegs on the beam. [4]

A force of L N is applied vertically downwards at B. The beam is in equilibrium but is now on the point of tipping.

- (ii) Calculate the value of L . [3]

- (b) Fig. 4.2 shows a framework in a vertical plane constructed of light, rigid rods AB, BC and CA. The rods are freely pin-jointed to each other at A, B and C and to a fixed point at A. The pin-joint at C rests on a smooth, horizontal support. The dimensions of the framework are shown in metres. There is a force of 340 N acting at B in the plane of the framework. This force and the rod BC are both inclined to the vertical at an angle α , which is defined in triangle BCX. The force on the framework exerted by the support at C is R N.

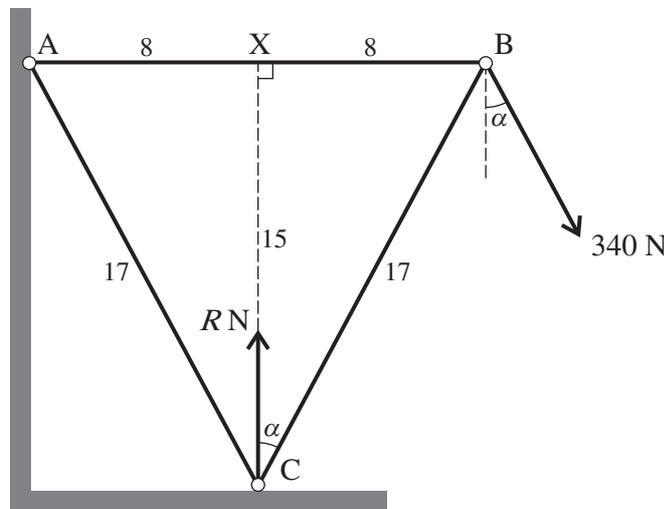


Fig. 4.2

- (i) Show that $R = 600$. [4]
- (ii) Draw a diagram showing all the forces acting on the framework and also the internal forces in the rods. [2]
- (iii) Calculate the internal forces in the three rods, indicating whether each rod is in tension or in compression (thrust). [Your working in this part should correspond to your diagram in part (ii).] [6]

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Q 1	Mark	Sub
(i) either $m \times 2u = 5F$ so $F = 0.4mu$ in direction of the velocity or $a = \frac{2u}{5}$ so $F = 0.4mu$ in direction of the velocity	M1 Use of $I = Ft$ A1 A1 Must have reference to direction. Accept diagram. M1 Use of <i>suvat</i> and N2L A1 May be implied A1 Must have reference to direction. Accept diagram.	3
(ii) PCLM $\rightarrow 2um + 3um = mv_p + 3mv_Q$ NEL $\rightarrow v_Q - v_p = 2u - u = u$ Energy $\frac{1}{2}m \times (2u)^2 + \frac{1}{2}(3m) \times u^2$ $= \frac{1}{2}m \times v_p^2 + \frac{1}{2}(3m) \times v_Q^2$ Solving to get both velocities $v_Q = \frac{3u}{2}$ $v_p = \frac{u}{2}$	M1 For 2 eqns considering PCLM, NEL or Energy A1 One correct equation A1 Second correct equation M1 Dep on 1 st M1. Solving pair of equations. E1 If Energy equation used, allow 2 nd root discarded without comment. A1 [If AG subst in one equation to find other velocity, and no more, max SC3]	6
(iii) either After collision with barrier $v_Q = \frac{3eu}{2} \leftarrow$ so $\rightarrow m \frac{u}{2} - 3m \frac{3eu}{2} = -4m \frac{u}{4}$ so $e = \frac{1}{3}$ At the barrier the impulse on Q is given by $\rightarrow 3m \left(-\frac{3u}{2} \times \frac{1}{3} - \frac{3u}{2} \right)$ so impulse on Q is $-6mu \rightarrow$ so impulse on the barrier is $6mu \rightarrow$	B1 Accept no direction indicated M1 PCLM A1 LHS Allow sign errors. Allow use of $3mv_Q$. A1 RHS Allow sign errors A1 M1 Impulse is $m(v - u)$ F1 $\pm \frac{3u}{2} \times \frac{1}{3}$ F1 Allow \pm and direction not clear. FT only e . A1 cao. Direction must be clear. Units not required.	9
	18	

Q 1	continued	mark		sub
(iii)	<p>or</p> <p>After collision with barrier $v_Q = \frac{3eu}{2} \leftarrow$</p> <p>Impulse – momentum overall for Q</p> $\rightarrow 2mu + 3mu + I = -4m \times \frac{u}{4}$ $I = -6mu$ <p>so impulse of $6mu$ on the barrier \rightarrow</p> <p>Consider impact of Q with the barrier to give speed v_Q after impact</p> $\rightarrow \frac{3u}{2} \times 3m - 6mu = 3mv_Q$ <p>so $v_Q = -\frac{u}{2}$</p> $e = \frac{u}{2} \div \frac{3u}{2} = \frac{1}{3}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>F1</p> <p>F1</p> <p>A1</p>	<p>All terms present</p> <p>All correct except for sign errors</p> <p>Direction must be clear. Units not required.</p> <p>Attempt to use I - M</p> <p>cao</p>	<p>9</p>

Q 2	Mark	Sub
(i) $R = 80g \cos \theta$ or $784 \cos \theta$ $F_{\max} = \mu R$ so $32g \cos \theta$ or $313.6 \cos \theta$ N	B1 M1 A1	Seen 3
(ii) Distance is $\frac{1.25}{\sin \theta}$ WD is $F_{\max} d$ so $32g \cos \theta \times \frac{1.25}{\sin \theta}$ $= \frac{392}{\tan \theta}$	B1 M1 E1	Award for this or equivalent seen 3
(iii) $\Delta \text{GPE is } mgh$ so $80 \times 9.8 \times 1.25 = 980$ J	M1 A1	Accept 100g J 2
(iv) either $P = Fv$ so $(80g \sin 35 + 32g \cos 35) \times 1.5$ $= 1059.85 \dots$ so 1060 W (3 s. f.) or $P = \frac{WD}{\Delta t}$ so $\frac{980 + \frac{392}{\tan 35}}{\left(\frac{1.25}{\sin 35}\right) \div 1.5}$ $= 1059.85 \dots$ so 1060 W (3 s. f.)	M1 B1 A1 A1 M1 B1 B1 A1	Weight term All correct cao Numerator FT their GPE Denominator cao 4
(v) either Using the W-E equation $0.5 \times 80 \times v^2 - 0.5 \times 80 \times \left(\frac{1}{2}\right)^2 = 980 - \frac{392}{\tan 35}$ $v = 3.2793 \dots$ so yes or N2L down slope $a = 2.409973 \dots$ distance slid, using <i>uvast</i> is 1.815372... vertical distance is $1.815372 \dots \times \sin 35$ $= 1.0412 \dots < 1.25$ so yes	M1 B1 B1 A1 A1 M1 A1 A1 M1 A1	Attempt speed at ground or dist to reach required speed. Allow only init KE omitted KE terms. Allow sign errors. FT from (iv). Both WD against friction and GPE terms. Allow sign errors. FT from parts above. All correct CWO All forces present valid comparison CWO 5
	17	

Q 3		Mark		Sub
(i)	$\bar{y}: 250 \times 4 + 125 \left(8 + \frac{30}{2} \cos \alpha \right) = 375\bar{y}$ $\bar{y} = \frac{28}{3} = 9\frac{1}{3}$ $\bar{z}: (250 \times 0 +) 125 \times \frac{30}{2} \sin \alpha = 375\bar{z}$ $\bar{z} = 3$	M1 B1 M1 B1 B1 E1 B1 E1	Correct method for \bar{y} or \bar{z} Total mass correct 15 cos α or 15 sin α attempted either part $\left(8 + \frac{30}{2} \cos \alpha \right)$ 250 \times 4 Accept any form LHS	8
(ii)	Yes. Take moments about CD. c.w moment from weight; no a.c moment from table	E1 E1	[Award E1 for $9\frac{1}{3} > 8$ seen or 'the line of action of the weight is outside the base']	2
(iii)	c.m. new part is at (0, 8 + 20, 15) $375 \times \frac{28}{3} + 125 \times 28 = 500\bar{y}$ so $\bar{y} = 14$ $375 \times 3 + 125 \times 15 = 500\bar{z}$ so $\bar{z} = 6$	M1 M1 E1 E1	Either y or z coordinate correct Attempt to 'add' to (i) or start again. Allow mass error.	4
(iv)	Diagram Angle is $\arctan \frac{6}{14}$ = 23.1985... so 23.2° (3 s. f.)	B1 B1 M1 A1	Roughly correct diagram Angle identified (may be implied) Use of tan. Allow use of 14/6 or equivalent. cao	4
		18		

Q 4		mark		sub
(a)	Let the \uparrow forces at P and Q be R_p and R_Q			
(i)	c.w. moments about P $2 \times 600 - 3R_Q = 0$ so force of 400 N \uparrow at Q a.c. moments about Q or resolve $R_p = 200$ so force of 200 N \uparrow at P	M1 A1 M1 A1	Moments taken about a named point.	4
(ii)	$R_p = 0$ c.w. moments about Q $2L - 1 \times 600 = 0$ so $L = 300$	B1 M1 A1	Clearly recognised or used. Moments attempted with all forces. Dep on $R_p = 0$ or R_p not evaluated.	3
(b)				
(i)	$\cos \alpha = \frac{15}{17}$ or $\sin \alpha = \frac{8}{17}$ or $\tan \alpha = \frac{8}{15}$ c.w moments about A $16 \times 340 \cos \alpha - 8R = 0$ so $R = 600$	B1 M1 A1 E1	Seen here or below or implied by use. Moments. All forces must be present and appropriate resolution attempted. Evidence of evaluation.	4
(ii)	Diagram (Solution below assumes all internal forces set as tensions)	B1 B1	Must have 600 (or R) and 340 N and reactions at A. All internal forces clearly marked as tension or thrust. Allow mixture. [Max of B1 if extra forces present]	2
(iii)	$B \downarrow 340 \cos \alpha + T_{BC} \cos \alpha = 0$ so $T_{BC} = -340$ (Thrust of) 340 N in BC $C \rightarrow T_{BC} \sin \alpha - T_{AC} \sin \alpha = 0$ so $T_{AC} = -340$ (Thrust of) 340 N in AC $B \leftarrow T_{AB} + T_{BC} \sin \alpha - 340 \sin \alpha = 0$ so $T_{AB} = 320$ (Tension of) 320 N in AB Tension/ Thrust all consistent with working	M1 A1 F1 M1 A1 F1	Equilibrium at a pin-joint Method for T_{AB} [Award a max of 4/6 if working inconsistent with diagram]	6
		19		

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General Comments

Many excellent responses to this paper were seen and the majority of the candidates made progress worth marks on every question. There was some evidence that some candidates found the paper long but, on the whole, these were candidates who were using inefficient methods of solution, particularly in Q1 and in Q3. The standard of presentation in many cases was poor; diagrams were either absent or badly drawn and poor notation led to errors. The standard of algebraic manipulation of a significant minority of the candidates was a cause for concern. The majority of candidates appeared to understand the concepts required but did not always make it clear which principle or process was being used; as has happened in previous sessions those parts of the questions that were least well done were usually those requiring candidates to *show* a given answer. A minority of candidates wasted time by attempting to work back from the given answers rather than trying to find and employ the principles required to solve the problem.

Comments on Individual Questions

- 1 The majority of candidates understood the principles required and applied them well. However, as has happened in previous series with this type of question, those candidates who drew a diagram and indicated direction clearly were more successful than those who failed to do so.
- (i) This part was well done by almost all of the candidates. However, very, very few of the responses included the direction of the force.
 - (ii) Most candidates obtained several marks for this part. They appreciated the need for two equations, obtained them and solved them simultaneously. It was encouraging to see far fewer sign errors in the use of Newton's experimental law this series than have been seen in the past. Those candidates who chose to use an energy equation along with either conservation of linear momentum or Newton's experimental law tended to be less successful than those who used both conservation of momentum and Newton's experimental law. Difficulties were encountered by some candidates when trying to solve a pair of equations one of which was quadratic; algebraic errors were common.
 - (iii) It was pleasing to see that many candidates made real progress with this part of the question. The main errors were usually sign related and again those candidates who drew a diagram were more successful than those who did not. Only those whose answers were all of high standard gave the direction of the impulse on the *barrier*.

- 2 More excellent answers were seen to this question than for similar questions set in previous series. The majority of the candidates appeared to understand the underlying concepts and were able to apply them correctly.
- (i) Almost all of the candidates gained full marks for this part.
 - (ii) A minority of candidates had difficulty with this part. The main problem appeared to be with the trigonometry required to find the distance and an apparent lack of knowledge of the identity $\tan \theta = \frac{\sin \theta}{\cos \theta}$
 - (iii) Very few incorrect solutions were seen to this part.
 - (iv) Correct solutions to this part were common with the majority of candidates opting to use $P = Fv$ rather than $P = \frac{\text{Work Done}}{\text{Time}}$. Of those using $P = Fv$ the most common mistake was to omit the component of the weight.
 - (v) The vast majority of candidates managed to score marks here. The mistakes that did occur were usually sign related.
- 3 This question was not as well done as similar questions in previous series. Candidates did not seem to appreciate that their responses had to show ALL the relevant steps when trying to show a given answer.
- (i) The majority of candidates gained many of the marks for this part. However, the working offered by a minority was muddled and inconsistent and did not show what it purported to show.
 - (ii) Only the strongest candidates gained full marks for this. Most candidates appreciated that, because of the position of the centre of mass, the fish-slice would topple but failed to explain that an unbalanced moment would be produced when placed as shown.
 - (iii) Again this part involved showing a given answer. Many correct solutions were seen but a sizeable minority of the candidates failed to explain exactly what they were doing and how they arrived at the lengths and masses used in working out the new centre of mass.
 - (iv) Many of the diagrams offered in this part were poor. Some were too small to be of any real use and others did not show the information requested. Those candidates who drew a clear diagram were usually successful in obtaining the required angle and gained full marks.

- 4 Many candidates could gain several marks for their solutions to this question. Part a) was usually done better than part b).
- (a) (i) Few candidates had problems with this part.
- (ii) Again well done by the majority of candidates. A small number of them did not appreciate that the reaction at P would be zero and tried setting up simultaneous equations. Others assumed that the values of the reactions would be the same as in the previous part, even though the conditions had changed.
- (b) (i) Most candidates successfully took moments about A and obtained the given answer. However, some employed resolution and stated $R = 2 \times 340 \cos \alpha$ without giving any explanation as to why this was so. In order to obtain full marks for this method, candidates had to refer to the symmetry of the system and produce a clear argument.
- (ii) Diagrams rarely obtained full marks often because internal forces were inadequately labelled and/or the forces in the hinge at A were omitted.
- (iii) Most candidates obtained some marks for this part. Almost all appreciated the need to resolve at the pin joints but sign errors were common with inconsistencies between equations and/or the diagram. A small number of candidates unwisely chose to resolve at A but did not appreciate that they had to include the force at the hinge.