OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4763

Mechanics 3

Tuesday **10 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

1 **(a)** **(i)** Write down the dimensions of force. [1]

The period, t, of a vibrating wire depends on its tension, F, its length, l, and its mass per unit length, σ.

(ii) Assuming that the relationship is of the form $t = kF^{\alpha} l^{\beta} \sigma^{\gamma}$, where k is a dimensionless constant, use dimensional analysis to determine the values of α, β and γ. [6]

Two lengths are cut from a reel of uniform wire. The first has length 1.2 m, and it vibrates under a tension of 90 N. The second has length 2.0 m, and it vibrates with the same period as the first wire.

(iii) Find the tension in the second wire. (You may assume that changing the tension does not significantly change the mass per unit length.) [4]

(b) The midpoint M of a vibrating wire is moving in simple harmonic motion in a straight line, with amplitude 0.018 m and period 0.01 s.

(i) Find the maximum speed of M. [3]

(ii) Find the distance of M from the centre of the motion when its speed is 8 m s^{-1}. [4]

2 **(a)** A moon of mass 7.5×10^{22} kg moves round a planet in a circular path of radius 3.8×10^{8} m, completing one orbit in a time of 2.4×10^{6} s. Find the force acting on the moon. [4]

(b) Fig. 2 shows a fixed solid sphere with centre O and radius 4 m. Its surface is smooth. The point A on the surface of the sphere is 3.5 m vertically above the level of O. A particle P of mass 0.2 kg is placed on the surface at A and is released from rest. In the subsequent motion, when OP makes an angle θ with the horizontal and P is still on the surface of the sphere, the speed of P is v m s^{-1} and the normal reaction acting on P is R N.

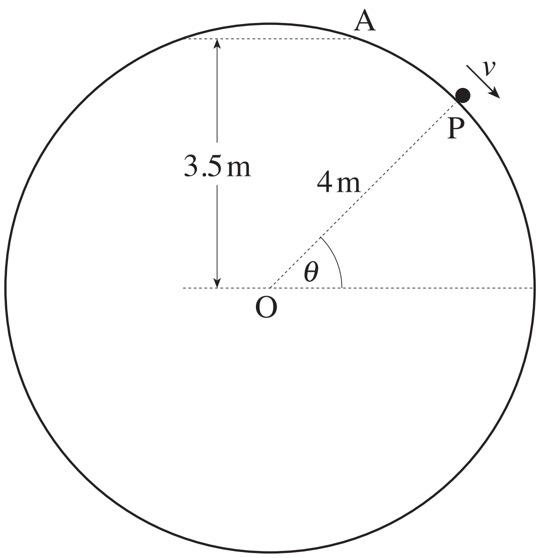

Fig. 2

(i) Express v^2 in terms of θ. [3]

(ii) Show that $R = 5.88 \sin\theta - 3.43$. [4]

(iii) Find the radial and tangential components of the acceleration of P when $\theta = 40°$. [4]

(iv) Find the value of θ at the instant when P leaves the surface of the sphere. [3]

[Turn over

3 A light elastic rope has natural length 15 m. One end of the rope is attached to a fixed point O and the other end is attached to a small rock of mass 12 kg.

When the rock is hanging in equilibrium vertically below O, the length of the rope is 15.8 m.

(i) Show that the modulus of elasticity of the rope is 2205 N. [2]

The rock is pulled down to the point 20 m vertically below O, and is released from rest in this position. It moves upwards, and comes to rest instantaneously, with the rope slack, at the point A.

(ii) Find the acceleration of the rock immediately after it is released. [3]

(iii) Use an energy method to find the distance OA. [5]

At time t seconds after release, the rope is still taut and the displacement of the rock *below the equilibrium position* is x metres.

(iv) Show that $\frac{d^2x}{dt^2} = -12.25x$. [4]

(v) Write down an expression for x in terms of t, and hence find the time between releasing the rock and the rope becoming slack. [4]

4 The region between the curve $y = 4 - x^2$ and the x-axis, from $x = 0$ to $x = 2$, is occupied by a uniform lamina. The units of the axes are metres.

(i) Show that the coordinates of the centre of mass of this lamina are $(0.75, 1.6)$. [9]

This lamina and another exactly like it are attached to a uniform rod PQ, of mass 12 kg and length 8 m, to form a rigid body as shown in Fig. 4. Each lamina has mass 6.5 kg. The ends of the rod are at P$(-4, 0)$ and Q$(4, 0)$. The rigid body lies entirely in the (x, y) plane.

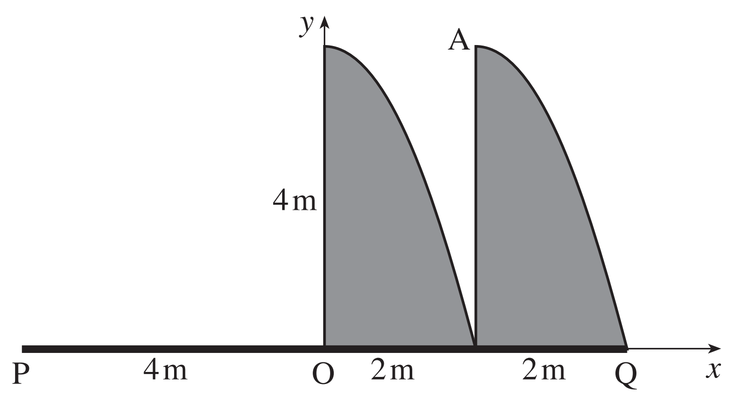

Fig. 4

(ii) Find the coordinates of the centre of mass of the rigid body. [5]

The rigid body is freely suspended from the point A$(2, 4)$ and hangs in equilibrium.

(iii) Find the angle that PQ makes with the horizontal. [4]

Mark Scheme 4763
January 2006

1(a)(i)	$\mathrm{M\,L\,T^{-2}}$	B1 **1**	Allow $\mathrm{kg\,m\,s^{-2}}$
(ii)	$(\mathrm{T}) = (\mathrm{M\,L\,T^{-2}})^{\alpha}\,(\mathrm{L})^{\beta}\,(\mathrm{M\,L^{-1}})^{\gamma}$ Powers of M: $\alpha + \gamma = 0$ of L: $\alpha + \beta - \gamma = 0$ of T: $-2\alpha = 1$ $\alpha = -\frac{1}{2},\quad \beta = 1,\quad \gamma = \frac{1}{2}$	B1 M1 M2 A2 **6**	For $\mathrm{M\,L^{-1}}$ For three equations Give M1 for one equation Give A1 for one correct
(iii)	$kF_1^{\alpha} l_1^{\beta} \sigma^{\gamma} = kF_2^{\alpha} l_2^{\beta} \sigma^{\gamma}$ $F_1^{-\frac{1}{2}}\, l_1 = F_2^{-\frac{1}{2}}\, l_2$	M1 A1	 Equation relating F_1, F_2, l_1, l_2
	OR $F^{\alpha} l^{\beta}$ is constant M1 F is proportional to l^2 A1		or equivalent
	$F_2 = 90 \times \dfrac{2.0^2}{1.2^2}$ $= 250$ (N)	M1 A1 **4**	
(b)(i)	$\dfrac{2\pi}{\omega} = 0.01$ $\omega = 200\pi$ Maximum speed is $A\omega = 0.018 \times 200\pi$ $= 11.3\ (\mathrm{m\,s^{-1}})$	B1 M1 A1 **3**	 Accept 3.6π
(ii)	Using $v^2 = \omega^2 (A^2 - x^2)$ $8^2 = (200\pi)^2 (0.018^2 - x^2)$ $x = 0.0127$ (m)	M1 M1 A1 A1 **4**	 Substituting values
	OR $v = 3.6\pi \cos(200\pi t) = 8$ M1 when $200\pi t = 0.785$ A1 ($t = 0.001249$) $x = 0.018 \sin(200\pi t) = 0.018 \sin(0.785)$ M1 $= 0.0127$ A1		*Condone the use of degrees in this part*

2 (a)	$\omega = \dfrac{2\pi}{2.4\times 10^6}$ $(= 2.618\times 10^{-6})$ Acceleration $a = r\omega^2$ (or $\dfrac{v^2}{r}$) $= 2.604\times 10^{-3}$ Force is $ma = 7.5\times 10^{22}\times 2.604\times 10^{-3}$ $= 1.95\times 10^{20}$ (N)	B1 M1 M1 A1 **4**	or $v = \dfrac{2\pi\times 3.8\times 10^8}{2.4\times 10^6}$ $(= 994.8)$ M0 for $F - mg = ma$ etc Accept 1.9×10^{20} or 2.0×10^{20}
(b)(i)	Change in PE is $mg(3.5 - 4\sin\theta)$ By conservation of energy $\frac{1}{2}mv^2 = mg(3.5 - 4\sin\theta)$ $v^2 = 68.6 - 78.4\sin\theta$	B1 M1 A1 **3**	or as separate terms Accept $7g - 8g\sin\theta$
(ii)	$0.2\times 9.8\sin\theta - R = 0.2\times\dfrac{v^2}{4}$ $1.96\sin\theta - R = 0.05(68.6 - 78.4\sin\theta)$ $R = 5.88\sin\theta - 3.43$	M1 M1 A1 E1 **4**	Radial equation of motion (3 terms) Substituting from part (i) Correctly obtained
(iii)	When $\theta = 40°$, $v^2 = 18.21$ Radial acceleration is $\dfrac{v^2}{4} = 4.55$ $(\mathrm{m\,s^{-2}})$ Tangential acceleration is $9.8\cos 40$ $= 7.51$ $(\mathrm{m\,s^{-2}})$	M1 A1 M1 A1 **4**	or $0.2g\sin 40 - R = ma$ Accept 4.5 or 4.6 M0 for $a = mg\cos 40$ etc
(iv)	Leaves surface when $R = 0$ $\sin\theta = \dfrac{3.43}{5.88}$ $\theta = 35.7°$	M1 M1 A1 cao **3**	Accept $36°$, 0.62 rad

3 (i)	$\frac{\lambda}{15}\times 0.8 = 12\times 9.8$ $\lambda = 2205$ (N)	M1 E1 **2**	
(ii)	$\frac{2205}{15}\times 5 - 12\times 9.8 = 12a$ $a = 51.45$ (m s^{-2})	M1 A1 A1 **3**	Equation of motion including tension Accept 51 or 52
(iii)	Loss of EE is $\frac{1}{2}\times\frac{2205}{15}\times 5^2$ $(=1837.5)$	M1 A1	Calculating elastic energy
	By conservation of energy $12\times 9.8\times h = 1837.5$ $h = 15.625$ OA $= 20 - h = 4.375$ (m)	M1 F1 A1 **5**	Equation involving EE and PE
	OR $12\times 9.8\times 5 + \frac{1}{2}\times 12\times v^2 = 1837.5$ M1 $v^2 = 208.25$ $0 = 208.25 - 2\times 9.8\times H$ F1 $H = 10.625$ OA $= 15 - H = 4.375$ (m) A1		Equation involving EE, PE and KE
(iv)	$T = \frac{2205}{15}(0.8 + x)$ $12\times 9.8 - \frac{2205}{15}(0.8 + x) = 12\frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} = -12.25x$	B1 M1 A1 E1 **4**	or $T = \frac{\lambda}{l}(x_0 + x)$ Equation of motion with three terms or $mg - \frac{\lambda}{l}(x_0 + x) = m\frac{d^2x}{dt^2}$ provided that $mg = \frac{\lambda}{l}x_0$ appears somewhere Correctly obtained *No marks for just writing* $-\frac{2205}{15}x = 12\frac{d^2x}{dt^2}$ *or just using the formula* $\omega^2 = \frac{\lambda}{ml}$ *If x is clearly measured upwards, treat as a mis-read*
(v)	$x = 4.2\cos(3.5t)$ Rope becomes slack when $x = -0.8$ $4.2\cos(3.5t) = -0.8$ $t = 0.504$ (s)	M1 A1 M1 A1 **4**	For $\cos(\sqrt{12.25}\,t)$ or $\sin(\sqrt{12.25}\,t)$ Accept 0.50 or 0.51

4 (i)	$\int y\,dx = \int_0^2 (4 - x^2)\,dx \quad = \left[4x - \frac{1}{3}x^3\right]_0^2 \quad (= \frac{16}{3})$	B1		
	$\int xy\,dx = \int_0^2 x(4 - x^2)\,dx$ $= \left[2x^2 - \frac{1}{4}x^4\right]_0^2 \quad (= 4)$	M1 A1		
	$\bar{x} = \frac{4}{\frac{16}{3}}$ $= 0.75$	M1 E1		Correctly obtained
	$\int \frac{1}{2}y^2\,dx = \int_0^2 \frac{1}{2}(16 - 8x^2 + x^4)\,dx$ $= \left[8x - \frac{4}{3}x^3 + \frac{1}{10}x^5\right]_0^2 \quad (= \frac{128}{15})$	M1 A1		
	OR $\int yx\,dy = \int_0^4 y\sqrt{4 - y}\,dy$ M1 $= \left[-\frac{2}{3}y(4-y)^{\frac{3}{2}} - \frac{4}{15}(4-y)^{\frac{5}{2}}\right]_0^4$ A1			Valid method of integration or $\left[-\frac{8}{3}(4-y)^{\frac{3}{2}} + \frac{2}{5}(4-y)^{\frac{5}{2}}\right]_0^4$
	$\bar{y} = \frac{\frac{128}{15}}{\frac{16}{3}}$ $= 1.6$	M1 E1	**9**	Correctly obtained *SR* If $\frac{1}{2}$ is omitted, marks for $\bar{y}$ are M1A0M0E0
(ii)	$\bar{x} = \frac{12 \times 0 + 6.5 \times 0.75 + 6.5 \times 2.75}{12 + 6.5 + 6.5}$ $= \frac{22.75}{25} = 0.91$	M1 M1 A1		For $6.5 \times 0.75 + 6.5 \times 2.75$ Using $\left(\sum m\right)\bar{x} = \sum mx$
	$\bar{y} = \frac{12 \times 0 + 6.5 \times 1.6 + 6.5 \times 1.6}{25}$ $= \frac{20.8}{25} = 0.832$	M1 A1	**5**	Using $\left(\sum m\right)\bar{y} = \sum my$
(iii)	$\tan\theta = \frac{2 - 0.91}{4 - 0.832} \quad (= \frac{1.09}{3.168})$	M1 M1 A1		For CM vertically below A For trig in a triangle containing θ, or finding the gradient of AG Correct expression for $\tan\theta$ or $\tan(90 - \theta)$
	$\theta = 19.0°$	A1	**4**	Accept 0.33 rad

4763: Mechanics 3

General Comments

The scripts were generally of a high standard, with half the candidates scoring 60 marks or more out of 72. The questions were usually answered confidently and accurately, and the only topics which caused significant difficulty were circular motion and simple harmonic motion.

Comments on Individual Questions

1) This question, on dimensions and simple harmonic motion, was very well answered, and the average mark was about 16 out of 18.
In part (a), almost all candidates knew the dimensions of force, and understood how to find the indices α, β and γ, although some made errors when solving the equations. The tension in the second wire was very often found correctly, although some candidates assumed that the mass, rather than the mass per unit length, remained constant.
The simple harmonic motion problem in part (b) was usually answered correctly, with most candidates using $v^2 = \omega^2(A^2 - x^2)$. Those working from $x = A\sin\omega t$ were more inclined to make arithmetic errors.

2) This question, on motion in a circle, had an average mark of about 14 out of 18.
In part (a), about two-thirds of the candidates found the force correctly. The principles required were very well known, but many candidates made errors in the calculations, especially when finding the angular velocity.
In part (b)(i), some candidates did not realise that conservation of energy was needed, but generally this was well answered apart from frequent errors in the potential energy term.
In part (b)(ii), most candidates correctly considered forces in the radial direction, the most common error being omission of the weight.
In part (b)(iii), very many candidates did not know what was required, and correct answers for the tangential acceleration were not at all common.
In the final part (b)(iv), the condition for the particle to leave the surface was well understood.

3) This question, on elasticity and simple harmonic motion, was the worst answered question, with an average mark of about 12 out of 18.
In part (i), the modulus of elasticity was correctly obtained by almost every candidate.
In part (ii), about one third of the candidates omitted the weight when calculating the acceleration.
In part (iii), some candidates made errors when calculating the elastic energy, but the majority set up the energy equation correctly. Having found the vertical distance moved by the rock, a significant number omitted the final calculation of the distance OA.
In part (iv), the correct expression $147(0.8 + x)$ for the tension usually appeared, and about half the candidates derived the differential equation correctly. There was some confusion over signs, but the main reason for failure was not writing down an equation of motion with three terms.
Part (v) was rarely answered correctly. Although the form $x = A\cos\omega t$ was well known, many had $A = 5$ instead of $A = 4.2$ or $\omega = 12.25$ instead of $\omega = 3.5$. Only the best candidates realised that the rope became slack when $x = -0.8$.

4) This question, on centres of mass, was very well answered, with an average mark of about 15 out of 18.

In part (i), the methods for finding the centre of mass of a lamina were well understood, and the integrals were evaluated accurately. Most candidates scored full marks.

In part (ii), the principles were very well understood, although the centre of mass of the second lamina was often taken to be $(1.5,\ 1.6)$ instead of $(2.75,\ 1.6)$.

In part (iii), almost all candidates realised that the centre of mass was vertically below A, but most were unable to obtain the required angle accurately. Some gave the angle between PQ and the vertical instead of the horizontal.