OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4763**

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 19 June 2009
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 A fixed solid sphere has centre O and radius 2.6 m. A particle P of mass 0.65 kg moves on the smooth surface of the sphere.

The particle P is set in motion with horizontal velocity $1.4\,\text{m s}^{-1}$ at the highest point of the sphere, and moves in part of a vertical circle. When OP makes an angle θ with the upward vertical, and P is still in contact with the sphere, the speed of P is $v\,\text{m s}^{-1}$.

(i) Show that $v^2 = 52.92 - 50.96\cos\theta$. **[3]**

(ii) Find, in terms of θ, the normal reaction acting on P. **[4]**

(iii) Find the speed of P at the instant when it leaves the surface of the sphere. **[4]**

The particle P is now attached to one end of a light inextensible string, and the other end of the string is fixed to a point A, vertically above O, such that AP is tangential to the sphere, as shown in Fig. 1. P moves with constant speed $1.2\,\text{m s}^{-1}$ in a **horizontal** circle with radius 2.4 m on the surface of the sphere.

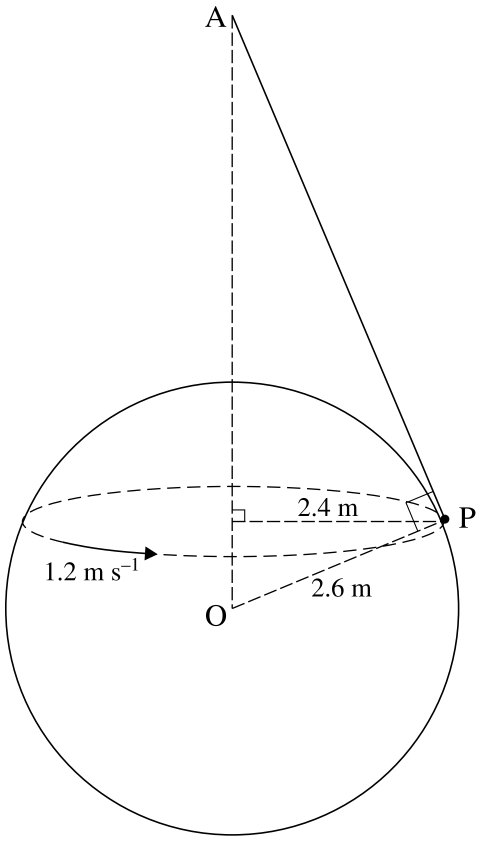

Fig. 1

(iv) Find the tension in the string and the normal reaction acting on P. **[8]**

2 In trials for a vehicle emergency stopping system, a small car of mass 400 kg is propelled towards a buffer. The buffer is modelled as a light spring of stiffness $5000\,\mathrm{N\,m^{-1}}$. One end of the spring is fixed, and the other end points directly towards the oncoming car. Throughout this question, there is no driving force acting on the car, and there are no resistances to motion apart from those specifically mentioned.

At first, the buffer is mounted on a horizontal surface, and the car has speed $3\,\mathrm{m\,s^{-1}}$ when it hits the buffer, as shown in Fig. 2.1.

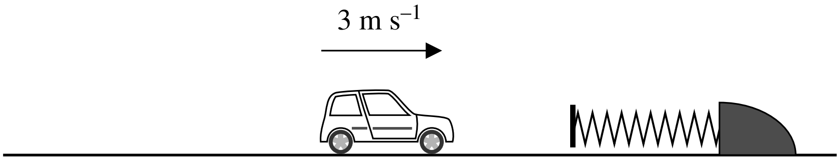

Fig. 2.1

(i) Find the compression of the spring when the car comes (instantaneously) to rest. **[3]**

The buffer is now mounted on a slope making an angle θ with the horizontal, where $\sin\theta = \frac{1}{7}$. The car is released from rest and travels 7.35 m down the slope before hitting the buffer, as shown in Fig. 2.2.

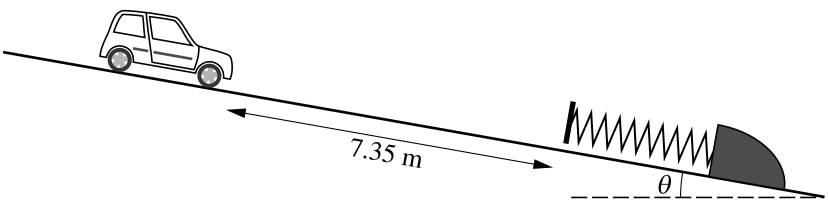

Fig. 2.2

(ii) Verify that the car comes instantaneously to rest when the spring is compressed by 1.4 m. **[4]**

The surface of the slope (including the section under the buffer) is now covered with gravel which exerts a constant resistive force of 7560 N on the car. The car is moving down the slope, and has speed $30\,\mathrm{m\,s^{-1}}$ when it is 24 m from the buffer, as shown in Fig. 2.3. It comes to rest when the spring has been compressed by x metres.

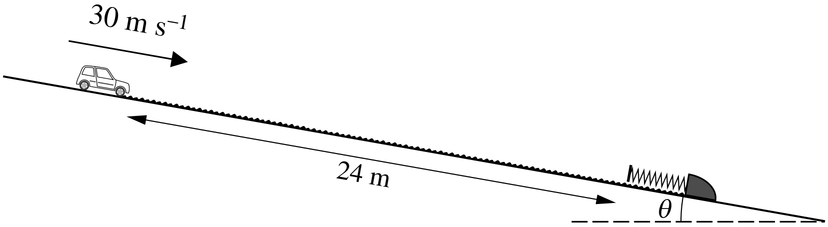

Fig. 2.3

(iii) By considering work and energy, find the value of x. **[10]**

3 **(a)** **(i)** Write down the dimensions of velocity, force and density (which is mass per unit volume). **[3]**

A vehicle moving with velocity v experiences a force F, due to air resistance, given by

$$F = \frac{1}{2}C\rho^{\alpha}v^{\beta}A^{\gamma}$$

where ρ is the density of the air, A is the cross-sectional area of the vehicle, and C is a dimensionless quantity called the drag coefficient.

(ii) Use dimensional analysis to find α, β and γ. **[5]**

(b) A light rod is freely pivoted about a fixed point at one end and has a heavy weight attached to its other end. The rod with the weight attached is oscillating in a vertical plane as a simple pendulum with period 4.3 s. The maximum angle which the rod makes with the vertical is 0.08 radians. You may assume that the motion is simple harmonic.

(i) Find the angular speed of the rod when it makes an angle of 0.05 radians with the vertical. **[5]**

(ii) Find the time taken for the pendulum to swing directly from a position where the rod makes an angle of 0.05 radians on one side of the vertical to the position where the rod makes an angle of 0.05 radians on the other side of the vertical. **[5]**

4 **(a)** A uniform lamina occupies the region bounded by the x-axis, the y-axis, the curve $y = e^x$ for $0 \leqslant x \leqslant \ln 3$, and the line $x = \ln 3$. Find, in an exact form, the coordinates of the centre of mass of this lamina. **[9]**

(b) A region is bounded by the x-axis, the curve $y = \dfrac{6}{x^2}$ for $2 \leqslant x \leqslant a$ (where $a > 2$), the line $x = 2$ and the line $x = a$. This region is rotated through 2π radians about the x-axis to form a uniform solid of revolution.

(i) Show that the x-coordinate of the centre of mass of this solid is $\dfrac{3(a^3 - 4a)}{a^3 - 8}$. **[6]**

(ii) Show that, however large the value of a, the centre of mass of this solid is less than 3 units from the origin. **[3]**

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1 (i)	$\frac{1}{2}m(v^2-1.4^2)=m\times 9.8(2.6-2.6\cos\theta)$ $v^2-1.96=50.96-50.96\cos\theta$ $v^2=52.92-50.96\cos\theta$	M1 A1 E1 **3**	Equation involving KE and PE
(ii)	$0.65\times 9.8\cos\theta-R=0.65\times\frac{v^2}{2.6}$ $6.37\cos\theta-R=0.25(52.92-50.96\cos\theta)$ $6.37\cos\theta-R=13.23-12.74\cos\theta$ $R=19.11\cos\theta-13.23$	M1 A1 M1 A1 **4**	Radial equation involving v^2/r Substituting for v^2 *Dependent on previous M1* *Special case:* $R=13.23-19.11\cos\theta$ *earns* M1A0M1SC1
(iii)	Leaves surface when $R=0$ $\cos\theta=\frac{13.23}{19.11}$ $(=\frac{9}{13})$ $(\theta=46.19°)$ $v^2=52.92-50.96\times\frac{9}{13}$ Speed is 4.2 m s^{-1}	M1 A1 M1 A1 **4**	(ft if $R=a+b\cos\theta$ and $0<-\frac{a}{b}<1$) *Dependent on previous M1*
(iv)	$T\sin\alpha+R\cos\alpha=0.65\times 9.8$ $T\cos\alpha-R\sin\alpha=0.65\times\frac{1.2^2}{2.4}$	M1 A1 M1 A1	Resolving vertically (3 terms) Horiz eqn involving v^2/r or $r\omega^2$
	OR $T-mg\sin\alpha=m\left(\frac{1.2^2}{2.4}\right)\cos\alpha$ M1A1 $mg\cos\alpha-R=m\left(\frac{1.2^2}{2.4}\right)\sin\alpha$ M1A1		
	$\sin\alpha=\frac{2.4}{2.6}=\frac{12}{13}$, $\cos\alpha=\frac{5}{13}$ $(\alpha=67.38°)$ Tension is 6.03 N Normal reaction is 2.09 N	M1 M1 A1 A1 **8**	Solving to obtain a value of *T* or *R* *Dependent on necessary M1s* *(Accept 6 , 2.1)* *Treat* $\omega=1.2$ *as a misread, leading to* $T=6.744$, $R=0.3764$ *for 7 / 8*

2 (i)	$\frac{1}{2}\times5000x^2=\frac{1}{2}\times400\times3^2$ Compression is 0.849 m	M1 A1 A1 **3**	Equation involving EE and KE Accept $\frac{3\sqrt{2}}{5}$
(ii)	Change in PE is $400\times9.8\times(7.35+1.4)\sin\theta$ $=400\times9.8\times8.75\times\frac{1}{7}$ $=4900\text{ J}$ Change in EE is $\frac{1}{2}\times5000\times1.4^2$ $=4900\text{ J}$ Since Loss of PE = Gain of EE, car will be at rest	M1 A1 M1 E1 **4**	Or $400\times9.8\times1.4\sin\theta$ *and* $\frac{1}{2}\times400\times4.54^2$ Or $784+4116$ M1M1A1 can also be given for a correct equation in *x* (compression): $2500x^2-560x-4116=0$ Conclusion required, or solving equation to obtain $x=1.4$
(iii)	WD against resistance is $7560(24+x)$ Change in EE is $\frac{1}{2}\times5000x^2$ Change in KE is $\frac{1}{2}\times400\times30^2$ Change in PE is $400\times9.8\times(24+x)\times\frac{1}{7}$	B1 B1 B1 B1	($=181440+7560x$) ($=2500x^2$) ($=180000$) ($=13440+560x$)
	OR Speed 7.75 m s^{-1} when it hits buffer, then WD against resistance is $7560x$ B1 Change in EE is $\frac{1}{2}\times5000x^2$ B1 Change in KE is $\frac{1}{2}\times400\times7.75^2$ B1 Change in PE is $400\times9.8\times x\times\frac{1}{7}$ B1		($=2500x^2$) ($=12000$) ($=560x$)
	$-7560(24+x)=\frac{1}{2}\times5000x^2-\frac{1}{2}\times400\times30^2$ $-400\times9.8\times(24+x)\times\frac{1}{7}$ $-7560(24+x)=2500x^2-180000-560(24+x)$ $-3.024(24+x)=x^2-72-0.224(24+x)$ $x^2+2.8x-4.8=0$ $x=\frac{-2.8+\sqrt{2.8^2+19.2}}{2}$ $=1.2$	M1 F1 M1 A1 M1 A1 **10**	Equation involving WD, EE, KE, PE Simplification to three term quadratic

3(a)(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$ [Density] $= \mathrm{M\,L^{-3}}$	B1 B1 B1 **3**	*Deduct 1 mark for $\mathrm{m\,s^{-1}}$ etc*
(ii)	$\mathrm{M\,L\,T^{-2}} = (\mathrm{M\,L^{-3}})^{\alpha}(\mathrm{L\,T^{-1}})^{\beta}(\mathrm{L^2})^{\gamma}$ $\alpha = 1$ $\beta = 2$ $-3\alpha + \beta + 2\gamma = 1$ $\gamma = 1$	B1 B1 M1A1 A1 **5**	(ft if equation involves α, β and γ)
(b)(i)	$\frac{2\pi}{\omega} = 4.3$ $\omega = \frac{2\pi}{4.3}$ ($= 1.4612$)	M1 A1	
	$\dot{\theta}^2 = 1.4612^2(0.08^2 - 0.05^2)$ Angular speed is $0.0913\ \mathrm{rad\,s^{-1}}$	M1 F1 A1 **5**	Using $\omega^2(A^2 - \theta^2)$ For RHS (b.o.d. for $v = 0.0913\ \mathrm{m\,s^{-1}}$)
	OR $\dot{\theta} = 0.08\omega\cos\omega t$ M1 $= 0.08 \times 1.4612\cos 0.6751$ F1 $= 0.0913$ A1		Or $\dot{\theta} = (-)\ 0.08\omega\sin\omega t$ $= (-)\ 0.08 \times 1.4612\sin 0.8957$
(ii)	$\theta = 0.08\sin\omega t$ When $\theta = 0.05$, $0.08\sin\omega t = 0.05$ $\omega t = 0.6751$ $t = 0.462$ Time taken is 2×0.462 $= 0.924\ \mathrm{s}$	B1 M1 A1 cao M1 A1 cao **5**	or $\theta = 0.08\cos\omega t$ Using $\theta = (\pm)0.05$ to obtain an equation for *t* *B1M1 above can be earned in (i)* or $t = 0.613$ from $\theta = 0.08\cos\omega t$ or $t = 1.537$ from $\theta = 0.08\cos\omega t$ Strategy for finding the required time (2×0.462 or $\frac{1}{2} \times 4.3 - 2 \times 0.613$ or $1.537 - 0.613$) *Dep on first M1* *For $\theta = 0.05\sin\omega t$, max B0M1A0M0 (for $0.05 = 0.05\sin\omega t$)*

4 (a)	Area is $\int_0^{\ln 3} e^x \, dx = \left[e^x \right]_0^{\ln 3}$	M1	
	$= 2$	A1	
	$\int x y \, dx = \int_0^{\ln 3} x e^x \, dx$	M1	
	$= \left[x e^x - e^x \right]_0^{\ln 3}$	M1 A1	Integration by parts For $x e^x - e^x$
	$= 3\ln 3 - 2$		
	$\bar{x} = \dfrac{3\ln 3 - 2}{2} = \frac{3}{2}\ln 3 - 1$	A1	ww full marks (B4) Give B3 for 0.65
	$\int \frac{1}{2} y^2 \, dx = \int_0^{\ln 3} \frac{1}{2}(e^x)^2 \, dx$	M1	For integral of $(e^x)^2$
	$= \left[\frac{1}{4} e^{2x} \right]_0^{\ln 3}$	A1	For $\frac{1}{4} e^{2x}$
	$= 2$		
	$\bar{y} = \dfrac{2}{2} = 1$	A1 **9**	If area wrong, SC1 for $\bar{x} = \dfrac{3\ln 3 - 2}{area}$ and $\bar{y} = \dfrac{2}{area}$
(b)(i)	Volume is $\int \pi y^2 \, dx = \int_2^a \pi \dfrac{36}{x^4} dx$	M1	*π may be omitted throughout*
	$= \pi \left[-\dfrac{12}{x^3} \right]_2^a = \pi \left(\dfrac{3}{2} - \dfrac{12}{a^3} \right)$	A1	
	$\int \pi x y^2 \, dx = \int_2^a \pi \dfrac{36}{x^3} dx$	M1	
	$= \pi \left[-\dfrac{18}{x^2} \right]_2^a = \pi \left(\dfrac{9}{2} - \dfrac{18}{a^2} \right)$	A1	
	$\bar{x} = \dfrac{\int \pi x y^2 \, dx}{\int \pi y^2 \, dx}$	M1	
	$= \dfrac{\pi \left(\frac{9}{2} - \frac{18}{a^2} \right)}{\pi \left(\frac{3}{2} - \frac{12}{a^3} \right)} = \dfrac{3(a^3 - 4a)}{a^3 - 8}$	E1 **6**	
(ii)	Since $a > 2$, $4a > 8$ so $a^3 - 4a < a^3 - 8$ Hence $\bar{x} = \dfrac{3(a^3 - 4a)}{a^3 - 8} < 3$ i.e. CM is less than 3 units from O	M1 A1 E1 **3**	*Condone $\geq$ instead of $>$ throughout* Fully acceptable explanation *Dependent on M1A1*
	OR As $a \to \infty$, $\bar{x} = \dfrac{3(1 - 4a^{-2})}{1 - 8a^{-3}} \to 3$ M1A1 Since $\bar{x}$ increases as *a* increases, $\bar{x}$ is less than 3 E1		Accept $\bar{x} \approx \dfrac{3a^3}{a^3} \to 3$, etc (M1 for $\bar{x} \to 3$ stated, but A1 requires correct justification)

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General Comments

This paper was found to be slightly more difficult than last year's, but most candidates responded well by showing what they could do, and presenting their work clearly. There appeared to be sufficient time to complete the paper, and there was a good spread of marks: about 20% of the candidates scored 60 marks or more (out of 72), and about a quarter scored fewer than half marks. Questions 1 and 2 were found harder than questions 3 and 4.

Comments on Individual Questions

1) *(Circular motion)*
The average mark on this question was about 11 (out of 19).

(i) This was usually answered correctly, although some candidates had incorrect signs in their energy equation.

(ii) Most candidates knew that they should form a radial equation of motion and substitute the given expression for v^2. However, there were many sign errors; in particular, the normal reaction was often taken to be acting towards the centre instead of away from it.

(iii) Almost all candidates considered when the normal reaction became zero, and a good proportion obtained the speed correctly. Very many obtained the value of θ (unnecessarily, as only $\cos\theta$ was needed); and some stopped here and forgot to find the speed.

(iv) This problem involving motion in a horizontal circle was found very difficult, and there were few fully correct solutions. It was common for the normal reaction to be omitted when resolving vertically. The horizontal equation of motion was quite often given correctly, although sign errors occurred frequently and a component of the weight was sometimes included. Some candidates did try to consider motion in the radial and transverse directions, but this approach was even more prone to errors. In many cases, a clear diagram would have been helpful, for both the candidate and the examiner.

2) *(Elastic energy)*
This question had an average mark of about 10 (out of 17).

(i) Most candidates applied the conservation of energy successfully here.

(ii) This was also quite well answered, although very many candidates made it more complicated than necessary. The simplest method was to verify that the loss of gravitational potential energy is equal to the gain in elastic energy, but a more popular method was to form an equation for the compression. Some candidates split the motion into two parts: constant acceleration until the car hits the buffer, followed by consideration of kinetic, gravitational and elastic energy while the spring compresses. The most common error in this part was to forget the contribution to gravitational potential energy made while the car is in contact with the buffer.

(iii) Most candidates knew basically what needed to be done, but there were many opportunities to go wrong. Common errors included omitting the work done against the resistive force, forgetting the contribution made by the compression *x* to the work done or the gravitational potential energy, and incorrect signs when forming the work-energy equation.

3) *(Dimensional analysis and simple harmonic motion)*
The average mark on this question was about 12 (out of 18). Part (a) on dimensions was answered extremely well, and part (b) on simple harmonic motion rather poorly.

(a)(i) Almost all candidates gave the dimensions correctly.

(a)(ii) Dimensional analysis was very well understood, and most candidates obtained the powers correctly.

(b)(i) Most candidates could use the period to find the constant (ω) for the simple harmonic motion, although a surprising number made no progress beyond this. Those who were familiar with formulae such as $\dot{\theta}^2 = \omega^2(A^2 - \theta^2)$ usually obtained the correct value of $\dot{\theta}$. Some preferred to use $\theta = A\sin\omega t$ and $\dot{\theta} = A\omega\cos\omega t$, which was quite efficient in this case, as the value of *t* could then be used in the next part.

(b)(ii) Most candidates obtained a relevant value of *t*, but a very large number did not have a correct strategy for finding the required time interval. Some misinterpreted the question and thought that the amplitude had now changed to 0.05 radians.

4) *(Centres of mass)*
This was the best answered question, with an average mark of about 13 (out of 18).

(a) The method for finding the centre of mass of a lamina was well understood, and very often carried out accurately. The only common errors were slips in evaluating the definite integrals, and omitting the factor ½ from the *y*-coordinate. A few candidates appeared to be unfamiliar with integration by parts; knowledge of topics in Core 1 to 4 is of course assumed in this unit.

(b)(i) Finding the centre of mass of a solid of revolution was also well understood, and most candidates were able to derive the given result.

(b)(ii) Those who started with $a > 2 \Rightarrow a^3 - 4a < a^3 - 8$ were able to establish $\bar{x} < 3$ quite easily. A popular method was to state that $\bar{x}$ tends to 3 as *a* tends to infinity, but full marks were rarely obtained this way; this statement was often not proved properly (for example, there was a lot of work involving $\infty^3 - 4\infty$ and so on), and hardly any candidates mentioned that $\bar{x}$ is an increasing function of *a*.