OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4764

Mechanics 4

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Thursday 11 June 2009
Morning

Duration: 1 hour 30 minutes

4764

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Turn over

Section A (24 marks)

1 A raindrop increases in mass as it falls vertically from rest through a stationary cloud. At time t s the velocity of the raindrop is $v\,\mathrm{m\,s^{-1}}$ and its mass is m kg. The rate at which the mass increases is modelled as $\dfrac{mg}{2(v+1)}\,\mathrm{kg\,s^{-1}}$. Resistances to motion are neglected.

(i) Write down the equation of motion of the raindrop. Hence show that

$$\left(1-\frac{1}{v+2}\right)\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{1}{2}g.$$ **[5]**

(ii) Solve this differential equation to find an expression for t in terms of v. Calculate the time it takes for the velocity of the raindrop to reach $10\,\mathrm{m\,s^{-1}}$. **[5]**

(iii) Describe, with reasons, what happens to the acceleration of the raindrop for large values of t. **[2]**

2 A uniform rigid rod AB of mass m and length $4a$ is freely hinged at the end A to a horizontal rail. The end B is attached to a light elastic string BC of modulus $\frac{1}{2}mg$ and natural length a. The end C of the string is attached to a ring which is small, light and smooth. The ring can slide along the rail and is always vertically above B. The angle that AB makes below the rail is θ. The system is shown in Fig. 2.

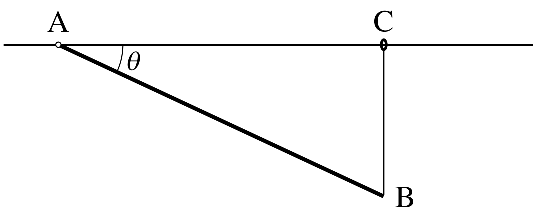


Fig. 2

(i) Find the potential energy, V, of the system when the string is stretched and show that

$$\frac{\mathrm{d}V}{\mathrm{d}\theta}=4mga\cos\theta(2\sin\theta-1).$$ **[5]**

(ii) Hence find any positions of equilibrium of the system and investigate their stability. **[7]**

Section B (48 marks)

3 A uniform circular disc has mass M and radius a. The centre of the disc is at point C.

(i) Show by integration that the moment of inertia of the disc about an axis through C and perpendicular to the disc is $\frac{1}{2}Ma^2$. **[6]**

The point A on the disc is at a distance $\frac{1}{10}a$ from its centre.

(ii) Show that the moment of inertia of the disc about an axis through A and perpendicular to the disc is $0.51Ma^2$. **[2]**

The disc can rotate freely in a vertical plane about an axis through A that is horizontal and perpendicular to the disc. The disc is held slightly displaced from its stable equilibrium position and is released from rest. In the motion that follows, the angle that AC makes with the downward vertical is θ.

(iii) Write down the equation of motion for the disc. Assuming θ remains sufficiently small throughout the motion, show that the disc performs approximate simple harmonic motion and determine the period of the motion. **[6]**

A particle of mass m is attached at a point P on the circumference of the disc, so that the centre of mass of the system is now at A.

(iv) Sketch the position of P in relation to A and C. Find m in terms of M and show that the moment of inertia of the system about the axis through A and perpendicular to the disc is $0.6Ma^2$. **[5]**

The system now rotates at a constant angular speed ω about the axis through A.

(v) Find the kinetic energy of the system. Hence find the magnitude of the constant resistive couple needed to bring the system to rest in n revolutions. **[5]**

4 A parachutist of mass 90 kg falls vertically from rest. The forces acting on her are her weight and resistance to motion R N. At time t s the velocity of the parachutist is $v\,\mathrm{m\,s^{-1}}$ and the distance she has fallen is x m.

While the parachutist is in free-fall (i.e. before the parachute is opened), the resistance is modelled as $R = kv^2$, where k is a constant. The terminal velocity of the parachutist in free-fall is $60\,\mathrm{m\,s^{-1}}$.

(i) Show that $k = \dfrac{g}{40}$. **[2]**

(ii) Show that $v^2 = 3600\left(1 - \mathrm{e}^{-\frac{gx}{1800}}\right)$. **[7]**

When she has fallen 1800 m, she opens her parachute.

(iii) Calculate, by integration, the work done against the resistance before she opens her parachute. Verify that this is equal to the loss in mechanical energy of the parachutist. **[7]**

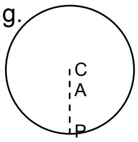
As the parachute opens, the resistance instantly changes and is now modelled as $R = 90v$.

(iv) Calculate her velocity just before opening the parachute, correct to four decimal places. **[1]**

(v) Formulate and solve a differential equation to calculate the time it takes after opening the parachute to reduce her velocity to $10\,\mathrm{m\,s^{-1}}$. **[7]**

4764 MEI Mechanics 4

1(i)	$\frac{d}{dt}(mv) = mg$	B1	Seen or implied	
	$\Rightarrow \frac{dm}{dt}v + m\frac{dv}{dt} = mg$	M1	Expand	
	$\Rightarrow \frac{mg}{2(v+1)}v + m\frac{dv}{dt} = mg$	M1	Use $\frac{dm}{dt} = \frac{mg}{2(v+1)}$	
	$\Rightarrow \frac{dv}{dt} = g\left(1 - \frac{v}{2(v+1)}\right) = g\left(\frac{v+2}{2(v+1)}\right)$			
	$\Rightarrow \left(\frac{v+1}{v+2}\right)\frac{dv}{dt} = \frac{1}{2}g$	M1	Separate variables (oe)	
	$\Rightarrow \left(1 - \frac{1}{v+2}\right)\frac{dv}{dt} = \frac{1}{2}g$	E1		
				5
(ii)	$\int\left(1 - \frac{1}{v+2}\right)dv = \int\frac{1}{2}g\,dt$	M1	Integrate	
	$v - \ln\lvert v+2\rvert = \frac{1}{2}gt + c$	A1	LHS	
	$t = 0, v = 0 \Rightarrow -\ln 2 = c$	M1	Use condition	
	$v - \ln\lvert v+2\rvert = \frac{1}{2}gt - \ln 2$			
	$t = \frac{2}{g}(v - \ln\lvert v+2\rvert + \ln 2)$	A1		
	$v = 10 \Rightarrow t \approx 1.68$	B1		
				5
(iii)	As t gets large, v gets large	M1		
	So $\frac{dv}{dt} \to \frac{1}{2}g$ (i.e. constant)	A1	Complete argument	
				2
2(i)	$V = -mg \cdot 2a\sin\theta + \frac{\frac{1}{2}mg}{2a}(4a\sin\theta - a)^2$	B1	GPE	
		M1 A1	Reasonable attempt at EPE EPE correct	
	$\frac{dV}{d\theta} = -2mga\cos\theta + \frac{mg}{2a}(4a\sin\theta - a)\cdot 4a\cos\theta$	M1	Differentiate	
	$= -2mga\cos\theta + 2mga\cos\theta(4\sin\theta - 1)$			
	$= 4mga\cos\theta(2\sin\theta - 1)$	E1	Complete argument	
				5
(ii)	$\frac{dV}{d\theta} = 0$	M1	Set derivative to zero	
	$\Leftrightarrow \cos\theta = 0$ or $\sin\theta = \frac{1}{2}$	M1	Solve	
	$\Leftrightarrow \theta = \frac{1}{2}\pi$ or $\frac{1}{6}\pi$	A1	Both	
	$\frac{d^2V}{d\theta^2} = 4mga\cos\theta(2\cos\theta) - 4mga\sin\theta(2\sin\theta - 1)$	M1	Second derivative (or alternative method)	
	$V''\left(\frac{1}{2}\pi\right)(= -4mga) < 0 \Rightarrow$ unstable	M1	Consider sign	
		A1	One correct conclusion validly shown	
	$V''\left(\frac{1}{6}\pi\right)\left(= 4mga\cdot\frac{\sqrt{3}}{2}(\sqrt{3})\right) > 0 \Rightarrow$ stable	A1	Complete argument	
				7

3(i)	Mass of 'ring' $\approx 2\pi r \delta r \rho$	B1	May be implied	
	$\Rightarrow I_C = \int_0^a r^2 \cdot 2\pi\rho r \, dr$	M1	Set up integral	
		A1	All correct	
	$= \left[2\pi\rho \cdot \frac{1}{4}r^4\right]_0^a = \frac{1}{2}\pi a^4 \rho$	M1	Integrate	
	$M = \pi a^2 \rho$	M1	Use relationship between ρ and M	
	$\Rightarrow I_C = \frac{1}{2}Ma^2$	E1	Complete argument	
				6
(ii)	$I_A = I_C + M\left(\frac{1}{10}a\right)^2$	M1	Use parallel axis theorem	
	$= \frac{1}{2}Ma^2 + \frac{1}{100}Ma^2 = 0.51Ma^2$	E1	Convincingly shown	
				2
(iii)	$I_A\ddot{\theta} = -Mg \cdot \frac{1}{10}a\sin\theta$	B1	LHS	
		B1	RHS	
	$\Rightarrow \ddot{\theta} = -\frac{g}{5.1a}\sin\theta$	M1	Expression for $\ddot{\theta}$	
	θ small $\Rightarrow \sin\theta \approx \theta$	M1	Use small angle approximation	
	$\Rightarrow \ddot{\theta} = -\frac{g}{5.1a}\theta$, i.e. SHM	E1	Complete argument and conclude SHM	
	Period $2\pi\sqrt{\frac{5.1a}{g}} \approx 4.53\sqrt{a}$	F1	Follow their SHM equation	
				6
(iv)	e.g. 	B1	Show PAC in straight line (in any direction)	
	$mg \cdot \frac{9}{10}a = Mg \cdot \frac{1}{10}a$	M1	Moments or $(\Sigma m)\bar{x} = \Sigma mx$ (oe)	
	$\Rightarrow m = \frac{1}{9}M$	A1	Method may be implied	
	$I = 0.51Ma^2 + m\left(\frac{9}{10}a\right)^2$	M1		
	$= 0.6Ma^2$	E1	Convincingly shown	
				5
(v)	KE $= \frac{1}{2}I\omega^2 = \frac{1}{2}(0.6Ma^2)\omega^2$	M1	Attempt to find KE	
	$= 0.3Ma^2\omega^2$	A1		
	$C \cdot n \cdot 2\pi = 0.3Ma^2\omega^2$	M1	Work-energy equation	
		A1	Correct equation	
	$\Rightarrow C = \frac{0.3Ma^2\omega^2}{2n\pi}$	A1		
				5

4(i)	At terminal velocity, $\Sigma F = 0 \Rightarrow k \cdot 60^2 = 90g$	M1	Equilibrium of forces	
	$\Rightarrow k = \frac{1}{40}g$	E1	Convincingly shown	
				2
(ii)	$90v\frac{dv}{dx} = 90g - \frac{1}{40}gv^2$	M1	N2L	
		A1		
	$\int \frac{90v}{90g - \frac{1}{40}gv^2}\,dv = \int dx$	M1	Separate and integrate	
	$-\frac{1800}{g}\ln\left\|90g - \frac{1}{40}gv^2\right\| = x + c_1$	A1	LHS	
	$90g - \frac{1}{40}gv^2 = Ae^{-\frac{gx}{1800}}$			
	$v^2 = \frac{40}{g}\left(90g - Ae^{-\frac{gx}{1800}}\right)$	M1	Rearrange, dealing properly with constant	
	$x = 0, v = 0 \Rightarrow A = 90g$	M1	Use condition	
	$v^2 = 3600\left(1 - e^{-\frac{gx}{1800}}\right)$	E1	Complete argument	
				7
(iii)	WD against $R = \int_0^{1800} kv^2\,dx$	B1		
	$= \int_0^{1800} 90g\left(1 - e^{-\frac{gx}{1800}}\right)dx$			
	$= \left[90g\left(x + \frac{1800}{g}e^{-\frac{gx}{1800}}\right)\right]_0^{1800}$	M1	Integrate	
	$= 162000(g + e^{-g} - 1)$	A1		
	$x = 1800 \Rightarrow v^2 = 3600(1 - e^{-g})$	B1		
	Loss in energy			
	$= 90g \cdot 1800 - \frac{1}{2} \cdot 90 \cdot 3600(1 - e^{-g})$	M1	GPE	
		M1	KE	
	$= 162000(g + e^{-g} - 1) =$ WD against R	E1	Convincingly shown (including signs)	
				7
(iv)	$v = 60\sqrt{1 - e^{-g}} \approx 59.9983$	B1		
				1
(v)	$90\frac{dv}{dt} = 90g - 90v$	M1	N2L	
		A1		
	$\int \frac{dv}{g - v} = \int dt \quad \left[\text{or } \int_{59.9983}^{10} \frac{dv}{g - v} = \int_0^t dt\right]$	M1	Separate and integrate	
	$-\ln\|g - v\| = t + c_2$	A1		
	$t = 0, v = 59.9983 \Rightarrow c_2 = -3.91598$	M1	Use condition (or limits)	
	$v = 10 \Rightarrow t = -\ln 0.2 + 3.91598$	M1	Calculate t	
	≈ 5.53 s	A1		
				7

4764 Mechanics 4

General Comments

The standard was very high in general. Candidates showed a good understanding of the syllabus, though many found the work on rotation difficult.

Comments on individual questions

1) (i) Generally well answered, though many candidates produced much more work than was required, either when rearranging their expression or by unnecessarily deriving the expansion of $\frac{\mathrm{d}}{\mathrm{d}t}(mv)$ from first principles.

(ii) Most candidates produced good work for this question.

(iii) Some candidates gave part of the reasoning, but very few gave a full account. This was often left unanswered.

2) (i) The concepts were well understood here, but details were often incorrect in the algebraic manipulation and calculus.

(ii) Again the concepts were well understood, but details were often missing in the second half of the question. In most cases this led only to the last accuracy mark being withheld. Many candidates gave extra values for θ which were not physically possible for the given system; these were ignored.

3) (i) The expression for mass of an elementary ring and the resulting integral were usually found accurately. Those candidates that did so then tended to get the correct moment of inertia. Again, many of the candidates gave more working than necessary by deriving the expression for mass of the disc by integration.

(ii) Almost universally correct.

(iii) Most candidates knew the form which the equation of motion should take, though the detail was often incorrect. In many cases the incorrect detail was introduced while deriving the equation of motion from the energy equation rather than writing it down as the question asked. It was very common to see equations of the form $\ddot{\theta} = k\theta$, with k positive, being given as SHM. Finding the period from their SHM equation was usually done well.

(iv) This was generally well answered.

(v) The kinetic energy was usually found correctly. Many candidates chose to continue by using the constant acceleration formulae rather than the energy considerations asked for in the question. Those that used the work-energy equation generally gave the correct magnitude for C, though many gave much more working than is necessary when dealing with a constant couple.

4) (i) This was usually correct.

(ii) Most candidates were able to set up the differential equation, separate variables and recognise the form of the integrand. The subsequent manipulation was usually well done; candidates that simplified their integrand before proceeding were more often successful.

(iii) This was sometimes done very well. Many candidates lost a term by disregarding the lower limit or gave the velocity as 60 when calculating the kinetic energy. It was very common to see candidates using rounded values to show equality rather than retaining the exact forms. Some candidates found the work done against mg-kv2, which caused difficulties if they included GPE in their calculation of mechanical energy.

(iv) Almost universally correct.

(v) Not many accurate solutions given. Most candidates omitted the modulus signs required in the integral (v being greater than g in the motion under consideration). Of these, some simplified their expression for t in such a way as to produce the correct answer, but many candidates' either ignored the negative or stopped when they found that they needed to evaluate the logarithm of a negative number.

4764 Mechanics 4

General Comments

The standard was very high in general. Candidates showed a good understanding of the syllabus, though many found the work on rotation difficult.

Comments on individual questions

1) (i) Generally well answered, though many candidates produced much more work than was required, either when rearranging their expression or by unnecessarily deriving the expansion of $\frac{d}{dt}(mv)$ from first principles.

(ii) Most candidates produced good work for this question.

(iii) Some candidates gave part of the reasoning, but very few gave a full account. This was often left unanswered.

2) (i) The concepts were well understood here, but details were often incorrect in the algebraic manipulation and calculus.

(ii) Again the concepts were well understood, but details were often missing in the second half of the question. In most cases this led only to the last accuracy mark being withheld. Many candidates gave extra values for θ which were not physically possible for the given system; these were ignored.

3) (i) The expression for mass of an elementary ring and the resulting integral were usually found accurately. Those candidates that did so then tended to get the correct moment of inertia. Again, many of the candidates gave more working than necessary by deriving the expression for mass of the disc by integration.

(ii) Almost universally correct.

(iii) Most candidates knew the form which the equation of motion should take, though the detail was often incorrect. In many cases the incorrect detail was introduced while deriving the equation of motion from the energy equation rather than writing it down as the question asked. It was very common to see equations of the form $\ddot{\theta} = k\theta$, with k positive, being given as SHM. Finding the period from their SHM equation was usually done well.

(iv) This was generally well answered.

(v) The kinetic energy was usually found correctly. Many candidates chose to continue by using the constant acceleration formulae rather than the energy considerations asked for in the question. Those that used the work-energy equation generally gave the correct magnitude for C, though many gave much more working than is necessary when dealing with a constant couple.

4) (i) This was usually correct.

(ii) Most candidates were able to set up the differential equation, separate variables and recognise the form of the integrand. The subsequent manipulation was usually well done; candidates that simplified their integrand before proceeding were more often successful.

(iii) This was sometimes done very well. Many candidates lost a term by disregarding the lower limit or gave the velocity as 60 when calculating the kinetic energy. It was very common to see candidates using rounded values to show equality rather than retaining the exact forms. Some candidates found the work done against mg-kv2, which caused difficulties if they included GPE in their calculation of mechanical energy.

(iv) Almost universally correct.

(v) Not many accurate solutions given. Most candidates omitted the modulus signs required in the integral (v being greater than g in the motion under consideration). Of these, some simplified their expression for t in such a way as to produce the correct answer, but many candidates' either ignored the negative or stopped when they found that they needed to evaluate the logarithm of a negative number.