

**ADVANCED SUBSIDIARY GCE**  
**MEI STATISTICS**  
Statistics 2 (Z2)

**G242**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**

None

**Monday 1 June 2009**  
**Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

- 1 A wine producer wishes to determine whether the quality of the grapes harvested on his estate is associated with the part of the estate on which they are grown. To investigate this, the estate is divided into three different areas. A random sample of 300 bunches of grapes is taken from across the entire estate. The area from which each bunch is taken is noted and the quality of each bunch is classified as excellent, good or satisfactory. The results are as follows.

	Area A	Area B	Area C
Excellent	36	18	30
Good	36	50	42
Satisfactory	21	31	36

- (i) State null and alternative hypotheses for a test to examine whether these data provide any evidence of an association between these classification factors. [1]

The following tables show some of the expected frequencies and contributions to the test statistic.

Expected frequencies

	Area A	Area B	Area C
Excellent	26.04	27.72	30.24
Good	39.68		
Satisfactory	27.28		

Contributions to the test statistic

	Area A	Area B	Area C
Excellent	3.8096	3.4083	0.0019
Good	0.3413		
Satisfactory	1.4457		

- (ii) Calculate the remaining expected frequencies and contributions to the statistic for the  $\chi^2$  test, and carry out the test at the 5% significance level. [10]
- (iii) With reference to the values in the above tables, discuss briefly how the quality of grapes harvested differs between areas. [3]

- 2 (i) A coal merchant supplies smokeless fuel in bags labelled as containing 25 kg. A machine is used to fill the bags. The quantity delivered to each bag,  $X$  kg, may be modelled using a Normal distribution with mean 25.2 kg and standard deviation 0.1 kg.
- (A) Find the probability that the machine delivers less than 25 kg of smokeless fuel to a bag. [3]
- (B) Find the probability that, in a customer's order of five bags, there is at least one bag containing less than 25 kg. [3]
- (ii) The coal merchant also sells house coal in bags labelled as containing 25 kg. The coal merchant investigates whether or not the machine used to fill bags with house coal is delivering the nominal amount of 25 kg on average. The amount,  $y$  kg, in each of a random sample of 50 bags is measured. The results are summarised as follows.

$$\Sigma y = 1295 \quad \Sigma y^2 = 33\,544$$

- (A) Use these data to show that the sample variance is  $0.071\,43 \text{ kg}^2$ , correct to 4 significant figures. [2]
- (B) Find a two-sided 95% confidence interval for the mean amount delivered by this machine. [4]
- (C) With reference to the confidence interval found in part (B), describe what the coal merchant could conclude about the amount delivered by this machine. [3]
- 3 A practice manager is monitoring the length of time, during evening surgeries, that patients wait to see a doctor. She has found the median waiting time to be 23 minutes. She decides to change the system of allocating appointment times.

Following the introduction of a new system, she measures the waiting times, during evening surgeries, of a random sample of 12 patients. The results, in minutes, for the sample are as follows.

14    33    12    11    6    16    27    18    29    8    9    20

- (i) Use a Wilcoxon test to examine, at the 5% significance level, whether the new appointment booking system has been successful in reducing the median waiting time. State your null and alternative hypotheses clearly. [12]
- (ii) Suppose that it could be assumed that the underlying distribution of these waiting times is Normal. State, with a reason, what the most appropriate test procedure would be. [2]

- 4 A botanist is investigating how the seeds of the Black Nightshade plant spread. He believes that seeds spread randomly, and that  $X$ , the number of seedlings in plots of a given area, may be modelled using a Poisson distribution. To test his belief, the botanist divides a region into plots of equal area, and then counts the number of Black Nightshade seedlings in each of a random sample of 120 plots.

The botanist's results are as follows.

Number of seedlings, $x$	0	1	2	3	4	5	6	$\geq 7$
Observed frequency	16	25	24	18	12	13	12	0

The sample standard deviation is 1.880 correct to 3 decimal places.

- (i) (A) Verify that the sample mean is 2.6. [2]
- (B) Do the sample statistics provide any reason to doubt the appropriateness of the Poisson model? Justify your answer. [2]
- (C) Does the pattern of observed frequencies provide any reason to doubt the appropriateness of the Poisson model? Justify your answer. [2]

The botanist wishes to carry out a test of the goodness of fit of the Poisson model. He uses 2.6 as an estimate for the mean of the underlying population. The following tables show some of the expected frequencies and corresponding contributions to the test statistic.

Expected frequencies

Number of seedlings, $x$	0	1	2	3	4	5	$\geq 6$
Expected frequency	8.916	23.172	30.120	26.112	16.968		

Contributions to the test statistic

Number of seedlings, $x$	0	1	2	3	4	5	$\geq 6$
Contribution	5.6284	0.1442	1.2435	2.5201			6.3698

- (ii) Use the appropriate cumulative probability tables to find  $P(X = 5)$  and  $P(X \geq 6)$  and hence calculate the remaining expected frequencies and contributions. Carry out the test at the 5% level of significance. [10]

- 5 A golf club manufacturer is testing clubs made with a new alloy to find out if the average ball striking distance is greater than for clubs made with the standard alloy. A random sample of 10 observations from a test using the new alloy is as follows. All distances are in yards.

269 275 279 266 273 274 276 275 267 276

- (i) Use these data to estimate the mean and variance of the underlying population. [2]

In previous tests it has been found that, when using clubs made with the standard alloy, the ball striking distance is Normally distributed with a mean of 268 yards.

- (ii) Use a  $t$  test at the 5% level of significance to examine whether this sample provides evidence that the mean striking distance for clubs made with the new alloy is greater than 268 yards. State your null and alternative hypotheses clearly. [9]
- (iii) When testing golf clubs, a robot arm is used to ensure that all balls are struck with a club head speed of 100 mph. Tests are carried out at an outdoor golf driving range. Describe briefly two other factors that the golf club manufacturer could consider to help ensure fairness in the testing procedure. [2]

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## G242 Statistics 2

Q1																																			
(i)	$H_0$ : there is no association between area and quality $H_1$ : there is an association between area and quality	B1	1																																
(ii)	<p>Expected frequencies</p> <table border="1"> <thead> <tr> <th></th> <th>Area A</th> <th>Area B</th> <th>Area C</th> </tr> </thead> <tbody> <tr> <td>Excellent</td> <td>26.04</td> <td>27.72</td> <td>30.24</td> </tr> <tr> <td>Good</td> <td>39.68</td> <td><b>42.24</b></td> <td><b>46.08</b></td> </tr> <tr> <td>Satisfactory</td> <td>27.28</td> <td><b>29.04</b></td> <td><b>31.68</b></td> </tr> </tbody> </table> <p>Contributions to <math>X^2</math></p> <table border="1"> <thead> <tr> <th></th> <th>Area A</th> <th>Area B</th> <th>Area C</th> </tr> </thead> <tbody> <tr> <td>Excellent</td> <td>3.8096</td> <td>3.4083</td> <td>0.0019</td> </tr> <tr> <td>Good</td> <td>0.3413</td> <td><b>1.4256</b></td> <td><b>0.3613</b></td> </tr> <tr> <td>Satisfactory</td> <td>1.4457</td> <td><b>0.1323</b></td> <td><b>0.5891</b></td> </tr> </tbody> </table> <p> <math>X^2 = 11.5150</math>  4 degrees of freedom  Critical value for 5% significance level is 9.488  As <math>11.5150 &gt; 9.488</math> the result is significant </p> <p>There is evidence of an association between the area where grapes grow and their quality.</p>		Area A	Area B	Area C	Excellent	26.04	27.72	30.24	Good	39.68	<b>42.24</b>	<b>46.08</b>	Satisfactory	27.28	<b>29.04</b>	<b>31.68</b>		Area A	Area B	Area C	Excellent	3.8096	3.4083	0.0019	Good	0.3413	<b>1.4256</b>	<b>0.3613</b>	Satisfactory	1.4457	<b>0.1323</b>	<b>0.5891</b>	M1 A1  M1 M1  A1 CAO B1 B1 M1A1  A1	10
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(iii)	<p>Large contribution of 3.8096 shows there were more excellent quality grapes than expected in Area A. Large contribution of 3.4083 shows there were fewer excellent grapes than expected in Area B. Low contributions for Area C show that grape quality was more or less as expected.</p>	E1  E1  E1	3																																
Q2																																			
(i)(A)	$P(X < 25) = P(Z < \frac{25 - 25.2}{0.1}) = P(Z < -2)$ $1 - \Phi(2) = 1 - 0.9772 = 0.0228$	M1 standardising  M1 correct tail A1	3																																
(i)(B)	$1 - p^5$ where $p = 0.9772$ $= 0.1089$	M1 M1 A1	3																																
(ii)(A)	$\frac{33544 - \frac{1295^2}{50}}{49} = 0.07143$ (AG)	M1  A1	2																																
(ii)(B)	$25.9 \pm 1.96 \times \frac{\sqrt{0.07143}}{\sqrt{50}}$ (25.83, 25.97)	M1 centred on 25.9 M1 structure (S.E.) B1 (1.96) A1 CAO	4																																
(ii)(C)	<p>This interval does not contain 25 kg.  This suggests that the mean amount of coal delivered could be greater than 25 kg.</p>	E1 E1 (mean) E1 (greater) allow sensible alternatives	3																																

Q3			
(i)	<p><math>H_0</math>: population median = 23  <math>H_1</math>: population median &lt; 23</p> <p>Actual differences            -9 +10 -11 -12 -17 -7 +4 -5 +6 -15 -14 -3</p> <p>Associated ranks            6 7 8 9 12 5 2 3 4 11 10 1</p> <p><math>T = 6 + 8 + 9 + 12 + 5 + 3 + 11 + 10 + 1 = 65</math>  <math>T^+ = 7 + 2 + 4 = 13</math>  <math>\therefore T = 13</math></p> <p>From tables – at the 5% level of significance in a one-tailed Wilcoxon signed rank test, the critical value of <math>T</math> is 17</p> <p><math>13 &lt; 17 \therefore</math> the result is significant</p> <p>The evidence suggests a decrease in the median waiting time with the new appointments system</p>	<p>B1            B1</p> <p>B1</p> <p>M1 A1</p> <p>B1            B1            B1</p> <p>B1</p> <p>M1 A1</p> <p>E1</p>	<b>12</b>
(ii)	<p>Sample small and population variance unknown.  <math>t</math> test</p>	<p>B1            B1</p>	<b>2</b>

Q4			
(i)A	<p>Sample mean = <math>312 \div 120</math> (=2.6 AG)</p>	M1 A1	<b>2</b>
(i)B	<p>Variance = <math>1.880^2 = 3.5344</math>            Comparison of sample mean and variance with conclusion about suitability of Poisson model.</p>	<p>B1            E1 dep</p>	<b>2</b>
(i)C	<p>The observed frequencies would tail-off more in a Poisson model (or the observed frequency for <math>x = 5</math> is too high) – hence a Poisson model may not be suitable.</p>	<p>E1            E1</p>	<b>2</b>
(ii)	<p><math>H_0</math>: The Poisson model is suitable            Missing expected frequencies are 8.832 (<math>x = 5</math>), and 5.88 (<math>x \geq 6</math>)            Missing contributions are 1.4546 (<math>x = 4</math>) and 1.9670 (<math>x = 5</math>)  <math>\chi^2 = 19.32(76)</math>            There are <math>7 - 1 - 1 = 5</math> degrees of freedom.            At the 5% significance level the critical value is 11.07            The result is significant            Evidence suggests that the Poisson model is inappropriate.</p>	<p>M1 A1 A1</p> <p>M1A1            A1            B1            B1            B1            B1</p>	<b>10</b>

<b>Q5</b>			
<b>(i)</b>	Estimate of population mean = 273 Estimate of population variance = 18.222	B1 B1	<b>2</b>
<b>(ii)</b>	$H_0 : \mu = 268$ & $H_1 : \mu > 268$ Where $\mu$ represents the population mean ball striking distance using clubs made from the new alloy. $t = \frac{273 - 268}{SD / \sqrt{10}} = 3.704 \text{ (4s.f.)}$ 9 degrees of freedom At 5% level, critical value of $t$ is 1.833 $3.704 > 1.833$ so the result is significant. Evidence suggests the clubs made with the new alloy have a mean striking distance greater than 268 yards.	B1 (both) B1  M1 A1  B1 B1 M1A1 A1	<b>9</b>
<b>(iii)</b>	Any suitable comments – e.g. wind speed/length of grass/type of ball will affect the distance travelled.	E1 E1	<b>2</b>

## G242 Statistics 2

The fourth sitting of this AS Statistics module has seen another increase in the number of candidates yet the total entry remains small. The overall standard of entry was encouraging; a broad range of marks were seen, with the majority of candidates being suitably prepared for the examination.

It is encouraging to see that many candidates are now comfortably applying appropriate techniques and can present solutions using suitable phrases. However, in questions requiring candidates to choose a technique, or in less routine questions, only the better prepared succeeded. Candidates would benefit from familiarising themselves with the differences between the statistical distributions/tests used and when to apply them. Candidates must have a good understanding of the differences between one-tailed and two-tailed hypothesis tests and the corresponding differences when looking up critical values in tables. Currently, too many candidates seem unsure how to identify the correct number of degrees of freedom when needed, and get mixed up by allowing for degrees of freedom when not needed.

### Question 1 Chi-squared test for Association

In part (i), some candidates mixed up the hypotheses - this sort of mistake is expensive, as any resulting conclusions are usually contradictory. It is also a requirement that the hypotheses are written 'in context' - simply stating 'no association/association' is not enough.

In part (ii), most candidates scored well. Mistakes in calculating expected frequencies were surprisingly common. Attempts at calculating 'contributions' were on the whole more successful. Most candidates managed to correctly identify the number of degrees of freedom and the corresponding critical value, then carry on to make a sensible comparison (using either an inequality or sketch) and conclusion. Again, context must be included in the conclusion - simply stating 'significant' is not enough for the final mark.

Part (iii) was poorly answered. The requirement was for candidates to look at the table of contributions and identify either large contributions (indicating evidence of association) or contributions close to zero (indicating evidence of independence). In cases where contributions are large, candidates should also indicate whether there was a greater or smaller observed number of grapes than expected.

### Question 2 Normal distribution and confidence interval

Part (i)(A) was well answered - some candidates calculated the probability for the 'other tail'. Part (i)(B) was poorly answered - many recognised the binomial nature of the situation but were unsure what calculation was needed.

Part (ii)(A) was well answered.

Part (ii)(B) poorly answered - frequent mistakes included use of an incorrect  $z$  value (not 1.96), and use of variance in place of standard deviation.

In part (ii)(C), candidates were expected to point out that the (correct) interval calculated earlier did not contain 25kg and realise that this was an indication that the true value of the mean was perhaps greater than 25kg.

### Question 3 Wilcoxon test

Part (i) was quite well answered but common errors were seen. These include omitting 'population' when stating hypotheses, ranking actual (not absolute value of) differences, adding the differences instead of the ranks, and using ' $n - 1$ ' when looking up the critical value. Despite this, many candidates scored well on this question.

In part (ii), most recognised that a t test was suitable, quoting the fact that it was a small sample, but few also mentioned that the population variance being unknown was relevant too.

#### **Question 4 Chi-Squared test for goodness of fit**

This proved to be a difficult question.

In part (i)(A), most candidates realised that evidence of calculation was necessary and were on the whole successful. A surprising number stated that 'my calculator says that the mean is 2.6' or words to that effect, and scored no marks.

In part (i)(B) many candidates compared the mean with the standard deviation, rather than with the variance, and achieved no credit. In such questions candidates are required to calculate the variance (using the given sample standard deviation) to make their comparison clear.

Part (i)(C) proved to be difficult for all but the most able; most candidates seemed to miss the instruction to look at the 'pattern of observed frequencies' and made general comments about 'independence'

In part (ii), many seemed unsure how to calculate the missing expected frequencies - attempts at  $X = 5$  were more successful than for  $X = 6$  or more. Few candidates identified the correct number of degrees of freedom, not realising the need to allow for a restriction due to the mean being estimated from the sample.

#### **Question 5 t test**

In part (i) most provided a correct estimate for the population mean and variance - mistakes with variance were more common.

In part (ii) many good answers were seen. It was pleasing to see  $\mu$  identified as representing the population mean in many scripts. Attempts at finding the test statistic were good, although it was not unusual to see variance used in place of standard deviation. ' $n - 1$ ' seems to be the 'default setting' for looking up numbers in tables, for many candidates, and so in this case most were successful in identifying the correct critical value.

In part (iii) most candidates made sensible comments.