MEI Insights 10
Scheme of Work

by **Bernard Murphy, Avril Steele** and **Stephen Lee**

Introduction

Mathematics in Education and Industry (MEI) is an independent charity, committed to improving mathematics education. It is widely known as a leading curriculum development organization and has AS and A level Mathematics and Further Mathematics specifications examined through the OCR awarding body. MEI also provides support, through CPD and teaching and learning resources, for all AS and A level Mathematics specifications. This, the tenth article in the MEI Insights series, looks at MEI's work to develop a Scheme of Work for the 2017 A levels.

Background

The new A level in mathematics for first teaching from September 2017 will inevitably present schools and teachers with both opportunities and challenges. In planning to deliver this new specification you may begin by asking what is changing. A key feature of the new A level is the requirement for students to demonstrate the knowledge and skills that are described in the following three *Overarching Themes*: mathematical argument, language and proof (OT1); mathematical problem solving (OT2); and mathematical modelling (OT3).

In addition the DfE guidance document states that '*the use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the study of AS and A level mathematics*' (DfE, 2014). These requirements, along with the move to a linear format in which the entire A level content will be examined at the end of a two-year course, will mean that a simple reshuffle of schemes of work developed for the current modular specification is unlikely to be sufficient. As this isn't a trivial task it is a good opportunity to ask at the outset what, for you, are the important features of a scheme of work? Teaching order and scheduling are likely to be a priority for most, but would you also include links to the specifications, learning objectives, resources and teaching activities; or perhaps even lesson plans and textbook references? What else would you like to see included? There may already be structures in place for developing and sharing long-, medium-, and short-term planning at KS3/4 that you could follow, but you may want to consider whether an A level scheme of work needs to incorporate any different features.

In this article we will describe MEI's approach to its freely available A level Scheme of Work (SoW) and offer suggestions for how you might use and adapt it in your department to suit your needs. Since the content of A level Mathematics is 100% prescribed by the DfE, the SoW will be suitable for use by all centres regardless of the awarding body that has been chosen.

Features of MEI's Scheme of Work

The SoW is presented in 43 units, each focusing on a particular topic (see: **www.mei.org.uk/2017-sow**). The decision to do this was taken in recognition of the careful thought that had gone into designing the textbooks (see: **www.hoddereducation.co.uk/Mathematics**); each unit corresponds to a chapter in these textbooks. Whilst the order of the 43 units presents a coherent path through AS and A level Mathematics, it should be emphasized that teachers are not necessarily expected to use this order. For example, the units covering the mechanics topics in AS are presented together after all the AS pure units; this is to highlight that the mechanics ideas form a coherent body of mathematics in their own right. However, teachers are encouraged to think about teaching some mechanics alongside other topics such as calculus.

Each unit follows the same three-page structure:

- the content statements from the DfE document 'Mathematics AS and A level content' (DfE, 2014) along with a commentary on some points of conceptual or historical interest;
- two sample resources addressing the *Overarching Themes* and the effective use of technology;
- some suggestions related to the following issues that a department needs to consider: prerequisites for

the unit, links with other topics, questions and prompts for mathematical thinking, opportunities for proof (in pure units) or modelling (in applied units), and common errors.

Some of these are illustrated below.

Sample resource

The aim is to provide a range of types of resource across the SoW with each one promoting one or more of the *Overarching Themes*. The resource on page 14 is taken from the A level 'Trigonometry' unit. Using the definition of radian measure (arc length divided by radius), students are challenged to draw appropriate diagrams to help them complete the grid. Is it necessary to provide the formulae for arc length or area of sector or might they work these out for themselves? Challenging students to tackle a task which is less structured and requiring them to justify and explain their placement of the numerical values supports elements of the *Overarching Themes* of mathematical argument, language and proof (OT1) and problem solving (OT2).

Effective use of technology

Across the SoW, a range of software, both teacher-led and student-led, is used, including *Autograph*, *GeoGebra* and spreadsheets, along with suggested uses of graphing calculators. At MEI, as stated in our MEI Insights 2 article on 'The use of technology in mathematics education': '*we believe technology should be embedded within the teaching and learning of mathematics in a way that enhances students' mathematical learning by helping them to develop their understanding of mathematical concepts and by allowing them to access more mathematical ideas …*' The resource on page 14 is a student-centred use of graphing calculators taken from the AS level 'Polynomials' unit. Through exploring numeric, graphic and algebraic features of a cubic function, students gain a deeper understanding of the factor theorem. For a free download of one year's subscription to the Casio fx-CG20 emulator visit **www.casio.co.uk/emulator/mei** and for additional free resources visit **www.casio.co.uk/resources/mei** .

Questions and prompts for mathematical thinking

The examples included in almost all of the units are inspired by the highly recommended ATM publication 'Questions and Prompts for Mathematical Thinking' (Watson and Mason, 1998). The following are taken from the AS level 'Surds and Indices' unit. How would your students respond? And how does the final question promote language, argument and proof (OT1)?

- Give me an example of a number that is equal to $3\sqrt{2}$ … and another … and a peculiar example.
- Change one digit in $\left(2+\sqrt{8}\right)\left(4-\sqrt{2}\right)$ so that the product is a rational number.
- Give me an example of a number between $5\sqrt{6}$ and $6\sqrt{5}$.
- $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$. Always true, sometimes true, never true?

Opportunities for proof

Although one unit is devoted to proof, the SoW identifies opportunities to engage with proof in all the pure units. These are the suggestions from the A level 'Sequences and Series' unit.

- Prove that the infinite arithmetic sequence 3,7,11,15, ... contains no square numbers.
- Prove the formulae for the sum of arithmetic and geometric series.
- Prove that, for every triangular number T, $8T+1$ is a square number.

Can you find a geometric proof for the final one? (Nelsen, 1993). How would you use these alternative proofs with your students?

Applications and modelling

Mathematical modelling is the third *Overarching Theme* in the new A level. In several units of the SoW, suggestions are included for designing or commenting on a mathematical model. One example, from the A level 'Differential Equations' unit, takes the June 2012 OCR (MEI) Core 4 Comprehension paper (MEI, 2014) entitled 'The World's Population', and explores the logistic equation, $\frac{\mathrm{d}y}{\mathrm{d}x}=ay\left(b-y\right)$ to model a scenario which has naturally occurring asymptotic behaviour.

Adapting the Scheme of Work for your Department

There is no single correct way to utilize the SoW. Indeed the intention is that it is flexible and can be adapted to suit your department's needs. In tailoring it to fit your requirements a number of key questions are likely to arise and some key decisions will need to be taken. There will be organizational issues to address, such as whether the content will be delivered by a single teacher or by two (or more) teachers. How would you split the units from the SoW between teachers? If the option (or requirement) to sit the AS exam in Year 12 is to be retained for your students then some elements of both mechanics and statistics must be included in the first year of study. The total applied content remains at around a third so having a pure teacher and an applied teacher is not a good match. Remembering that the *Overarching Themes* of problem solving and modelling must be addressed, could you embed mechanics and statistics with the pure maths content in order to facilitate this? Or would you prefer to deliver them in series and build more opportunity to apply pure maths in applied contexts as you go along?

No specific scheduling or time allocation is included in the SoW, so it has the freedom to be adapted to suit your timetable. How much time will you allocate to each unit? Which units with a common theme might you combine? Whilst the scheduling of units will clearly be designed to deliver the content in a logical order there are other factors that ought to be considered, making the exercise something of an iterative process. One factor might be the range of skills and expertise available in your department. Are all teachers prepared to teach the mechanics or statistics elements? Will all of your classes be able to access computing suites or other resources at appropriate stages to ensure that the use of technology permeates their studies? Another factor might be the impact of your choices on the delivery of Further Maths in parallel with Maths. The provision of pathways for Further Maths is likely be one of the initial structural decisions to be made, but having a good sense of how these might fit with your schemes of work could have a bearing on these decisions. Will your Further Maths students be in separate classes or integrated with Maths students? What would be the implications of teaching Further Maths in parallel with Maths? Or in series? Have you considered offering AS Further Maths to Year 13 students?

Teaching a linear course at A level will necessitate giving some thought to the way that revision is approached. Some units build upon and consolidate earlier units, but some key mathematical techniques, such as completing the square and the binomial expansion, may be covered in Year 12 and not necessarily revisited in Year 13. Will experience of building linear schemes for GCSE or planning for progression across Key Stages 3 and 4 be useful here? (See for example: **www.ncetm.org.uk/resources/24350** – free login required to access) And what about the monitoring of progress and assessment? What are your departmental and school policies and how can these be catered for?

The MEI Scheme of Work as a Professional Development Tool

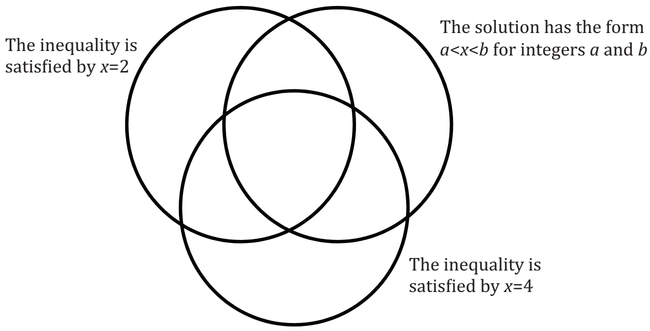
The final page in each unit is deliberately incomplete; it is designed for colleagues in departments to adapt together, sharing ideas and expertise. Below are two suggestions of how the SoW might be used in a departmental meeting.

Take any unit and spend time, first individually, then in pairs, then as a group, thinking of suggestions for each of the following:

- Prerequisites. What are the skills and knowledge you would expect students to bring to this topic?
- Links with other topics. Which topics overlap with this one? And what are the implications for teaching these units?
- Questions and prompts for mathematical thinking. Use a stem from the book (Watson and Mason, 1998) to design questions to promote mathematical thinking. For example, *Change one aspect of … so that …*, or *Give me an example of …, and another …, and another.*
- Opportunities for proof. Are there any proofs students can engage with in this (pure) topic? What would be more effective than simply showing them the proof?
- Applications and modelling. What opportunities are there in this topic to illustrate the modelling cycle? Can you think of a situation in which a likely initial model could be improved?
- Common errors. What mistakes do students often make? Why do they make them and what are the messages for your teaching?

Add your ideas to the editable SoW to start the process of making it your own.

An alternative use of departmental planning time might be to consider which resources can be adapted to fit another context. For example, how can a Venn Diagram, as used in the AS level ‘Equations and Inequalities’ unit, be used in other topics?



In Conclusion

The SoW developed by MEI is available at: **www.mei.org.uk/2017-sow**.

It provides resources, ideas and guidance for the 2017 mathematics AS and A levels, which can be used, modified and developed by teachers to suit their own individual and departmental circumstances. We hope you find the SoW useful and we would be delighted to hear from teachers who are using it.

References

DfE 2014 *GCE AS and A level Subject Content for Mathematics*, DFE-00706-2014.

MEI 2014 *June 2012 OCR (MEI) Core 4 Comprehension Examination Paper*. Available from: **www.mei.org.uk/files/papers/c4_june_2012.pdf**

Nelsen, R. 1993 *Proofs without Words: Exercises in Visual Thinking*, Mathematical Association of America, New York.

Watson, A. and Mason, J. 1998 *Questions and Prompts for Mathematical Thinking*, ATM.

Keywords: Scheme of work; A levels; Maths; Further maths.

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Arcs and Sectors

The numbers in the grid below are 12 of the 13 numbers in the grid on the right. What is the 13th missing number?

5	6	14	15.5	0.4	4
19.2	24	0.6	1.1	4.8	4

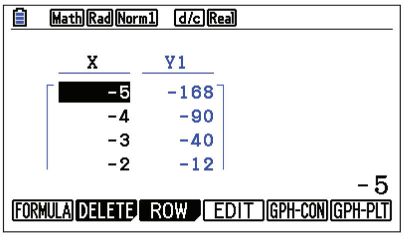
[Available from: **www.mei.org.uk/files/sow/22-trigonometry-res.pdf**]

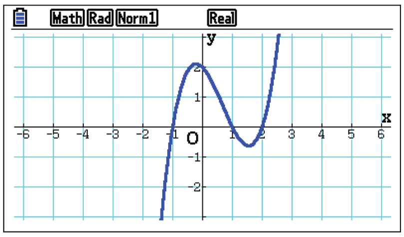
θ (in radians)	r (cm)	Arc Length (cm)	Perimeter of Sector (cm)	Area of Sector (cm^2)
	8		20.8	
	10			
		5.5		13.75
1.5				12

MEI Casio Tasks for AS Pure

Task 8: The Factor Theorem

1. Go into Table mode: MENU 7
2. Add Y1 = $x^3 - 2x^2 - x + 2$: X,θ,T ^ 3 ▶ − 2 X,θ,T x^2 − X,θ,T + 2 EXE
3. Use SET to set the table to Start: –5, End: 5, Step: 1: F5 (−) 5 EXE EXIT
4. Display the table: F6
5. Go into Graph mode and plot the graph of this function: MENU 5 F6





Questions

- How do this table and graph confirm that $x^3 - 2x^2 - x + 2 = (x + 1)(x - 1)(x - 2)$?
- Can you find the factors of the following cubics:
  $y = x^3 + 4x^2 + x - 6$ $y = x^3 - 4x^2 - 11x + 30$
  $y = x^3 - x^2 - 8x + 12$ $y = x^3 - 7x^2 + 36$

Problem (*Try the question with pen and paper first then check it on your calculator)*

Show that $(x - 2)$ is a factor of $f(x) = x^3 + 4x^2 - 3x - 18$. Hence find all the factors of $f(x)$.

Further Tasks

- Find examples of cubics that only have one real root.
- Investigate using the factor theorem for polynomials of other degrees, e.g. quadratics or quartics.
- Investigate the polynomial solver: MENU ALPHA X,θ,T F2.

[Available from: **www.mei.org.uk/files/ict/mei-casio-tasks-as-core.pdf#page=8**]