Exploring the New A-levels in Mathematics

Background

Across all subject areas AS and A-level qualifications have been undergoing reform. Teaching of many of the reformed qualifications is well underway, and the first A-level examinations took place in England this summer for a range of subjects including biology, physics, chemistry, computer science, and economics. For Mathematics and Further Mathematics it was considered that more extensive restructuring and reform was required and this resulted in the formation in 2013 of an A-level Content Advisory Board (ALCAB) panel. The purpose of the panel was to:

> ... provide advice to Government and to the Office of Qualifications and Examinations Regulation (Ofqual) on the content of A-levels in the subjects of particular importance for preparation for the leading universities. [1]

The panel consisted predominantly of university representatives and was chaired by Professor Richard Craster of Imperial College, London. ALCAB's report was published in July 2014 [2].

This article considers the key changes to AS and A-level qualifications in Mathematics and Further Mathematics in England. An article on the wider context, including the Smith Report, will be published in a future issue of *Mathematics Today*.

Detail about changes to these qualifications in Wales and Northern Ireland can be found via the UCAS website [3] and broader information about the pre-higher education Mathematics curriculum can be found on the MEI website [4].

Creation of the new 2017 Mathematics A-level

Some of the key aspects of the changes to AS and A-level Mathematics and Further Mathematics qualifications which resulted from the panel's recommendations are:

- the content of AS and A-level Mathematics is *fully prescribed* [5], meaning that no matter which specification[1] is studied the subject content will be the same. Alongside the *pure mathematics* content which makes up two-thirds of the subject content, there will be equally weighted compulsory content for both *mechanics* and *statistics*; decision/discrete mathematics will now only be available as an option within Further Mathematics.
- in order to be accredited by Ofqual, the awarding organisations will provide *at least one route* through which a student can study AS Further Mathematics alongside their AS or A-level Mathematics course. The importance of maintaining and growing participation in Further Mathematics was recognised by the ALCAB panel, which stated: 'The AS levels in Mathematics and Further Mathematics are welcomed by university departments and have considerable value in their own right. They should be supported and retained' [2, p. 2].
- 50% of the content of A-level Further Mathematics is prescribed pure mathematics content[1] which includes polar coordinates, further calculus and vectors, hyperbolic functions and differential equations; the remaining content can be chosen by the awarding organisation and may include additional optional topics within pure mathematics as well as applied options (mechanics, statistics, discrete/decision mathematics). The 50% prescribed pure content for A-level also contains the pure content which is prescribed for AS Further Mathematics, encompassing mainly complex numbers and matrices. Overall, a minimum of 30% of the AS Further Mathematics content must be selected from the prescribed A-level content, the remainder again being made up of options and/or additional compulsory pure content.

In addition, in England:

- along with all other reformed AS and A-level qualifications, the new qualifications in Mathematics and Further Mathematics will be *linear*, meaning that all assessment will take place at the end of the course of study.
- the AS and A-levels will also be *de-coupled*, meaning that any results from AS examinations do not count towards the final A-level grade; in other words, for both Mathematics and Further Mathematics, the two qualifications are completely distinct in terms of assessment and grading.
- the new qualifications are intended to be *equivalent* in demand to the current AS and A-levels in Mathematics and Further Mathematics and will have the *same grading structure*; this is unlike the new GCSE Mathematics qualification, first sitting summer 2017, which has intentionally been increased in demand from the legacy qualification and will now be graded on a 9–1 scale, with 9 being awarded to the very highest performing students [6].

First teaching commences in September 2017 with first examinations in 2018 (for AS Mathematics, A-level Mathematics and AS Further Mathematics) or 2019 (A-level Further Mathematics).

Overarching themes

The reformed AS and A-level qualifications for both Mathematics and Further Mathematics are underpinned by three overarching themes which are intended to permeate the study of the whole of the subject content.

Theme 1: Mathematical argument, language and proof

Specifications will require students to set out clear mathematical arguments, comprehend and critique these arguments, and use mathematical language. The subject content documents[1] [5] set out in appendices the notation that students will need to be familiar with and the formulae that they should know.

Theme 2: Mathematical problem solving

Specifications will embed problem solving, requiring students to extract information from a situation in order to solve a problem, recognising the underlying mathematical structure in a situation, and interpret and communicate the results.

Theme 3: Mathematical modelling

Students will need to translate a situation into a mathematical model or work with a given mathematical model, interpreting the results in the original context and recognise the modelling assumptions that have been made.

Use of technology

In addition, it is intended that:

> The use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the study of AS and A-level Mathematics. [4, p. 5]

Students will be required to use a calculator with an iterative function and which facilitates computation of summary statistics and access probabilities from standard statistical distributions.

AS and A-level Mathematics

In the reformed AS and A-level qualifications for Mathematics all students will study both mechanics and statistics. One of the biggest changes is the introduction of the study of large data sets. The Department for Education (DfE) subject content document [5, p. 5] states:

> Specifications should require students to explore the data set(s), and associated contexts, during their course of study to enable them to perform tasks that assume familiarity with the contexts, the main features of the data and the ways in which technology can help explore the data.

This includes the requirement for students to:

- become familiar with one or more *specific large data set(s)* in advance of the final assessment (these data must be real and sufficiently rich to enable the concepts and skills of data presentation and interpretation in the specification to be explored).
- use *technology* such as spreadsheets or specialist statistical packages to explore the data set(s).
- interpret *real* data presented in summary or graphical form.
- use data to investigate questions arising in real contexts.

This change is intended to achieve ALCAB's goal of moving the emphasis from routine calculation of statistical measures to 'place more emphasis upon understanding, interpretation of data and making inferences from data' [2, p. 8]. The data sets selected by the awarding organisations incorporate DEFRA data on the purchasing of food and drink in regions of England (AQA), Met Office weather data (Pearson/Edexcel), National Census data relating to age and method of travel to work (OCR A) and CIA World Factbook data and London 2012 Olympic Medal data (OCR B (MEI)).

Much of the pure subject content is unchanged from the previous Core 1 to 4 modules. A summary of some of the changes to AS and A-level is provided below.

AS Mathematics

New content:

- the functions $y=e^x$, $y=\ln x$
- using exponential and logarithmic graphs in modelling
- differentiation from first principles
- 2D vectors

Radians, all work on series, and the trapezium rule are now A-level topics only. The remainder theorem has been removed from AS and A-level.

A-level Mathematics

New content:

- Proof by contradiction, with examples given in subject content – infinity of primes and the irrationality of $\sqrt{2}$
- Newton-Raphson method
- proofs of addition formulae
- small angle approximations for sin, cos and tan

Volumes of revolution, the vector equation of a line, and the scalar product have been removed from the qualification (but appear in the prescribed content for A-level Further Mathematics).

AS and A-level Further Mathematics

In legacy AS and A-level Further Mathematics specifications there was a high degree of flexibility in terms of module choices. Schools and colleges were able to provide routes which specialised in one area of applied mathematics or offered a breadth across more than one application, alongside the compulsory pure content. Examination results for the applied modules could be swapped between the Mathematics and Further Mathematics qualifications in order to determine the best overall grades for the student. This is now no longer possible, with Mathematics and Further Mathematics being awarded independently. This offers schools and colleges scope, if they wish, to use different awarding organisations for Mathematics and Further Mathematics qualifications.

At the time of writing (June 2017) the last of the eight AS and A-level Further Mathematics specifications and associated sample assessment materials had just been accredited by Ofqual. An overview of the assessment structure for each qualification can be seen at http://furthermaths.org.uk/2017-fm. In addition to the 50% compulsory pure content most awarding organisations have offered additional pure mathematics options, together with options in statistics, mechanics, decision/discrete mathematics or a combination of these. Additionally OCR B (MEI) offers options in numerical methods and further pure with technology, the latter allowing the use of Computer Algebra Systems in the examination. It is likely that schools and colleges will tailor their provision taking into account the future intended study routes for their students, staff knowledge and available professional development opportunities, and the structure and emphasis of the sample assessment materials.

A full breakdown of the subject content of the components of each qualification can be seen on the relevant awarding organisation website.

Implications for Higher Education

From 2019 onwards, all students entering higher education with AS or A-levels in Mathematics or Further Mathematics will have studied the reformed qualifications. Whilst much of the subject content for the AS and A-level in Mathematics is unchanged, all students will have completed study of the same subject content [5], which will be beneficial for universities in planning first year courses of study. Similarly, A-level Further Mathematics students will all have studied a common 50% pure mathematics content [7].

However, there are also likely to be challenges associated with the reforms to the curriculum and post-16 funding. Students starting the new AS and A-level Mathematics and Further Mathematics courses in September are also the first cohort through the new GCSE Mathematics specification. Some schools and colleges have expressed concern that the increase in the demand of the new Mathematics GCSE has lowered students' confidence and may put them off taking A-level. In addition the changes to post-16 funding are leading to many schools and colleges reducing the number of subjects most students can take in Year 12 from four to three.

These pressures, alongside the decoupling of AS and A-levels, could combine to see a reduction in the number of students opting for AS and A-levels in Mathematics and Further Mathematics unless there is action to promote and encourage participation. These factors may have a disproportionate impact on girls' participation. Research suggests girls have a more risk-averse attitude and often have a wider range of subject choices available to them post-16 due to attaining higher GCSE grades on average, across all subjects, than boys [8].

How can Higher Education help?

The Further Mathematics Support Programme (FMSP) is a DfE funded programme which supports schools and colleges at Key Stage 4 and post-16 level to increase teacher expertise and student participation in AS and A-level Mathematics and Further Mathematics. The FMSP considers it vital for Higher Education Institutions (HEIs) to play a key role in sustaining the growth in entries for AS and A-levels in Mathematics and Further Mathematics [8, p. 5–6], by providing:

- a clear, strong message about the importance and usefulness of studying AS or A-level Mathematics and Further Mathematics as evidence to both support teachers in maintaining provision and to encourage students to choose these courses
- a clear, strong message to government about the need to support provision for Further Mathematics, in particular AS level Further Mathematics

Previous reports, such as Hillman [9], also stressed the need for clear signalling from HEIs to students and their teachers about the nature and extent of mathematical and statistical knowledge and skills needed in undergraduate degree programmes, and a recommendation from HEIs on the benefits of continuing with mathematical/statistical study beyond the age of 16.

After the fall in numbers of students taking AS and A-level qualifications in Mathematics due to reforms under Curriculum 2000, the combined efforts of the mathematics community has seen numbers experience a period of sustained growth. In 2016, A-level Mathematics was the most popular subject with over 90,000 entries and A-level Further Mathematics, with over 15,000 entries, was more popular than subjects such as physical education and law. Continued direct support and intervention is likely to remain vital in the coming years to ensure the successes of the last decade are maintained through this period of substantial reform. Now is an important time for all stakeholders to reinforce the key messages about the importance of mathematics both for a wide range of careers and for progression to higher education.

Further Reading

You might also be interested to read the recent *MSOR Connections* article [10].

Notes

1 The three English awarding organisations with accredited specifications for AS and A-level Mathematics and Further Mathematics are AQA (www.aqa.org.uk/subjects/mathematics/as-and-a-level), Pearson/Edexcel (http://tinyurl.com/Pearson-maths2017), and OCR A and OCR B/MEI (http://tinyurl.com/OCR-maths2017).

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SIAM-IMA Student Chapter Annual Conference

The University of Reading SIAM-IMA student chapter hosted their annual conference on Friday 16 May. Over 40 staff and students attended from a variety of institutions, including the University of Bath, the University of Oxford, Imperial College London and Brunel University London.

Keynote talks were given by Dr Melina Freitag (University of Bath) on saddle point formulations for data assimilation and Dr Tim Rogers (University of Bath) on how to explain collaborative behaviour as a result of evolution using stochastic noise.

Student talks were given on a broad range of topics within applied mathematics, with applications including meteorology, finance and neuroscience. The prize for the best talk, judged by conference attendees, was given to Tom Bendall (Imperial College London); John Meeson presented his award (see photo). Florian Klimm (University of Oxford) was the runner-up.

The poster session provided another opportunity for students to present their work in an informal setting over lunch. Prizes were awarded to Dan Derrick and Nicola Thorn, both from the University of Reading. All prizes were sponsored by the IMA.

The event concluded with a conference dinner, which was a great chance for everyone to relax and reflect on the day. The organising committee would like to thank all of the speakers, presenters and attendees as well as SIAM, the IMA and the University of Reading who all supported the event.

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