Mathematics for all Post-16
An MEI position Paper

Background

The December 2010 Nuffield Foundation report, *Is the UK an Outlier?*, compared the provision and participation rates in upper-secondary mathematics in 24 developed countries and showed that England, Wales and Northern Ireland have the lowest participation rates in post-GCSE level mathematics of all 24 countries, with fewer than 20% of students studying any mathematics at this level.

There has been a low participation rate in post-16 mathematics in the UK for many years; this may be related to the negative perception of mathematics that many young people develop throughout their education, from primary school onwards. There is also a societal bias that denies the importance of mathematics; traditionally many people in our society feel it is acceptable, or even laudable, to be poor at, or to dislike, mathematics. The importance of mathematics and statistics in modern society has become undeniable, and it is now recognised explicitly that improving the mathematical and statistical skills of the population is crucial to future economic prosperity. Encouragingly, mathematics does seem to be growing in popularity, with A Level Mathematics entries on the increase and mathematics being portrayed more positively in the media.

In his speech at the Royal Society in June 2011 Michael Gove, Secretary of State for Education, stated:

“Only half the population has even basic maths skills, we are producing only about a quarter to a third of the number of pupils with the maths skills that our universities need, and economic trends mean that this gap will, unless we change, get wider and wider with all that entails for our culture and economy. That is why I think we should set a new goal for the education system so that within a decade the vast majority of pupils are studying maths right through to the age of 18.”


‘Students who are under 19 and do not have GCSE A*-C in English and/or Maths should be required, as part of their programme, to pursue a course which either leads directly to these qualifications, or which provide significant progress towards future GCSE entry and success.’

In June 2011, ACME’s *Mathematical Needs* project reported the following.

‘One of the main messages to come out of the two reports is simple. We need more young people to know more mathematics and to be confident, robust and fluent in their use of it. Not only are university courses in many disciplines increasingly quantitative in content, but there is also a steady shift in the employment market away from manual and low skill jobs and toward those requiring higher levels of management expertise and problem-solving skills, many of which are mathematical in nature.’
In August 2011, the Vorderman Report, *A world-class mathematics education for all our young people*, recommended compulsory mathematics to age 18.

“To bring this country into line with the rest of the world, mathematics, in some form, should be made compulsory to the age of 18. The implementation of this recommendation is a matter of urgency."

To support both the nation’s economic needs, and the aspirations of young people themselves, it would be beneficial for all students to continue studying mathematics to age 18. However, the mathematics they study must be seen to be useful, rather than an additional burden and the mathematics qualifications that students work towards must have real currency.

While recognising that improvements in mathematics education are needed throughout the system, from primary upwards, this paper is concerned with the short-term practicalities of what we can do now to develop the provision of post-16 mathematics. In view of the government’s plan to have all young people under the age of 18 in education or training by 2015, it is essential to plan for this now. Key Stage 4 mathematics is addressed in a separate MEI paper.

**Summary of Recommendations**

1. **Communication**
   
a. The need for students to keep studying mathematics beyond 16 must be communicated much more effectively to students, teachers, careers officers, school managers and parents.

b. Materials and resources for Information, Advice and Guidance are needed for teachers, schools and careers advisors in order to guide students effectively when choosing mathematics pathways.

c. Universities must specify the mathematics/statistics qualifications that prospective undergraduates should take in order to succeed on their degree courses, and should recommend or require these qualifications for entry onto their undergraduate programmes. This need for the specification of qualifications applies across a wide range of academic disciplines - social sciences as well as STEM subjects and should include the specification of AS courses and free-standing mathematics qualifications, where appropriate.

d. It would be helpful if employers specified the mathematics/statistics qualifications that prospective employees should take in order to succeed in different careers.

2. **Curriculum development**
   
a. In order to satisfy the needs of employers and to address differing mathematical temperaments, a GCSE Mathematics qualification designed for students aged 16+ is required. This qualification should be focused on the needs of employment and not constrained by the National Curriculum.
b. Level 3 qualifications which provide the skills and knowledge to apply basic statistics and mathematical modelling should be promoted and further developed, in consultation with higher education and employers. These qualifications must be accessible to students with grade C in GCSE Mathematics.

c. Level 3 mathematics qualifications should enable students to follow different pathways, suited to their different mathematical needs. These pathways should build upon one another to ensure flexibility, enabling students to move between them.

d. Coordinated curriculum development is needed between mathematics and other subjects, to help develop the teaching and learning of mathematics in context and to embed the use of mathematics and statistics in context into the curricula of other subjects at Level 3.

e. Extensive teaching and learning resources must be developed to support both teachers and students.

3. Professional development of teachers

a. Professional development courses and materials must be developed to enable teachers of subjects that use mathematics and statistics to teach the application of basic statistics and mathematical modelling at Level 3 in context.

4. Funding

a. Sufficient funding is needed to ensure that appropriate courses are developed and taught to a high standard; this will require specific funding for professional development.

b. Schools and colleges must be adequately funded to provide such courses as part of the standard curriculum for all post-16 students.

1. Level 2 Mathematics Post-16

The Wolf Report drew attention to the importance of Mathematics and English GCSEs for students’ future prospects, and to the small percentage of the cohort who are successful in attaining these qualifications between the ages of 16 and 18.

However, retaking the same kind of GCSE they have already failed has not proved beneficial to the large majority of students who fail to achieve a grade C in Key Stage 4; it serves instead mainly to reinforce negative views of mathematics amongst a large section of the population. Such students should be able to access a recognised, high quality, high currency qualification in mathematics, which they find relevant and engaging.
It is difficult to obtain up to date statistics about post-16 GCSE retakes. Statistics from an awarding body show that, for students in 2010 who took Mathematics GCSE on a linear specification in year 11 and then resat in November of year 12, only 19% of those who had not gained at least grade C in year 11 were successful in doing so in the November of year 12. The overwhelming majority of students who gained grade C in the November resit had achieved grade D in year 11 with only 1% of students with lower than grade D in year 11 achieving a C in November of year 12. This accords with anecdotal evidence from teachers that GCSE resits are often a poor and unsuccessful experience for both students and teachers.

GCSE Use of Mathematics was piloted from 2006 to 2010, designed specifically for post-16 students. This qualification was attractive to both centres and students. The June 2011 Evaluating Mathematics Pathways Final Report stated:

‘The decision by Ofqual that after the pilot period this qualification can no longer be designated a GCSE has struck a serious blow to one of the significant developments of the Pathways Project. Although the qualification will exist, re-badged as the AQA Certificate in Use of Mathematics, it is possible that without explicit support it will struggle to flourish in the qualifications marketplace because of the kudos accorded to the label of GCSE Mathematics and its specific role as a gatekeeper to subsequent opportunities.’

A GCSE Mathematics qualification designed for students aged 16 or above, focused on the needs of employment and not constrained by the National Curriculum, would be a positive way forward. Professor Alison Wolf, quoted in the Vorderman Report, stated:

“If we want everyone to study maths successfully, both in the years after GCSE and as adults, then we need to develop an interesting and varied curriculum. This is especially, though by no means only, true for people who have found maths difficult or who were badly taught at school and so may well have failed GCSE.

That is one reason why, several years ago, I was involved in developing free-standing maths qualifications which cover important parts of the maths syllabus in a different way from the standard GCSE. Qualifications of this sort can be very effective in helping people to progress towards GCSE, and well beyond, and I hope they will be used more in the future. Unfortunately, in the period since they were first developed, we have actually gone backwards in terms of what is available for post-16 students. GCSE Mathematics for adults has vanished, even though it was highly successful, and recognised that a single approach cannot work for all age groups. I would be delighted to see it re-established.”

A mature GCSE Mathematics qualification should include collaborative work, coursework and the use of IT and should enable students to understand how useful and relevant mathematics is to their future, in both their personal and working lives. It is especially important for students who have not yet succeeded in mathematics to appreciate its uses, rather than thinking of mathematics as a list of unconnected techniques, crammed for an examination, which they have been unable to make sense of or remember.

Employers understand what GCSEs are. They are unlikely to be familiar with other qualifications, no matter how well devised those qualifications are; “league table equivalence does not necessarily translate into real equivalence” (The Wolf Report). Consequently, it is important that “GCSE” is included in the qualification title; this is illustrated, in Appendix 2, by the current status of iGCSE Mathematics.
This paper does not offer detailed recommendations for a post-16 GCSE specification. Significant work should be undertaken, in consultation with employers, to develop such a specification.

2. Level 3 Mathematics Post-16

ACME’s Mathematical Needs project reported that there is a need for far more young people to study mathematics beyond GCSE level.

“We estimate that of those entering higher education in any year, some 330,000 would benefit from recent experience of studying some mathematics (including statistics) at a level beyond GCSE, but fewer than 125,000 have done so.”

To meet this need, it is important that suitable courses are available for students who do not wish to take a full A Level in Mathematics, or for whom AS/A Level Mathematics is not appropriate.

Universities are sometimes reluctant to require mathematics qualifications for entrance to courses because they fear putting off prospective students; this leads to both an impression that mathematics is not needed for many degree courses and a need for remedial provision for students who arrive at university having done no mathematics since the age of 16. Lack of necessary mathematics skills in undergraduates is difficult and expensive to put right.

2.1 AS/A Level Mathematics

The current A Level in Mathematics is suitable not just for those who plan to study mathematics or highly mathematical subjects at university, but also for a very wide range of students who will need to use Level 3 mathematics in their future studies and careers. One of the strengths of the current AS/A Level Mathematics provision is that its modular nature, with the choice of different applied modules, allows flexibility, so that it can be tailored to students’ needs. It is essential that students and teachers understand the implications of possible alternative pathways through AS/A Level Mathematics, where different applied mathematics options enable students to study mathematics appropriate for different university courses and careers; this is even more so for those who take Further Mathematics. Online resources and teaching, as demonstrated by the Further Mathematics Support Programme, can allow students in small sixth forms a choice of units where this would otherwise be impossible.

It is very encouraging that numbers taking A Level Mathematics (and Further Mathematics) are now growing strongly. However, it is now appropriate for AS and A Level Mathematics to be developed further.

AS/A Level Mathematics has not changed significantly for many years. In higher education and industry considerable use is made of computer power to develop mathematical models, analyse data and solve mathematical problems; currently, this is not reflected in AS/A Level Mathematics. New specifications should be developed that incorporate significant use of computer technology, particularly into applied mathematics and statistics, but also into many aspects of ‘pure’ mathematics (this does not mean compromising mathematical rigour). This would help make the AS/A Level Mathematics more directly relevant to real world applications and would link directly with uses of mathematics in higher education and employment. It would also make the qualification more appealing to students.
Mathematics differs from most subjects in that students with grade C in GCSE Mathematics rarely succeed at AS/A Level, whereas in other subjects AS/A Level success is common for students with grade C at GCSE. Increasingly, schools and colleges will only accept students onto AS Mathematics if they have an A or A* at GCSE.

2.2 Level 3 Free Standing Mathematics Qualifications

AQA’s Level 3 Free Standing Mathematics Qualifications (FSMQs) were developed to enable students who do not take AS/A Level Mathematics, but who are following a path that requires mathematics beyond GCSE level, to study some Level 3 mathematics. They are designed to be accessible to students with a grade C or above at GCSE, and aim to support the mathematics needed in other disciplines. Each FSMQ counts as one third of an AS Level and it is possible to combine them into AS ‘Use of Mathematics’. An A Level ‘Use of Mathematics’ qualification is currently being piloted.

So far uptake of AQA’s Level 3 FSMQs has been low, so the qualifications have limited currency with higher education and employers. See Appendix 3 for recent statistics.

A potential problem with AS/A Level ‘Use of Mathematics’ qualifications is that they may be seen as a ‘soft’ way to achieve an AS/A Level in mathematics. It is important that teachers, students, higher education and employers properly understand the distinction between AS/A Level Mathematics and AS/A Level Use of Mathematics.

2.3 How can participation in Level 3 mathematics be increased?

To increase participation in Level 3 mathematics by up to 200 000 students per year, as suggested by the ACME report, different routes or pathways are needed for different types of students. Four different pathways could be available for Level 3 students, relating to their mathematical needs and aspirations. It is important that students should be able to transfer smoothly between these pathways.

1. **Students who hope to progress to employment or higher education which involves little direct use of mathematics**: These should take Level 3 mathematics qualifications equivalent in size and scope to one or two of the current AS Level Mathematics units or Free Standing Mathematics Qualifications.

2. **Students who hope to progress to employment or higher education which involves some direct application of mathematics**: These should take Level 3 mathematics qualifications equivalent in size and scope to the current AS Level Mathematics or Use of Mathematics.

3. **Students who hope to progress to employment or higher education in a STEM discipline with significant mathematical content**: These should take Level 3 mathematics qualifications equivalent in size and scope to the current A Level Mathematics and, possibly, AS Further Mathematics.

4. **Students who are very keen and mathematically able, who hope to progress to higher education or a career in a highly mathematical field**: These should take Level 3 mathematics qualifications equivalent in size and scope to the current A Level Mathematics and AS/A Level Further Mathematics, and a significant subset of them
should also take an enrichment qualification focussed on high-level mathematical problem solving, along the lines of STEP or the Advanced Extension Award.

New Level 3 mathematics qualifications need to be developed, building on the experience of the FSMQs. These qualifications should enable all students who have at least a grade C in GCSE Mathematics to study mathematics at Level 3. See appendix 1 for outline curriculum ideas.

To avoid the danger of some pathways being perceived as having low status, it is essential that students are able to move smoothly between them as their needs, plans and aspirations change.

As well as the development of new Level 3 mathematics qualifications, coordinated curriculum development across subjects is required. Mathematics and statistics at Level 3 should be embedded in context into other subjects at A Level. This can help students to see the relevance of mathematics and help to motivate their learning.

Universities and employers must emphasise the need for students to keep studying mathematics and this must be communicated effectively to students, teachers, careers officers, school managers and parents.

To ensure that wider provision and increased participation in Level 3 mathematics is manageable for smaller sixth forms, there should not be a large number of new post-16 courses in mathematics. A common core of techniques, applied in different contexts to reflect different pathways, would seem a practical solution. This is explored further in appendix 1.

2.4 Extended Project Qualifications

In addition to a separate, specific assessment of Level 3 mathematics, it would be possible to extend the amount of mathematics done by students post-16 through the Extended Project Qualification (EPQ). The number of students taking the EPQ is growing rapidly and universities are now noticing its value and using it when selecting students. A modified EPQ could be developed to include explicitly the use of quantitative techniques. This would enable students to demonstrate the application of mathematics/statistics in context.

3. Next Steps

To ensure that the number of students studying appropriate mathematics post-16 increases dramatically, a considerable investment is required to design appropriate courses, develop teaching and learning resources, promote the courses effectively and provide suitable CPD for teachers. This investment is essential to future national well-being.

This paper does not deal with the specific detail of new courses but consideration of such detail will need to take place and MEI hopes to be in the forefront of such work.
Appendix 1

What mathematics should be included in new Level 3 qualifications suitable for a wide range of learners?

This appendix outlines broad ideas and principles. It is not intended to be a fully detailed model.

These new Level 3 qualifications should be suitable for students following the proposed pathway 1, but it should be possible for them to count towards any of the four proposed Level 3 mathematics pathways, maximising flexibility. It is not intended to specify a curriculum here; considerable specialised work and consultation will be required to do that. However, the following themes would seem important, and should be linked directly to applications wherever possible:

A. An appreciation of statistical inference and risk, including a basic understanding of the Normal distribution, statistical significance and conditional probability; this could include the use of statistical software to analyse real data sets.

B. Basic calculus and topics from operational research and numerical methods, developing a basic understanding of the importance of rates of change and the use of technology when solving quantitative problems.

C. An introduction to mathematical modelling, showing how mathematics is applied to solve real problems, analyse real situations, inform decisions and plan complex projects.

Examples of applications might include:
- analysis of data in sciences and social sciences; finance; medical screening; market research; quality management and control in manufacturing; insurance and risk; population growth; the planning of inspection and delivery systems; internet security; pollution control; planning complex events (e.g. the Olympic Games); computer simulation (e.g. to model traffic flows); computer graphics; internet search engines; mobile phones.

If the curriculum were built around a relatively small core of mathematical content from themes A and B, theme C could focus on different applications depending on the interests and aspirations of different groups of students (e.g. science, social science, engineering, humanities). However, the need to engage students must be balanced with the need to make courses manageable. Considerable investment in appropriate resources would be needed to make this work.

The detailed curriculum should build on work already carried out in the development of the current AS/A Level Mathematics specifications and the Level 3 FSMQs and should involve consultation with higher education and employers, to ensure the mathematics which is most useful and relevant to their needs is included.

The courses would build on the Level 2 mathematics from GCSE and would reinforce and develop students’ skills in elementary arithmetic (e.g. ratio & percentages) and algebra (e.g. working with formulae), which employers and universities feel are often lacking in students who have achieved a pass at grade C or above in GCSE Mathematics.
Course structure and resources

The courses could be modular with modules of similar size to current AS Level modules (60 guided learning hours each), with all students expected to do at least one module in each of years 12 and 13. (Students who achieve Level 2 at the end of year 12 would do at least one module in year 13). This would need 1.5 – 2 hours of timetabled classes per week for all students following pathway 1 and students following the other pathways spending correspondingly more time studying mathematics.

It would be appropriate to offer additional optional elements for some students; for example it is likely to benefit those specialising in social sciences or business to have the option of taking an additional Level 3 statistics unit.

Delivery of the courses must be workable across the wide range of post-16 providers, from small school sixth forms to very large colleges.

Specialised resources to support the teaching and learning of the courses could be developed and made available online. These resources could include some self-study elements, which would enhance the students’ learning experience and encourage the development of independent learning skills.

Who would teach the courses?

Working with specialised teaching and learning resources, the courses could be taught by a range of teachers with relatively strong numerical backgrounds; this could include science teachers and numerate social science teachers, as well as mathematics teachers, though there would be a substantial need for CPD.

How should the courses be assessed?

Alongside more traditional means of assessment, some elements could be assessed through online tests throughout the course.
Appendix 2

The importance of qualification titles

The importance of having ‘GCSE’ in the qualification title is illustrated by the current position of iGCSE Mathematics. The Mathematics iGCSE currently counts as an equivalent to GCSE Mathematics for school performance tables. However, this will no longer be allowed after 2013, when state schools are expected to move to the regulated qualification, which is called the ‘Level1-Level 2 Certificate in Mathematics’. The awarding bodies say that the content and assessment for this qualification are the same as for the iGCSE, however the title is different. This will not be obvious to employers or universities and so is likely to disadvantage state educated students. Independent schools are not driven by their position in school performance tables and, consequently, can continue to use the familiar and understood ‘GCSE Mathematics’ title. This goes directly against the spirit of press notice id 2010/0089 on the DfE website which quotes Nick Gibb, Minister for Schools, as follows.

“For too long, children in state-maintained schools have been unfairly denied the right to study for qualifications like the iGCSE, which has only served to widen the already vast divide between state and independent schools in this country.

By removing the red tape, state school pupils will have the opportunity to leave school with the same set of qualifications as their peers from the top private schools – allowing them to better compete for university places and for the best jobs.”
Appendix 3

**Number of students taking FSMQs in the Use of Mathematics suite**

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<tr>
<th>AQA Level 3 FSMQs available to all centres June 2011 entry</th>
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<td>FSMQ Algebraic and Graphical Techniques</td>
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<tr>
<td>FSMQ Modelling with Calculus</td>
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</tr>
<tr>
<td>FSMQ Using and Applying Decision Maths</td>
<td>208</td>
</tr>
<tr>
<td>FSMQ Using and Applying Statistics</td>
<td>540</td>
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<td>AS Use of Mathematics (based on FSMQs)</td>
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<th>Pilot AQA Level 3 FSMQs June 2011 entry</th>
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