Mathematics of Measurement Systems Analysis

Introduction

Product quality is crucial in manufacturing, and industries have many quality control processes in place to ensure that products for their customers will perform correctly. A critical part of quality control is dimensional measurement: making sure things are the right shape and size for reasons of safety, lifespan and performance.

Many systems are used to measure components in today’s industries, and these are also subject to their own type of quality control – measurement systems analysis (MSA). This is to ensure that when you measure:

A) The measurement system is accurate
B) The measurement system is repeatable
C) The measurement system is reproducible
( I’ll explain the difference between repeatable and reproducible later…)

There is no point in using a measurement system that gives you five incorrect results from measuring the same feature five times, or gives you different results depending on who is operating it. If you are familiar with high school science equipment, then you will probably already appreciate this.

In this document, I will introduce and explain some of the mathematics behind MSA, which has fundamental roots in GCSE and A-Level statistics. All this can be done on computer programs (notably Minitab Statistical Software), but I find that a manual appreciation of each statistic is vital for anyone deciding to use them.

Gauge Bias

Bias is the term for how far away the measurement system is from the true value. It can be documented in many different ways, but usually a percentage of the tolerance of the feature is preferable. The equation used is simply:

\[
\text{Bias} \% = 100 \times \left| \frac{\text{Mean of Measured Values} - \text{Reference Measurement}}{\text{Feature Tolerance}} \right|
\]

The Reference Measurement is the value generated by a measurement system known to have measured the feature acceptably. The Feature Tolerance is the total tolerance (upper and lower bounds) of the feature in question.

For example, if a specific length on a cast metal component had a tolerance of ±0.05mm, a mean measured value of 23.474mm and a reference value of 23.456mm, then the bias would be \((100 \times (23.474 - 23.456) / 0.1) = 18\%\) of the tolerance. Different industries will have different acceptance levels for bias, but at Rolls-Royce, we ideally require bias to be ≤ 10% of the feature tolerance.

Type 1 Gauge Study

A Type 1 study is the first formal investigation into the repeatability of the measurement system: how well the system can measure the same feature again and again. Variation is natural in the dimensions of things – after all, at an atomic level nothing stands still – but a good measurement system will be able to give close values when measuring the same feature multiple times.

To carry out a Type 1 study, a single operator must measure a single component’s features many times over and collect the results. Typically the component should be measured at least 25 times to ensure a good range of data is available which accurately represents how the measurement system copes with the component.
Key statistics for understanding a system’s repeatability are Cg values. Cg gives a picture of how well the range of the measurements (the study variation) fits into a desired percentage of the Feature Tolerance, and can be calculated as follows:

$$C_g = \frac{(P \div 100) \times Feature\ Tolerance}{n \times \sigma}$$

- **P** = the percentage of the Feature Tolerance you wish to assess
- **n** = the number of standard deviations you wish to include (default is 6)
- **σ** = the standard deviation, though hopefully you knew that

Remember, measured dimensions are continuous data, and tend to be normally distributed, hence why we use 6 standard deviations as a default (it covers 99.73%) of a normal distribution. Also 20 is typically decided as the percentage of Feature Tolerance to assess, however sometimes 10 is used on critical components.

The second key statistic is CgK, which is like Cg, except it incorporates any bias in the system. It can be calculated as follows:

$$C_{gK} = \frac{(P \div 200) \times Feature\ Tolerance - |Mean\ of\ Measured\ Values - Reference\ Measurement|}{n \times \sigma}$$

*Note how the equation for CgK is made up of the equations for Cg and bias*

Here, **n** = the number of standard deviations to represent half of the process spread (i.e. 3)

For example, if we take the same cast metal component from the bias studies before, and wanted to investigate it within 20% of its tolerance, and the standard deviation was 0.01, its Cg would be $20/100 \times 0.01 = 0.002$ and its CgK would be $\frac{(20/200) \times 0.01 - |23.474 - 23.456|}{3 \times 0.01} = 0.267$

The two statistics of Cg and CgK should be 1.33 or greater. A value of 1.33 indicated that the study variation fits into 75% of the tolerance band chosen (P). The larger the number, the smaller the percentage the study variance fits into, and hence the better the measurement system. Anything less than 1 is generally deemed unacceptable, as this indicated that the study variation does not fit within the specified tolerance band.

Cg will give you a value of 0 or greater, but CgK can give negative values which represent the bias: a small, negative value (i.e. -1.33 > X > 0) indicates that the measurement system is unrepeatable, and biased, whereas a large negative value (i.e. > -1.33) indicates that the measurement system is repeatable but biased.

**What next?**

Many of you will have figured that there is a high chance that a Type 1 study has some sort of Type 2 study. This is indeed correct, and also where we begin to introduce reproducibility. Reproducibility is simply a measure of how the measurement system will perform if we change the person operating it: a very reproducible system will be hardly affected by the operator. In fact, the Type 2 study is often referred to as a Full Gauge R&R Study (the R&R referring to Repeatability and Reproducibility).

The mathematics for this, while revolving around similar statistical principles, will not fit onto two sides of A4 however. There is also the whole discipline of measurement uncertainty that ties in as a part of MSA, though you could write a whole PhD dissertation on this... I won’t make you.

I hope this brief introduction to MSA mathematics has been informative, and has provided some much-needed context to the work you do at school - Work which, unfortunately, has quite a misleading habit of feeling very abstract. Believe me, it isn’t.