Solutions for AEA Section 1 – Constructing a Concise Argument

June 2006

1 (a) $(1-y)^{-2} = 1 + 2y + 3y^2 + 4y^3 + \dots$ B1 (1)

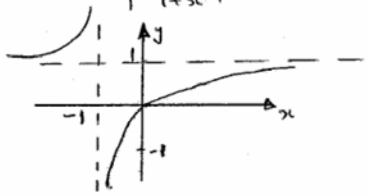
(b) $S = 1 + 2\left(\frac{x}{1+x}\right) + 3\left(\frac{x}{1+x}\right)^2 + \dots$ Identify $y = \frac{x}{x+1}$ M1

$\Rightarrow S = 1 + 2y + 3y^2 + \dots$ A1

$= \left(1 - \frac{x}{1+x}\right)^{-2}$

$= \underline{(1+x)^{+2}}$, so $\underline{a = 1,\ n = 2}$ M1, A1 (4)

(c) Need $\left|\frac{x}{1+x}\right| < 1$ correct condition B1



Critical value is $\frac{x}{x+1} = -1$ M1

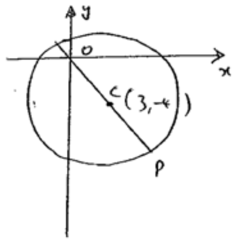
$\Rightarrow x = -\frac{1}{2}$

$\therefore \underline{x > -\frac{1}{2}}$ A1 (3)

July 2005

1 $(x-3)^2 + (y+4)^2 = 24 + 9 + 16 = 49$ M1

Curve is circle, centre $(3, -4)$, radius 7 A1



$OC = \sqrt{3^2 + 4^2} = 5$ M1

Greatest length OP = $5 + r$ (or least) M1

$= \underline{12}$ A1

Least length $= r - 5 = \underline{\underline{2}}$ A1 (6)

June 2003

6 (a) $(\text{LHS})^2 = (2 + \sqrt{3}) - 2\sqrt{(2+\sqrt{3})(2-\sqrt{3})} + (2 - \sqrt{3})$

$= 4 - 2\sqrt{4-3}$

$= 2 = (\text{RHS})^2$

[$2 + \sqrt{3} > 2 - \sqrt{3}$ we know LHS > 0]

(b) $\log_{\frac{1}{8}} \sqrt{2} = \dfrac{\log_2 \sqrt{2}}{\log_2 \frac{1}{8}}, = \dfrac{\log_2 2^{\frac{1}{2}}}{\log_2 2^{-3}}, = \dfrac{\frac{1}{2}}{-3} = -\dfrac{1}{6}$ *

(c) $\sqrt{a+\sqrt{15}} - \sqrt{a-\sqrt{15}} = \left(\frac{1}{n}\right)^{-\frac{1}{2}} = \sqrt{n}$

Squaring $\Rightarrow a + \sqrt{15} - 2\sqrt{a^2 - 15} + a - \sqrt{15} = n$

i.e. $\quad 2(a - \sqrt{a^2 - 15}) = n$

So we require $\quad a^2 - 15 = b^2$

$a^2 - b^2 = 15$

i.e $\quad (a - b)(a + b) = 15$

Or $\quad 15 = 3 \times 5$ or 1×15

But $3 \times 5 \Rightarrow \quad a - b = 3 \quad \therefore a = 4$

$a + b = 5$

$1 \times 15 \Rightarrow \quad a - b = 13 \quad \therefore a = 8$

$a + b = 15$

$a = 4, \quad n = 2(4 - \sqrt{4^2 - 15}) = 6$

$a = 8, \quad n = 2(8 - \sqrt{64 - 15}) = 2 \quad$ Pairs are $a = 4, n = 6$

$a = 8, n = 2$