

## Exam Advice

You will find helpful advice about common errors in the Examiner's Reports. Some specific examples are dealt with here.

### All modules

The OCR 'Report on the Units taken in June 2006' contains a statement of the rules examiners use when they see replaced or crossed out work. Our advice is that you should show this statement to your students. The final paragraph reads as follows.

*"If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others."*

A few candidates adopt a policy that, when in doubt between two alternative answers to a question, the best course of action is to give both, on the assumption that the examiner will select the better one. The wording above makes it clear that this is not the case; such a candidate will lose marks unless the better answer happens to be the last one.

This is a quite different situation from a candidate being genuinely unhappy with an answer and replacing it, whether or not the earlier work is crossed out. These eventualities are covered in two earlier paragraphs of the OCR statement.

*"If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests."*

*"If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out."*

It is normal practice to mark work that is crossed out and not replaced, assuming that the crossing out has not been so thorough as to make it illegible. The use of Tippex in examinations can be an expensive way of losing marks.

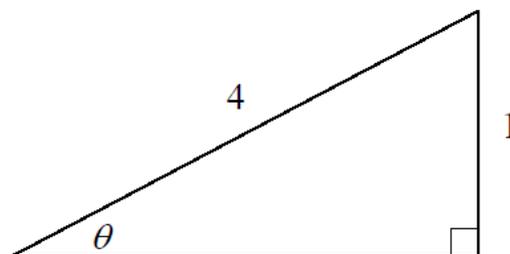
### Use of calculator 1

Many candidates are losing marks because they do not understand the meaning of the word exact. An example of this occurred in the Question 3 of the June 2006 C2 paper.

$\theta$  is an acute angle and  $\sin \theta = \frac{1}{4}$ . Find the exact value of  $\tan \theta$ .

Candidates were expected to draw a right-angled triangle, with hypotenuse 4 units and the side opposite the angle  $\theta$  1 unit, and then use Pythagoras' theorem to show that the length of the third (adjacent) side is

$\sqrt{15}$ . So the exact value of  $\tan \theta$  is  $\frac{1}{\sqrt{15}}$ .



Many candidates, however, used their calculators to find  $\arcsin(0.25)$  and then used them again to find  $\tan$  of the answer. Candidates who did this commonly gave their answers as 0.258 or 0.2581988897, neither of which is the exact answer which was asked for in the question.

The Examiner's Report says "It must be made clear to candidates, as part of the preparation for this paper, that a request for 'the exact value' of anything usually implies that calculators must not be used." The same advice is given on page 32 of the Students' Handbook.

### Use of calculator 2

The September 2006 Newsletter included a reminder that questions which ask for an **exact** value should be done without a calculator. Calculators are allowed in all A Level units other than C1, but candidates do lose marks in all units if they fail to show sufficient details of their working.

For example, a question in FP2 might ask candidates:

$$\text{Show that } \int_{-\ln 2}^{\ln 2} \cosh 2x \, dx = \frac{15}{18}$$

Having done the first step correctly  $\int_{-\ln 2}^{\ln 2} \cosh 2x \, dx = \left[ \frac{1}{2} \sinh 2x \right]_{-\ln 2}^{\ln 2}$  and got full credit for it,

candidates might turn to their calculators and complete the question thus:

$$\left[ \frac{1}{2} \sinh 2x \right]_{-\ln 2}^{\ln 2} = \frac{1}{2} \sinh(2 \ln 2) - \frac{1}{2} \sinh(-2 \ln 2) = \frac{15}{8}$$

Typing  $\frac{1}{2} \sinh(2 \ln 2) - \frac{1}{2} \sinh(-2 \ln 2)$  into many scientific calculators does give the exact

answer  $\frac{15}{8}$  but no credit can be given for this part of the question because the answer was

printed and candidates could just have copied it down.

Candidates are expected to show working, for example:

$$\sinh(2 \ln 2) = \frac{e^{2 \ln 2} - e^{-2 \ln 2}}{2} = \frac{e^{\ln 4} - e^{-\ln 4}}{2} = \frac{1}{2} \left( 4 - \frac{1}{4} \right) = \frac{15}{8} \quad \sinh(-2 \ln 2) = -\frac{15}{8}$$

$$\text{So } \left[ \frac{1}{2} \sinh 2x \right]_{-\ln 2}^{\ln 2} = \frac{1}{2} \left( \frac{15}{8} \right) - \frac{1}{2} \left( -\frac{15}{8} \right) = \frac{15}{8}.$$

The examiners' reports for the hyperbolic functions on FP2 show students often drop marks on this kind of question because they do not show sufficient working.

## Pure Modules

### Asymptotes in MEI Structured Mathematics

The word "asymptote" is not included in the national Subject Core for AS and A Level Mathematics and testing of asymptotes is excluded from C1. However, students are likely to have seen asymptotes already, for example, through sketching the curve  $y = \frac{1}{x}$  in GCSE.

In C2 students meet asymptotes when they draw graph of  $y = \tan \theta$ . This curve, together with transformations of it, is explicitly in the syllabus. So it is likely that, by now, students will have met the word "asymptote", at least in the context of vertical asymptotes, since teachers will have used the word when describing these curves.

When doing C3 coursework, students are expected to know that failure of a change of sign method can occur for solving  $f(x) = 0$  when  $y = f(x)$  has a vertical asymptote, as stated in Note C3e2 on page 61 of the specification.

In examination questions in C3 and C4, for example June 2007 C3 question 7, the correct technical term “asymptote” is used to avoid a lengthy explanation. Teachers are advised to look carefully at the style used in previous questions on these papers and to treat them as establishing precedent. Considerable care was taken when setting such questions to make sure that the meaning was clear from the accompanying text and/or diagrams and this will always be the case when the word “asymptote” is used on the C3 and C4 papers.

In FP1 vertical and horizontal asymptotes are explicitly in the syllabus (Note FP1C2 on page 75 of the specification) and are routinely tested in examination questions. Oblique asymptotes are first met in FP2 (Note FP2C4 on page 91).

### FP1

Students often drop marks unnecessarily in proof by induction questions by giving unstructured proofs and/or missing out vital steps. In FP1 these questions will always relate to proving formulae for the sums of series or for the  $n$ th term of a sequence. Below is an example question (adapted from a question in the FP1 textbook), together with a mark scheme. This is intended to highlight how the marks are allocated.

**Note that failure to state explicitly that the conjecture is assumed true for  $n = k$ , which is a fundamental logical step in a proof by induction, will cost a minimum of 3 marks, even if everything else is correct.**

#### Example question

A sequence of integers  $u_1, u_2, u_3, \dots, u_n$  is defined by

$$u_1 = 5 \text{ and } u_{n+1} = 3u_n - 2^n \text{ for } n \geq 1.$$

Prove by induction that  $u_n = 2^n + 3^n$  for all positive integers  $n$ .

#### Mark scheme

	Mark	Notes
$u_1 = 5$ and when $n = 1$ , $u_n = 2^n + 3^n = 5$ , so conjecture true for $n = 1$	B1	Clear demonstration for the case $n = 1$
Assume true for $n = k$ , so that $u_k = 2^k + 3^k$	E1	Assume true for $n = k$
$\Rightarrow u_{k+1} = 3(2^k + 3^k) - 2^k$	M1 A1	Use of inductive definition to find $u_{k+1}$ Correct substitution
$= 3 \times 2^k + 3^{k+1} - 2^k$ $= 2^{k+1} + 3^{k+1}$	A1	Correct simplification (correct answer only)
But this is the given result with $k + 1$ replacing $k$ . Therefore if it is true for $k$ , it is true for $k + 1$ .	E1	Dependent on previous E1 and immediately previous A1
Since it is true for $k = 1$ , it is true for $k = 1, 2, 3, \dots$ and so it is true for all positive integers.	E1	Dependent on B1 and both previous E marks

At the simplification stage, some teachers encourage the method of setting up a ‘target’ to aim for. This is perfectly acceptable and is marked equivalently.

## S2 and Z3

### Null hypothesis for correlation questions

For tests using Spearman's rank correlation coefficient, the null hypothesis should have the form:

$H_0$ : There is no association between the variables in the underlying population

There is no universally agreed way of writing this algebraically so the null hypothesis must be given in words.

There will be suitable corrections in the next reprint of the MEI S2 text book.

For tests using Pearson's product moment correlation coefficient, the null hypothesis can be written as:

$H_0$ :  $\rho = 0$ , where  $\rho$  is the population correlation coefficient.

Candidates should say what  $\rho$  stands for, emphasising that it refers to the underlying population. They should say what any letter they introduce into a question stands for but a mark is often given specifically for this in the case of hypothesis tests.

### t tests in S3

In the specification for S3, the notes about  $t$  tests outline when it is appropriate to use such a test for a mean: *"in situations where the sample is small and the population variance is unknown, but the population may be assumed to have a Normal distribution."*

Answers to some of the questions that students may ask about this are given below.

#### ***How small does the sample need to be use a t test?***

There is no exact answer, it depends on the situation but a reasonable rule of thumb is that samples of fewer than about 30 are small whereas samples of over 30 are large.

#### ***What if the sample is large?***

Here is no exact answer, it depends on the situation but a reasonable rule of thumb is that samples of fewer than about 30 are small whereas samples of over 30 are large.

#### ***What if the population variance is known?***

When the population is Normal and the variance is known, a Normal test should be used, whether the sample is large or small. Students learnt this in S2.

#### ***What if the population does not have a Normal distribution?***

If the sample size is large then the Central Limit Theorem implies that the sample mean will be approximately Normally distributed. The Normal test can be used, either with known or estimated variance.

#### ***How do we know whether the population has a Normal distribution?***

There are techniques for testing whether the underlying population is Normal. For example, using Normal probability paper or using a Kolmogorov-Smirnov test or a  $\chi^2$  goodness of fit test. Only the  $\chi^2$  goodness of fit test is in the S3 Syllabus. However, candidates may not have the data needed to use this test in an examination question. So, when candidates need

to decide what test to use, the examiners will indicate whether the variable can be modelled by a Normal distribution.

***Is it OK to use a  $t$  test if the sample is large, the variance is unknown and the population can be modelled by a Normal distribution?***

If the population is not exactly Normal (and, in real life, it rarely is), then it is better to use a Normal test for a large sample than a  $t$  test. For that reason, candidates should always use Normal tests for large samples, even though the tables available in examinations give percentage points of the  $t$  distribution for  $n = 50$  and  $n = 100$  as can be seen from the specification (S3i7 page 163), the same rules apply to confidence intervals.