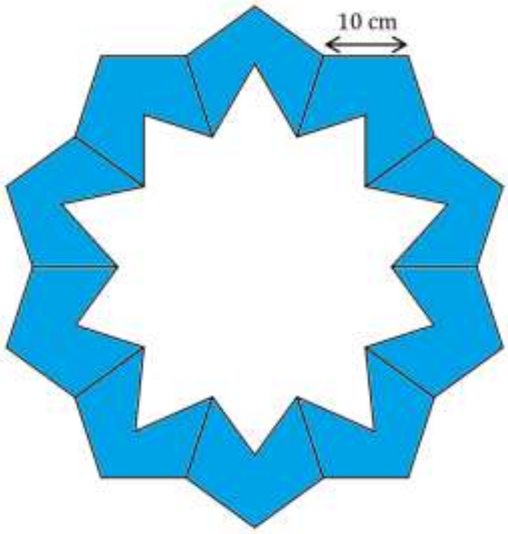
Mathematical Problem Solving
GCSE example

Solution to example 2



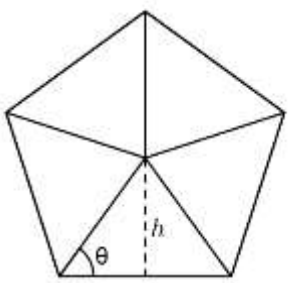
Find the shaded area.

The whole design is made from two different units:

A regular pentagon with $\frac{1}{5}$ removed

A regular pentagon with an equilateral triangle removed

To find the area of a complete regular pentagon:

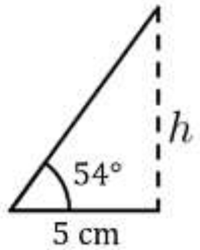


The pentagon is divided into five identical isosceles triangles.

Each triangle has one angle that is $360° \div 5 = 72°$ and two equal angles (θ in the diagram).

$$\theta = \frac{180 - 72}{2} = 54°$$

We can find the height h of the isosceles triangle by considering the right angled triangle



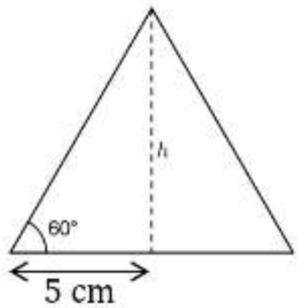
$h = 5\tan 54° = 6.88 \text{ cm} \ \ (3\ s.f.)$

The area of one of the isosceles triangles is therefore $5 \times 6.881 \ldots . = 34.4 \text{ cm}^2$

The area of a pentagon with $\frac{1}{5}$ removed is therefore $4 \times 34.4095 \ldots = 137.638192 \ldots = 138 \text{ cm}^2\ (3\ s.f.)$

The area of a complete pentagon is $5 \times 34.4095 \ldots . = 172.04774 \ldots = 172 \text{ cm}^2\ (3\ s.f.)$

The equilateral triangle that has been removed looks like this:



The area can be found quickly if the students know the formula $\frac{1}{2}ab\sin C$

Area $= \frac{1}{2} \times 10 \times 10 \times \sin 60° = 43.301270 \ldots = 43.3 \text{ cm}^2\ (3\ s.f.)$

If the students don't know that formula, a two stage process can be used:

$h = 5\tan 60° = 8.66025 \ldots = 8.66 \text{ cm}\ (3\ s.f.)$

Area $= 5 \times 8.66025 \ldots = 43.301270 \ldots = 43.3 \text{ cm}^2\ (3\ s.f.)$

The area of a pentagon with an equilateral triangle removed is

$172.04774 \ldots - 43.301270 \ldots = 128.746469 \ldots = 129\ cm^2\ (3\ s.f.)$

There are five of each type so the total area is

$5(137.638192 \ldots + 128.746469 \ldots) = 1331.92 \ldots = 1330 \text{ cm}^2\ (3\ s.f.)$