Solution to example 20

In this question you must show detailed reasoning.

Fig. 1 shows the circle with equation $(x - 3)^2 + (y - 4)^2 = 25$. The point C is the centre of the circle.

Fig 1
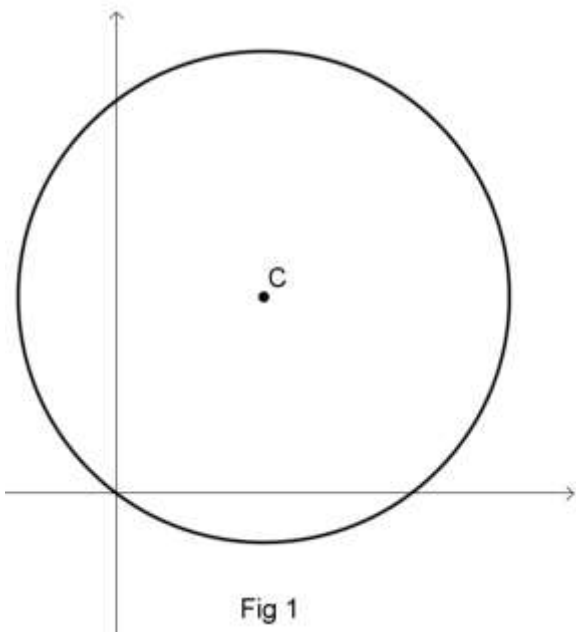
Find the area of the segment of the circle that is below the x-axis.

This problem requires students to find the associated sector of the circle first.

A sensible first step is to find out where the circle crosses the x axis.

One point is clearly $(0,0)$ from the diagram.

To find the second point symmetry can be used. The centre of the circle is $(3,4)$ so the circle has reflection symmetry in the line $x = 3$. The other x intercept is therefore at $(6,0)$.
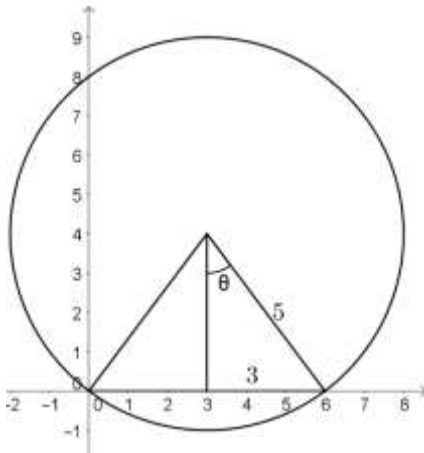
This can be confirmed by the following calculation

For $(x - 3)^2 + (y - 4)^2 = 25$ when $y = 0$, $(x - 3)^2 = 25 - 16$

$(x - 3)^2 = 9$

$x - 3 = \pm 3$

$x = 0$ or $x = 6$

Joining these two points up to the centre and drawing a perpendicular line along $x = 3$ reveals two $3, 4, 5$ triangles (since the radius of the circle is 5).

The angle $\theta = \sin^{-1} \frac{3}{5}$

The area of the whole sector shown $= 2 \times \frac{1}{2} \times 5^2 \times \theta = 25\theta = 16.0875 \dots$

The area of the whole triangle shown $= \frac{1}{2} \times 6 \times 4 = 12$

The area of the circle below the x axis is therefore $16.0875 - 12 = 4.09$ sq units to 3 s.f.