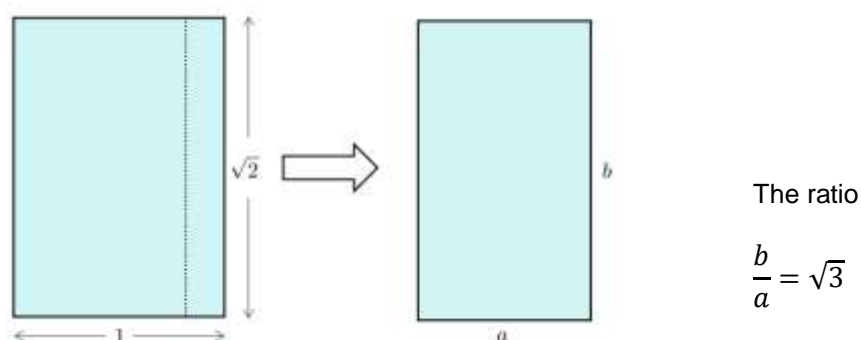# Mathematical Problem Solving
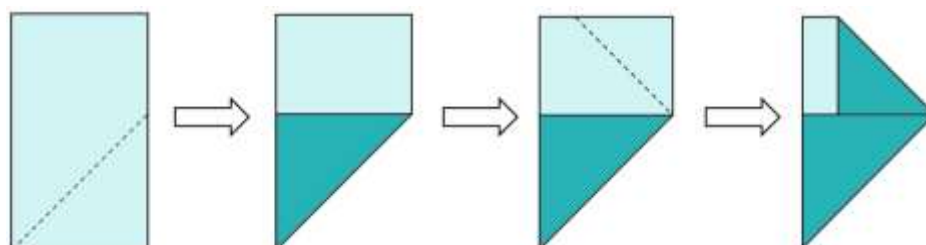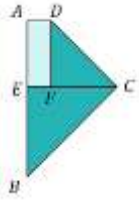## AS/A Level example

Solution to example 3

A piece of paper measures 1 unit by $\sqrt{2}$ units. A strip of paper is removed so that the dimensions of the paper are a and b as shown in the diagram below:

The ratio

$$\frac{b}{a} = \sqrt{3}$$

The piece of paper is now folded like this:

Find the perimeter and area of the final shape.

$BE = \frac{\sqrt{2}}{\sqrt{3}}$, $CE = \frac{\sqrt{2}}{\sqrt{3}}$ (45° triangle)

Since BEC is a 45° right-angled triangle, $BC = BE\sqrt{2}$

$BC = \frac{\sqrt{2}}{\sqrt{3}} \cdot \sqrt{2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$CF = DF = AE = \sqrt{2} - \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{18}-\sqrt{6}}{3} = \frac{3\sqrt{2}-\sqrt{6}}{3}$

$CD = DF\sqrt{2}$ (another 45° triangle)

$CD = \frac{3\sqrt{2}-\sqrt{6}}{3} \cdot \sqrt{2} = \frac{6-\sqrt{12}}{3} = \frac{6-2\sqrt{3}}{3}$

$AD = \frac{\sqrt{2}}{\sqrt{3}} - \frac{3\sqrt{2}-\sqrt{6}}{3} = \frac{3\sqrt{2}-3\sqrt{6}+\sqrt{18}}{3\sqrt{3}} = \frac{3\sqrt{2}-3\sqrt{6}+3\sqrt{2}}{3\sqrt{3}} = \frac{6\sqrt{2}-3\sqrt{6}}{3\sqrt{3}} = \frac{2\sqrt{2}-\sqrt{6}}{\sqrt{3}} = \frac{2\sqrt{6}-\sqrt{18}}{3} = \frac{2\sqrt{6}-3\sqrt{2}}{3}$

Perimeter $= AB + BC + CD + AD = \sqrt{2} + \frac{2\sqrt{3}}{3} + \frac{6-2\sqrt{3}}{3} + \frac{2\sqrt{6}-3\sqrt{2}}{3} = \frac{3\sqrt{2}+2\sqrt{3}+6-2\sqrt{3}+2\sqrt{6}-3\sqrt{2}}{3} = \frac{6+2\sqrt{6}}{3}$

Area $=$ rectangle $-$ two 45° triangles $= \frac{\sqrt{2}}{\sqrt{3}} \cdot \sqrt{2} - \frac{1}{2}\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 - \frac{1}{2}\left(\frac{3\sqrt{2}-\sqrt{6}}{3}\right)^2 = \frac{2}{\sqrt{3}} - \frac{2}{6} - \frac{18+6-6\sqrt{12}}{18}$

$= \frac{2\sqrt{3}}{3} - \frac{1}{3} - \frac{24-12\sqrt{3}}{18} = \frac{2\sqrt{3}}{3} - \frac{1}{3} - \frac{4-2\sqrt{3}}{3} = \frac{2\sqrt{3}-1-4+2\sqrt{3}}{3} = \frac{4\sqrt{3}-5}{3}$