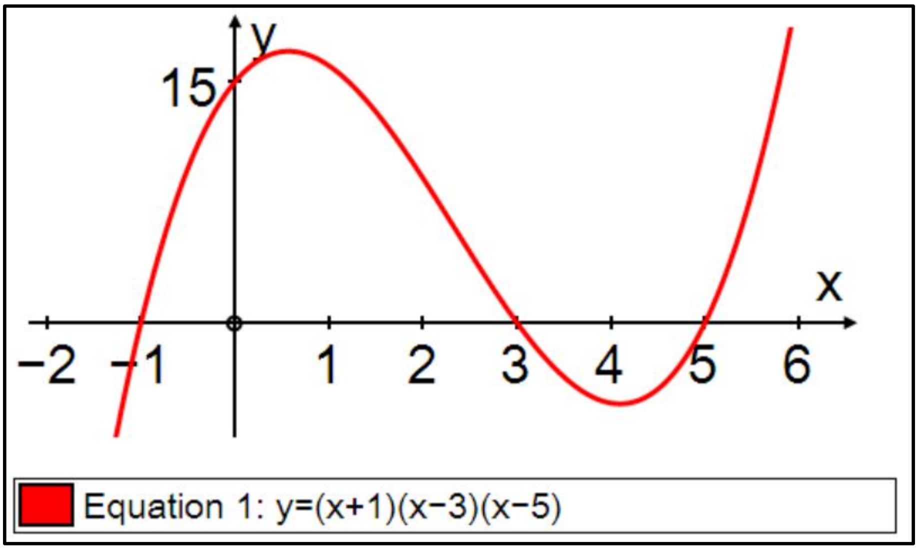
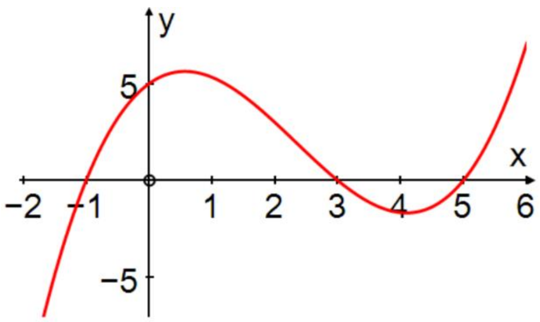
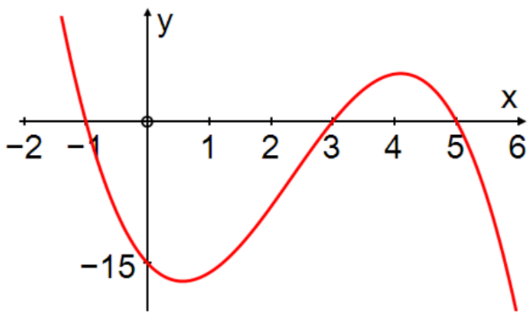
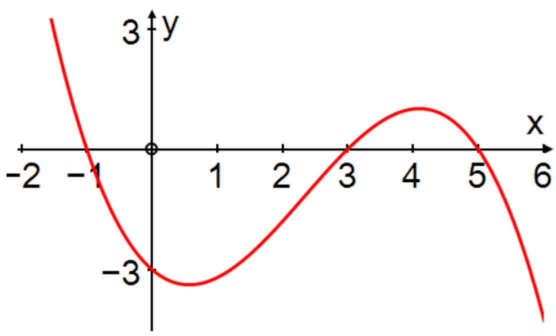
Equations of cubic curves

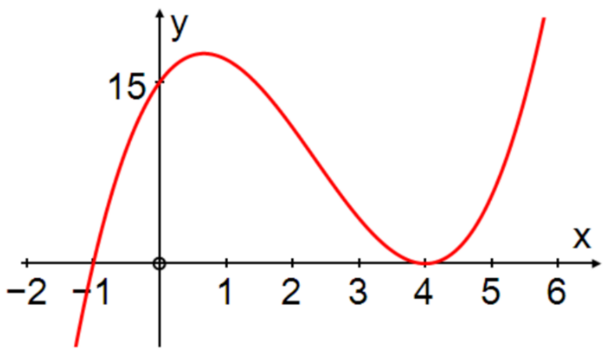


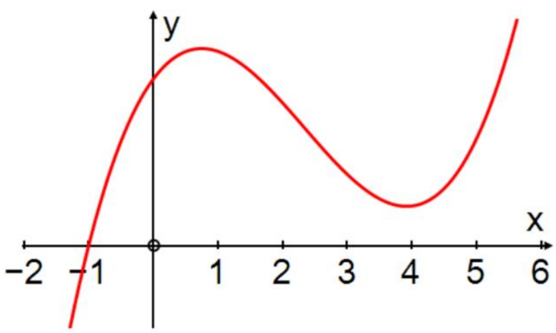
Given the information above, can you find the equations of these six graphs?

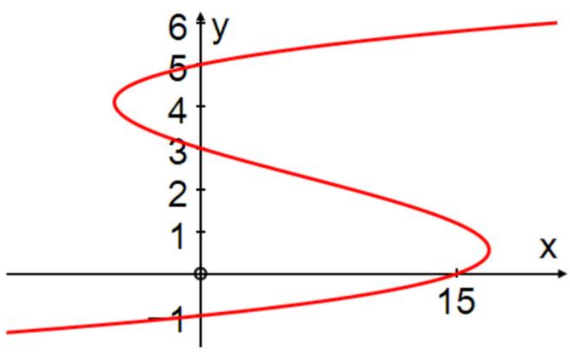












Equations of cubic curves

This activity is designed to help students make the link between the factorised form of a cubic and its graph. In particular, it helps to draw attention to the axes intercepts.

The activity is designed such that the relevant equations should be found with reference to the original graph and equation given at the top of the sheet.

The final two are tricky and it may well be worth letting students ponder this in between lessons!

The matching equations are as follows:

$$y=\frac{1}{3}(x+1)(x-3)(x-5) \qquad y=-(x+1)(x-3)(x-5)$$

$$y=-\frac{1}{5}(x+1)(x-3)(x-5) \qquad y=\frac{15}{16}(x+1)(x-4)^2$$

$$y=(x+1)\big((x-3)(x-5)+c\big) \quad c>1 \qquad x=(y+1)(y-3)(y-5)$$

Notes:

The first three encourage students to think about scalar multiples of the original function retaining the x-axis intercepts while varying the y-intercept. Also the general shape of positive and negative cubics are highlighted.

The fourth graph addresses the issue of the y-intercept remaining the same as in the original graph while (some) things change on the x-axis.

The fifth is posed in such a way as to allow a range of functions which satisfy the graph.
One approach would be to use the original function and adapt as shown above.
Once $c>1$ the graph only intersects the x-axis at one point, which must be $x=-1$.
However, as c increases the shape of the graph changes and an interesting debate could be to see which range of values for c students would allow. For example, look at the graph that is produced when $c=7$.
Please note that the nature of this question allows for a wide range of acceptable solutions. The suggested answer above adapts the original function and it will be up to you as to whether you insist on that! The question does give the opportunity for discussion with students explaining how they have tackled the problem.
Note that a key feature of any answer is that it must be of the form $y=(x+1)\left(ax^2+bx+c\right)$ where the quadratic term has no real roots, in order to avoid creating additional real roots of the cubic.

The sixth graph tackles the issue of expressing something as a function of y.