Polynomials (AS)

B6	Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem

Commentary

Addition and subtraction of polynomials is straightforward but there are opportunities for mathematical thinking. For example; which two polynomials have a sum of x^3+3x^2-x-1 and a difference of x^3+x^2-5x+9? How would you add $x^2(x+2)$ and $(1-x)(x+1)(x+2)$ (and what has happened here at a graphical level)? Give me two polynomials which meet on the x-axis and add to give x^3-2x+4.

Students will be familiar with multiplication of two brackets. Building on this can help with polynomial division. For example, after expanding $(x^3-2x^2+x-4)(2x+3)=2x^4-x^3-4x^2-5x-12$ ask students how they would have found $(2x^4-x^3-4x^2-5x-12)\div(2x+3)$ without seeing this expansion.

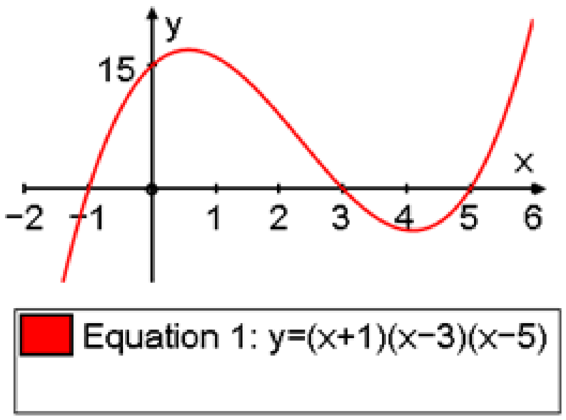
The factor theorem allows us to factorise a polynomial. Knowing a root, $x=a$, of $\mathrm{f}(x)=0$ guarantees that $(x-a)$ is present in the complete factorisation of $\mathrm{f}(x)$. Compare this with the unique prime factorisation of integers.

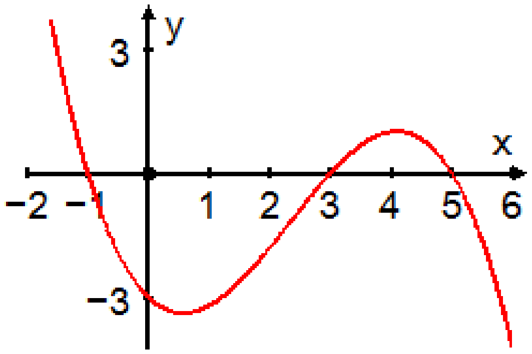
The following question makes another link between algebra and number. “I’m thinking of a polynomial, $\mathrm{P}(x)$, in which all the coefficients are integers between 0 and 9. You do not know the degree of the polynomial. You give me an integer n and I will tell you $\mathrm{P}(n)$. How many requests will you need in order to be sure you know my polynomial?” The surprising answer is that one well-chosen value would suffice. Can you see what it is? (see bottom of page 2 for the answer).

The linking of the algebraic and graphical representations of polynomials is important in helping students make connections about the various features of polynomials and make sense of what is happening. Their familiarity with the behaviour of quadratics and related graphs can be built on for other polynomials.

Sample MEI resource

'Equations of cubic curves' (which can be found at https://my.integralmaths.org/integral/sow-resources.php) is designed to help students make links between the factorised form of a cubic and its graph, particularly the axes intercepts. Working from some initial information students can deduce the equations of related graphs.



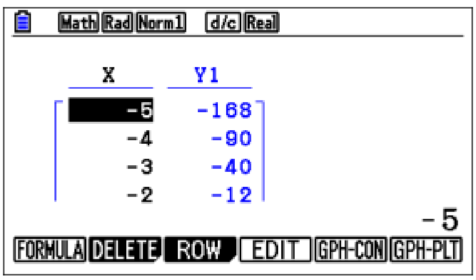


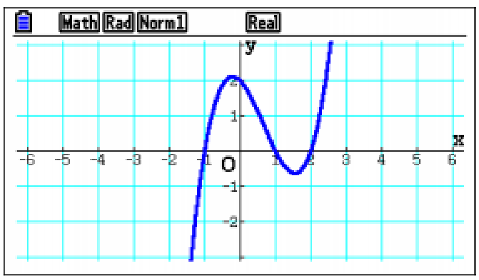
Effective use of technology

'The Factor Theorem' (which can be found at http://www.mei.org.uk/integrating-technology) is designed to support students in using graphing calculators to understand the numeric, graphic and algebraic features of the factor theorem. It comes with questions and further tasks.

Task 8: The Factor Theorem

1. Go into Table mode: [MENU] [7]
2. Add $\mathrm{Y1} = x^3 - 2x^2 - x + 2$: [X,θ,T] [^] [3] [▶] [−] [2] [X,θ,T] [x²] [−] [X,θ,T] [+] [2] [EXE]
3. Use SET to set the table to Start: –5, End: 5, Step: 1: [F5] [(−)] [5] [EXE] [EXIT]
4. Display the table: [F6]
5. Go into Graph mode and plot the graph of this function: [MENU] [5] [F6]





Hint: This is really a place value problem and so using a value of $n = 10$ should help…

Polynomials (AS)	**Time allocation:**

Pre-requisites
- GCSE: Expanding and factorising quadratics
- Quadratic equations and graphs
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Links with other topics
- Binomial expansion
- Further maths: Maclaurin series and other power series
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Questions and prompts for mathematical thinking
- How would you explain how to divide $2x^3 - 5x^2 + 3x - 2$ by $x - 2$?
- What is the same and what is different about the factor theorem and dividing polynomials?
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Opportunities for proof
- The sum of the roots of the cubic $y = x^3 - 3x^2 - 4x + 12$ is $2 + 3 + (-2) = 3$ and this is equal to the coefficient of x^2 multiplied by -1. Prove that this is true for all cubics of the form $y = x^3 + ax^2 + bx + c$ which have three real roots.
- Prove the factor theorem.
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Common errors
- In questions which specify 'by using the factor theorem' attempting to answer by other methods, such as polynomial division
- Division by $x + a$ when a is the known root
- Errors with signs when carrying out polynomial division
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