## Thinking about sequences

Students come across several new terms related to sequences and series (arithmetic, geometric, periodic, oscillating, converging, diverging) as well as new notation such as $u_{n}=3+5 n, \quad a_{n+1}=a_{n}+5, \quad \sum_{r=1}^{n} a_{r}$. Students can take most of this in their stride; they don't need to be taught it at length.

This activity encourages students to make links; after seeing the example in the first column and working in pairs they should be able to fill in the other cells. It is then left to the teacher to give hints where necessary and analyse where any difficulties lie.

Display the new terminology (Arithmetic, Geometric, Periodic, Oscillating, Converging, Diverging) and decide if you want to define these - their educated guesses might be a better approach.

Some of the questions are challenging but it's better for the students to persevere than to be helped too soon.

For $\sum_{n=1}^{10} b_{n}$ and $\sum_{n=1}^{12} c_{n}$ you might want to show some of them the 'writing backwards' trick for summing an arithmetic series if they haven't already seen it. (For example: $S_{5}=2+5+8+11+14$ then $S_{5}=14+11+8+5+2$ and adding term
by term gives $2 S_{5}=16+16+16+16+16=5 \times 16$ and so $S_{5}=\frac{5 \times 16}{2}=40$
To extend students, ask them to make up two columns of their own, one easy and one difficult, and to tell you what it is about a question that makes it difficult.

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|  | $a_{n}=3^{n}$ | $b_{n}=2 n+3$ | $c_{n}=$ | $d_{n}=$ | $u_{n}=$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $3,9,27,81, \ldots$ |  | $9,5,1,-3, \ldots$ |  |  |
|  | $\begin{gathered} a_{n+1}=3 \times a_{n}, \\ a_{1}=3 \end{gathered}$ |  |  | $\begin{aligned} & d_{n+1}=\frac{1}{2} d_{n}, \\ & d_{1}=4 \end{aligned}$ | $\begin{aligned} & u_{n+1}=-3 \times u_{n}, \\ & u_{3}=18 \end{aligned}$ |
|  | $a_{20}=3^{20}$ | $b_{100}=$ | $c_{50}=$ | $d_{10}=$ | $u_{21}=$ |
|  | Geometric <br> Diverging |  |  |  |  |
|  | $\sum_{n=1}^{5} a_{n}=363$ | $\sum_{n=1}^{10} b_{n}=$ | $\sum_{n=1}^{12} c_{n}=$ | $\sum_{n=4}^{8} d_{n}=$ | $\sum_{n=1}^{7} u_{n}-\sum_{n=1}^{6} u_{n}=$ |

