Tangents and Normals

This task uses the chain, product and quotient rules to find equations of tangents and normals.

The cards on the next page contain steps in the solutions to the two problems below but are muddled up.

Students should cut out the cards and arrange them in the appropriate order, inserting missing steps and writing notes of explanation.

Find the equation of the normal to the curve $y = x^2\sqrt{2x-1}$ at the point where $x = 1$.
Find the equation of the tangent to the curve $y = \dfrac{x}{\sqrt{2x-1}}$ at the point where $x = 5$.

gradient of tangent is 3	Remember to insert missing steps and lines of explanation
$v=\sqrt{2x-1}$	gradient of normal is $-\frac{1}{3}$
$y=-\frac{1}{3}x+\frac{4}{3}$	$x=1, \quad y=1$
$u=x^2$	gradient of tangent is $\frac{4}{27}$
At $x=5, \quad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3-\frac{5}{3}}{9}$	$3y+x=4$
$\frac{\mathrm{d}v}{\mathrm{d}x}=(2x-1)^{-\frac{1}{2}}$	$v=\sqrt{2x-1}$
$y=\frac{4}{27}x+\frac{25}{27}$	$x=5, \quad y=\frac{5}{3}$
$\frac{\mathrm{d}y}{\mathrm{d}x}=2x(2x-1)^{\frac{1}{2}}+x^2(2x-1)^{-\frac{1}{2}}$	$\frac{\mathrm{d}u}{\mathrm{d}x}=2x$
$\frac{\mathrm{d}v}{\mathrm{d}x}=(2x-1)^{-\frac{1}{2}}$	$u=x$
At $x=1, \ \frac{\mathrm{d}y}{\mathrm{d}x}=2(1)^{\frac{1}{2}}+\frac{1}{(1)^{\frac{1}{2}}}$	$\frac{\mathrm{d}u}{\mathrm{d}x}=1$
$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(2x-1)^{\frac{1}{2}}\times 1-x\times(2x-1)^{-\frac{1}{2}}}{(2x-1)}$	$27y=4x+25$