Further Differentiation

G1	Understand and use the derivative of $\sin x$ and $\cos x$ ~~Understand and use the second derivative in connection to convex and concave sections of curves and points of inflection~~
G2	Differentiate e^{kx} and a^{kx}, $\sin kx$, $\cos kx$, $\tan kx$ and related sums, differences and constant multiples Understand and use the derivative of $\ln x$
G5	Differentiate simple functions and relations defined implicitly ~~or parametrically~~, for first derivative only

Commentary

As with all differentiation, prior to carrying out differentials students should consider the gradient by first looking at the graph of the function and then the gradient function. Technology has an important role to play in this unit. Using graphing software, students can see that the graph of the gradient function of $y = k^x$ is similar to the function itself, and that when $k \simeq 2.718$ the function and its gradient function coincide, thus illustrating $\frac{\mathrm{d}}{\mathrm{d}x}\left(e^x\right) = e^x$. Exploring gradient functions of $y = e^{2x}$, $y = e^{3x}$ and so on then reinforces the chain rule. Similarly they can find the gradient functions of $\ln x, \sin x$ and $\cos x$ and appreciate the importance of radians in the final two.

How will your students engage with the proofs of the results in this unit? Will they be able to apply the first principle rules that they used in their first differentiation work?

It is important not to perform differentiation in isolation. For example, when asked to differentiate $y = x \sin x$, students might first attempt to sketch the curve (thinking about symmetry, behaviour near the origin, and relationship to the lines $y = x$ and $y = -x$) and then think what differentiation tells them about the turning points and the points where the curve touches the line $y = x$.

This unit represents almost the end of the road for differentiation but for integration the journey has hardly begun!

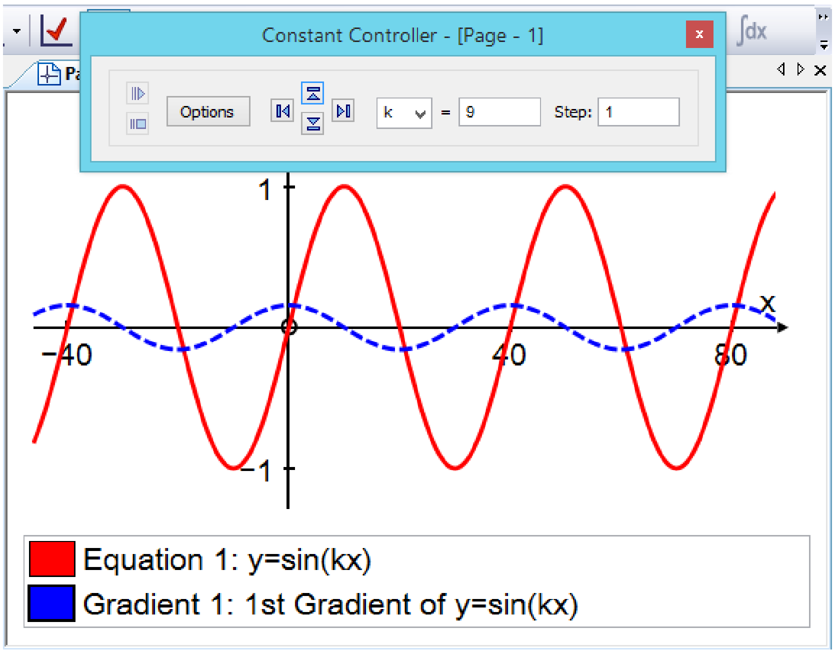
Sample MEI resource

‘Product and Quotient Rule Dominoes’ (which can be found at http://integralmaths.org/sow-resources.php) is a set of 20 dominoes which provides students with an opportunity to practise the product and quotient rules with trigonometric, exponential and logarithmic functions and to check their answers. Students should be encouraged to plot the graphs to help them make links between the calculus and the graphical forms.

START	$y = x^2 \ln x$
$\frac{dy}{dx} = 2x^2e^{2x} + 2xe^{2x}$	$y = x \ln x^2$
$\frac{dy}{dx} = x^2 \cos x + 2x \sin x$	$y = x \cos x$

Effective use of technology

Using the degrees mode in Autograph, plot the function $y = \sin(kx)$, predict its gradient function and then plot this. Use the constant controller to see how the gradient function changes with k. For what value of k does the gradient function have amplitude 1? What is the link with radian measure?



Further differentiation	Time allocation:

Pre-requisites

- Differentiation (AS): using differentiation techniques in problems
- Differentiation: familiarity with the chain, product and quotient rules
- The trigonometric, exponential and natural logarithm functions
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Links with other topics

- The binomial expansion: explore the implicit curve $x^3 + 3xy + y^3 = 1$
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Questions and prompts for mathematical thinking

- Tell me two ways of finding the gradient of the tangent to the circle $x^2 + y^2 = 5^2$ at the point $(3,4)$.
- How would you explain why $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$ is only true if x is measured in radians?
- Why do $\frac{\mathrm{d}}{\mathrm{d}x}(\ln x)$, $\frac{\mathrm{d}}{\mathrm{d}x}(\ln 3x)$, $\frac{\mathrm{d}}{\mathrm{d}x}(\ln 7x)$ all give the same answer?
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Opportunities for proof

- Prove that the graphs of $y = \tan x$ and $y = \cos x$ are perpendicular at all crossing points.
- Prove $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$
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Common errors

- Errors with signs in $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$ and $\frac{\mathrm{d}}{\mathrm{d}x}(\cos x) = -\sin x$
- Using the incorrect formula for the quotient rule e.g. using a ‘+’ in the numerator.
- Overlooking the need for the chain rule for some terms when differentiating a product or quotient.
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