Integrating Mathematical Problem Solving

Applying Mathematics and Statistics across the curriculum at level 3

End of Project Report

This project was managed by MEI
and funded by the Clothworkers' Foundation.

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Contents

1 Background

Integrating Mathematical Problem Solving (IMPS) was a one year pilot programme funded by the Clothworkers' Foundation and managed by Mathematics in Education and Industry (MEI) during the academic year 2011/12. The overall aim of the project was to investigate ways to provide post-16 students who had already achieved a grade C or above in GCSE Mathematics with a key, transferable skill: the ability to apply mathematics and statistics to analyse and solve problems across STEM subjects, social sciences and business.

Pilot work took place in the following areas:

- teaching and learning resources
- use of the GCE Extended Project
- working with university partners, engaging with students in schools and colleges
- teachers' professional development
- working with learned societies, professional bodies and industry
- curriculum development.

The outcomes of the project are detailed in Appendix 1. A full summary of the work carried out in each area is given in Appendix 2.

1.1 Why was there a need for this work?

Many young people embarking on higher education and employment are unable to apply mathematics and statistics effectively, even when they have achieved good grades in mathematics at school or college.

The aim of this project was to investigate ways to address this problem by enabling students to develop their skills in applied mathematics and statistics in contexts relevant to the subjects they are studying.

1.1.1 Meeting the country's mathematical needs

Evidence from the 2011 Advisory Committee for Mathematics Education (ACME) report[1] into the country's mathematical needs showed that the ability to apply mathematics and the ability to communicate mathematical ideas are skills that are highly valued in both higher education and employment. However, the research also found that significant numbers of students in higher education and employees in the workplace lack these skills.

1.1.2 The UK has very low participation in mathematics education beyond GCSE level

The December 2010 Nuffield Foundation report *Is the UK an Outlier*[2] shows that, compared with our economic competitors, the UK's participation rate in post-GCSE level mathematics is very low, with only 15% of the cohort studying level 3 mathematics.

> Our report does not criticise the standard of maths teaching post-16, but it does reveal that in terms of participation, the UK is out on a limb. The vast majority of young people's mathematics education ends with GCSE at age 15 or 16, leaving them inadequately prepared for further or higher education; or for employment.
>
> *Anthony Tomei, Nuffield Foundation Director*

1.1.3 Students are unaware of the importance of mathematics and statistics in a wide range of subjects

Students embarking on degrees in subject areas such as biology, business, chemistry, computing, economics and social sciences are often very surprised by the level of mathematics and statistics required for these courses. They had not understood the relevance of mathematics to these disciplines, and the present provision does not prepare them to cope with the mathematical demands placed upon them. This is discussed in detail in Appendix 3.

1.1.4 Changes at A level

Changes to A level Mathematics and A level Physics over the past 15 years mean that even students with these qualifications have only a limited experience of using applied mathematics and statistics to analyse and solve problems.

Since 1995, A level Mathematics and A level Physics have both undergone substantial curriculum changes and A level students now experience far less mathematical problem solving in both applied mathematics and physics than was previously the case. Changes to examination regulations mean that the applied mathematics content in A level Mathematics has been reduced by 33%, from three modules to two, and coursework, which was championed by MEI and was specifically designed as a means of teaching mathematical modelling, has almost completely disappeared. During the same period, A level Physics has become far less mathematical, with the problem-solving element of 'core physics' being substantially diminished. This means that even students with A levels in mathematics and physics have little experience of standard applied mathematics techniques and models and so have fewer mathematical tools available to them than in the past.

These changes have particularly concerned university departments of Engineering, Physics and Mathematics. These concerns were made clear in the 2009 report *Newton's Mechanics: Who Needs It?*[3], which stated the following:

> Universities report that many students currently admitted onto degree courses in physics, engineering and mathematics, even those with top grades in A level Mathematics, have an inadequate preparation in mathematical modelling and problem-solving skills and a poor knowledge of mechanics.

1.1.5 A level Mathematics is not considered accessible to students with C grades in Mathematics at GCSE

National Pupil Database extracts analysed by Cambridge Assessment in recent years showed that, in 2005, 82.6% of A level Mathematics candidates had A/A* in Mathematics at GCSE and 98.2% A*–B. In 2011, the percentages were 87.2% and 99.3% respectively. That is, hardly any students with grade C in Mathematics at GCSE embark on AS/A level Mathematics. Most schools and colleges have a policy of allowing only students with a grade B or higher to start AS/A level Mathematics; some schools insist on students having at least a grade A at GCSE before allowing them to start an AS/A level Mathematics course. This means that many students who would benefit from building on their GCSE mathematics are unable to do so.

1.1.6 The need for curriculum development

The Smith Enquiry Report, *Making Mathematics Count* (February 2004)[4], recommended that different mathematical pathways should be developed for students progressing to higher education and employment in different areas. The ACME report *Mathematical Needs*[5] shows that there is a need to improve the knowledge and understanding of new undergraduates and employees in how to apply mathematics and statistics to solve problems and communicate ideas.

In his speech at the Royal Society in June 2011[6], Michael Gove, Secretary of State for Education, said:

> ACME's research shows that about 330,000 16–18 students per year need to study maths and statistics at a level beyond GCSE (180,000 to a 'physics or engineering' level and another 150,000 to a 'social science' level). So our schools system is failing to provide anything like the number of suitably-equipped students to meet the needs of Higher Education.
> Only half the population has even basic maths skills, we are producing only about a quarter to a third of the number of pupils with the maths skills that our universities need, and economic trends mean that this gap will, unless we change, get wider and wider with all that entails for our culture and economy.
> That is why I think we should set a new goal for the education system so that within a decade the vast majority of pupils are studying maths right through to the age of 18.

The 2011 Vorderman report, *A world-class mathematics education for all our young people*[7], includes key recommendations relating to curriculum development in post-16 mathematics.

> 7.1 To bring this country into line with the rest of the world, mathematics, in some form, should be made compulsory to the age of 18. The implementation of this recommendation is a matter of urgency.

The growth in numbers taking AS and A level Mathematics has been encouraging. In 2003, 50,602 students took A level Mathematics; by 2012 this had increased to 85,714. However, as explained in section 1.1.5, most of the students taking A level Mathematics have grade A or A* in Mathematics at GCSE, with nearly all the rest having grade B. It is essential that students with grade C at GCSE are able to improve their mathematical understanding post 16.

Ofqual's international research in 2012[8] identified a number of issues including the following:

> Issue 3: Different levels of demand within mathematics – The number of different mathematics assessments at a variety of levels available to students in many education systems was also in contrast to A level Mathematics. Is there a need for A level Mathematics to have further lower-level options in addition to AS?

The time is ripe for curriculum development in mathematics to develop post-16 courses that build on GCSE and meet the mathematics and statistics needs of students across the level 3 curriculum.

1.1.7 Problems arising from the 'teach to the test' culture

There is great pressure on schools and colleges and on students to maximise performance in examinations and this has led to a 'teach to the test' culture. This emphasises teaching students how to answer examination questions – a **non-transferable** skill of little value beyond passing the examination – rather than to understand how to use mathematics and statistics to analyse and solve real problems – a vital **transferable** skill, necessary for higher education and employment.

1.1.8 Giving students an opportunity to distinguish themselves when applying for university

Winning university places is becoming increasingly competitive and it can be hard for students to distinguish themselves, even if they achieve top grades. In many disciplines, students who can provide evidence of their ability to use mathematics and statistics will have an advantage when applying for university.

1.1.9 Informing curriculum development

As explained in section 1.1.6, in the future it is likely that studying mathematics or statistics up to the age of 18 will become the norm for students. Applying mathematics and statistics in different contexts, tailored to students' aspirations, should be a key part of this.

The Integrating Mathematical Problem Solving (IMPS) project has aimed to initiate and trial ideas, working with qualification-awarding bodies, learned societies, professional bodies, industry and universities to show how applied mathematics and statistics in context could be embedded across the level 3 curriculum.

1.1.10 Enabling schools and colleges to have greater engagement with universities

Post-16 students and their teachers often have no real understanding of how the mathematics taught at schools and colleges is used to analyse and solve real problems. The ability to apply mathematics and statistics is valued across a wide range of degree disciplines and industries, and investigative project work can provide excellent opportunities for universities and industry to engage with their local schools and colleges.

As well as helping post-16 students to develop their applied mathematics and statistics skills, working on problems in context helps students to understand the relevance of their education, which increases their motivation and helps to ease their transition from school or college into higher education and employment.

2 Relevant reports published during the project period

2.1 Relevant reports

During the course of the IMPS project, a number of reports were published which underlined the need for students to develop mathematical and statistical problem-solving skills in context.

2.1.1 *Mind the Gap*[9], Institute of Physics (IoP)

Just as the IMPS project was starting, the IoP published the *Mind the Gap* report. One of the conclusions of this report was as follows:

> Academics perceived a lack of crossover between mathematics and physics at A-level, which was felt to not only leave students unprepared for the amount of mathematics in physics, but also led to them not applying their mathematical knowledge to their learning of physics and engineering. It was felt that this crossover should be highlighted, both to encourage those that enjoy mathematics to pursue physics, but also to ensure that students are able to link the two fields of thinking together.

2.1.2 *Mathematics within A-level Science 2010 examinations*[10], Science Community Representing Education (SCORE)

SCORE reported on the mathematics in biology, physics and chemistry A level assessments.

The SCORE report included the following comments:

> For biology, chemistry and physics, the analysis showed that the mathematical requirements that were assessed concentrated on a small number of areas (e.g. numerical manipulation) while many other areas were assessed in a limited way, or not at all.
>
> A perceived consequence, raised repeatedly by the science community in the online survey, is that if mathematical content areas are frequently not assessed then these areas will not be taught or practised in depth. If areas within the mathematical requirements are not taught or practised, it will limit students' access to the subject, their ability to understand scientific concepts and reduce their mathematical fluency.

2.1.3 *Mathematics in A level assessments*[11], Nuffield Foundation

The Nuffield Foundation reported on the mathematics in business studies, computing, economics, geography, psychology and sociology A level assessments.

The report included the following recommendations:

> The report shows that it is possible for students who are ostensibly following the same course of study to have widely different levels of exposure to quantitative approaches to their subject. Stakeholders should consider the implications of this, and its advantages and disadvantages.
>
> In some subjects the mathematical content or approaches are beyond those currently covered in GCSE. Where this is the case, it needs to be made explicit so that subject teachers can provide appropriate support for students and/or students can consider taking relevant post-16 mathematics qualifications.

2.1.4 *Fit for Purpose? The view of the higher education sector, teachers and employers on the suitability of A levels*[12], Ofqual

This Ofqual report expressed dissatisfaction with students' problem-solving skills.

> Lack of mathematical knowledge and lack of practice in mathematical modelling – using Mathematics to solve real world problems – were common complaints amongst the interviewees with a Physics background. The limited amount of Mathematics in the A level was also seen to give students the wrong impression of the subject. One respondent thought that more mathematically-minded students choose to study Mathematics instead of Physics at university as they do not understand how much Mathematics would be involved in the advanced study of Physics. Similarly, this lack of Mathematics means that universities have to add more Mathematics to their first year courses to bring students up to speed.

In addition to the lack of mathematics in A level subjects such as physics, the lack of problem-solving skills in students who have studied mathematics A level was identified as a cause for concern.

> One Mechanical Engineering lecturer at a Low-tier HEI felt students are now less willing to try things where they could make mistakes. The current tendency, he feels, is for students to either do things perfectly because they have been drilled in how to answer certain questions or to '*run away from problems which they can't solve immediately.*' (Lecturer, HEI, NI, Mechanical Engineering).

Problem-solving ability is prized by employers as well as by Higher Education.

> *'I think they have been coached at school and we have to convert them into people who [can think independently], if they cannot then they're not employable. The biggest transition that we have to make is to make them independent learners, [who are] confident and able to problem solve, all those kinds of things which the employers are looking for, and we're starting from a different position than we were.'*
>
> *Head of Department, HEI, England, Engineering*

2.1.5 *International Comparisons in Senior Secondary Assessment Report*[13], Ofqual

This study reviewed mathematics, chemistry, English and history qualifications that led to university entrance in a range of education systems.

The international comparison study found that there was insufficient emphasis on problem solving in A level Mathematics.

> In A level Mathematics the assessment objectives were however not met, although the content requirements were represented well. The questions did not reflect the problem-solving required by the assessment objectives.

By contrast, A level Further Mathematics was seen to be challenging in terms of problem solving, comparing well with the most demanding courses internationally.

> This specification [Further Mathematics] provides the broadest range of mathematics and its applications and the A2 modules in particular provide significant technical and problem-solving challenges for students, making this the most demanding programme of study reviewed.

Ofqual's international comparison work suggests that teacher assessment might allow an increased emphasis on problem solving, as well as enabling use of IT.

> Several of the qualifications involve teacher or school-based assessment where up to 50 per cent of the final grade is made up of such assessment tasks. Such assessment can broaden a qualification and allow students to be tested on the application of IT, extended problem-solving and the use of mathematics as a tool for modelling that cannot be tested in an examination environment.

2.1.6 *The Future of Statistics in our Schools and Colleges*[14], Royal Statistical Society and the Actuarial profession

This report also highlighted the importance of assessment and its influence on what is taught.

> Assessment has a strong influence on classroom practice. A consequence of the present arrangements is that, at almost all levels, most students do not engage in processes that are intrinsic to the nature of statistics, and so learn neither how to carry them out nor how important they are.
> Recommendation 11: The assessment techniques used should ensure that, at every level, students carry out work covering all the processes required to use statistics to solve problems and make decisions.

2.2 Themes arising from these reports

In addition to highlighting the need for mathematical and statistical problem-solving skills, two important themes emerge from these reports: the importance of assessment and the degree to which mathematical and statistical problem solving can be integrated into the teaching and learning of other subjects. Both of these themes are addressed more fully in *MEI Review of Mathematics in Other Subjects at Level 3* (see Appendix 3)[15].

2.2.1 Assessment

If mathematical and statistical problem-solving skills are not assessed, they are unlikely to be addressed in the classroom, so most students will not develop them.

2.2.2 Is it better to teach mathematics separately or to integrate it within other subjects?

Ofqual's international comparison work found that A level Chemistry was more mathematical than comparable qualifications in other countries.

> A strength of A level Chemistry was seen to be its high mathematical content in comparison with other systems. Is this balance correct? Would students and higher educationalists find it preferable to have further or deeper chemistry content within chemistry and move the bulk of mathematics currently covered in chemistry to mathematics subjects?

This finding contrasts with that from SCORE's survey, undertaken as part of the research for *Mathematics within A-level Science 2010 examinations*.

> Overall, participants were concerned about the levels of mathematical content in chemistry A-levels, with many feeling that students were being misled about the mathematical requirements of chemistry as a subject. Concern was also expressed that downgrading the mathematical requirements would also lead to a restriction in the chemistry content that could be assessed.

This illustrates the need to decide whether the mathematical requirements of different subjects can be adequately addressed within those subjects' A levels or whether there are some subjects for which students should be required to take appropriate mathematics qualifications alongside A levels in those subjects.

3 Teaching and learning resources

A major aim of the project was to investigate how teaching and learning resources could be developed that supported and encouraged teachers and students to integrate mathematical and statistical problem solving into the study of different subjects across the level 3 curriculum.

3.1 Review of current textbooks

As part of the planning for the project a small number of textbooks to support the mathematics in other subjects at early undergraduate level were reviewed. It was found that these often started by reviewing GCSE-level mathematics, mainly out of context, then proceeding quite rapidly to A level and beyond. Relevant contexts were used in these books but not usually integrated throughout them.

A small number of A level textbooks for other subjects were also reviewed. It was found that the mathematics and statistics was presented in abbreviated form, focusing more on getting students to follow 'recipes' rather than on understanding the processes involved. Occasional mathematical errors were also found in these books. Only a small sample of textbooks was surveyed, so the impression they gave may not be representative. However, mathematical and statistical errors can be problematic for teachers of other subjects who may not have sufficient mathematical understanding to realise when there is a mistake in the text.

3.2 Developing the resources

The resources for the IMPS project were designed to introduce mathematical and statistical techniques in context, making use of ICT, where appropriate, to enable understanding and help students to develop problem-solving skills. Teaching notes were provided so that the resources could either be used by mathematics teachers who wanted to show the techniques they were teaching being used in context or by teachers of other subjects who wanted to enable students to understand the mathematical and statistical techniques which they needed to use in their subjects.

3.2.1 The resources developed through the project

Resources have been written for each of the following:

- The Mathematics of Biology
- The Mathematics of Business and Finance
- The Mathematics of Chemistry
- The Mathematics of Geography
- The Mathematics of Physics
- The Mathematics of Psychology

The resources were made available to teachers who wished to try using them in the academic year 2011/12 and have been made freely available to all from September 2012, both through the national STEM Centre and through the MEI website[16]. An example resource is shown in Appendix 6.

There are many more resources which could be developed and resources could also be developed for subjects which are not included in the list above. This is considered further in section 8.

3.2.2 Issues associated with developing the resources

In developing such resources it is important to ensure that the context is relevant and correctly expressed in each subject area. The mathematical aspects must be clearly explained in such a way as to be understandable to a student who is not taking mathematics beyond GCSE level, but must avoid any over-simplification that might cause misunderstanding.

3.3 Articles based on the IMPS project resources

In addition to the resources produced for the project, a series of three articles based on the resources was written for the Mathematical Association's magazine for school teachers, *Mathematics in School*.

- *Would you like to take a later flight?* (statistical modelling in a business context)
- *Thank you for choosing to travel with us* (the mathematics of price and demand)
- *Keeping the coffee warm* (the mathematics of cooling)

3.4 Conference sessions and lectures based on the IMPS resources

The resources will also form the basis of two sessions at the Association of Teachers of Mathematics conference 2013: '*Maths for Real in A Level Science*' and '*Maths for Real in A Level Social Science*'.

Stella Dudzic, the project leader, is the Royal Statistical Society Guy Lecturer for the academic year 2012/13. This involves leading a series of interactive talks for sixth form students about uses of statistics. The theme of the lecture is measuring inequality and is related to one of the IMPS resources.

4 The GCE Extended Project Qualification (EPQ)

The GCE Extended Project Qualification (EPQ) is a level 3 qualification offered by all awarding bodies[17]. It is equivalent in size to half an A level but equivalent in standard to a full A level so it is possible for students to achieve an A* grade in an EPQ, unlike in an AS level.

The EPQ can provide an opportunity for students to use mathematical and statistical analysis and problem solving, but at present it is unusual for them to do so. It is hoped that the work of the IMPS project will enable more students to incorporate the use of mathematics and statistics within their EPQs.

4.1 Benefits to students of taking the EPQ

The benefits to students of undertaking an EPQ include:

- developing research skills
- developing planning skills
- developing problem-solving skills
- applying knowledge
- finding out about something they are interested in
- developing presentation skills
- carrying out an extended piece of work.

4.2 How the EPQ is assessed

Much of the weighting of the assessment of the EPQ is for the processes of planning, researching and producing the project. It is important for students, with appropriate mentoring, to make their own decisions regarding the content and direction of the project.

4.3 Uptake of the EPQ

The number of students taking EPQs have grown very rapidly since they were first generally available in 2009.

Year	Candidates
2009	5,094
2010	15,958
2011	24,099
2012	28,572

Numbers taking EPQs (JCQ figures)

4.4 Universities and the EPQ

Universities have become increasingly interested in including EPQs in their offers to potential students because of the research and problem-solving element in the projects. When talking with university lecturers as part of the IMPS project we found that they did not always know much about the EPQ, but when it was explained to them they were interested and keen to encourage students to undertake EPQs.

The University of Manchester is one of several universities including information about the EPQ on its website[18].

4.5 Supporting students to use mathematics and statistics within the EPQ

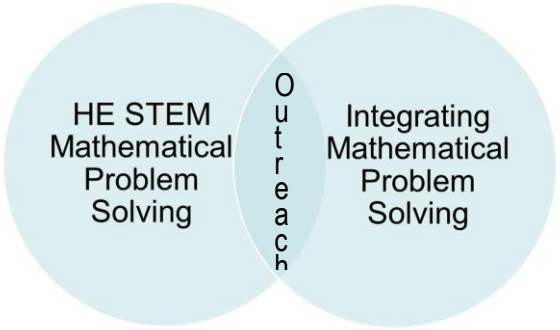
Few students currently base their extended projects around mathematics or make use of mathematics and statistics within their EPQ.

As part of the IMPS project, we wanted to show how mathematics and statistics could be used within EPQs, and to develop teaching and learning ideas and resources that could help students to do so.

The MEI website now includes a page of advice for teachers and students[19], together with links to other useful resources. Most of the resources produced as part of the IMPS project include links to further reading with the aim of giving interested students a starting point for further research.

5 Work with universities and A level students

The universities of Keele, Leeds, Manchester and the West of England have been working to develop mathematical modelling and problem solving at undergraduate level as part of an HE STEM programme, the Mathematical Modelling and Problem Solving project[20]. The aim of the IMPS project to work with universities to help engage pre-university students with mathematical modelling and problem solving overlapped with the outreach aims of the HE STEM project, so the IMPS project worked with the four universities to plan and deliver outreach days for A level students.



5.1 University outreach days

Four outreach days took place, one in each of the universities involved in the HE STEM programme's Mathematical Modelling and Problem Solving project. Each of the four outreach days had a different format; this allowed a variety of approaches to be trialled.

5.1.1 Keele University

This took the form of sixth formers at Newcastle-under-Lyme College working in groups on some unstructured problems, with support from undergraduates. The sixth formers were encouraged to present their solutions to others, perhaps by making a poster.

Approximately 20 A level students attended, along with 5 of their teachers.

5.1.2 University of Leeds

The theme of the Leeds day was modelling in mechanics, with the opportunity for students to undertake experimental work in mechanics, leading to mathematical modelling.

The day was attended by 49 A level students and 9 A level teachers from local schools and colleges.

5.1.3 University of Manchester

Students took part in an interactive lecture about applications of mathematics, delivered by Dr Louise Walker from the university mathematics department. They then worked on ideas for possible projects which could lead to a level 3 EPQ involving applications of mathematics. This session was supported by undergraduates. Teachers were given guidance about supervising EPQs from an AQA representative[21]. The day was attended by 35 A level students and 13 A level teachers from at least six schools and colleges.

5.1.4 University of the West of England (UWE)

The day was led by Dr Kevin Golden, of the UWE Department of Engineering Design and Mathematics. Students were given a variety of situations to model mathematically. Applications included simple kinematics and drug testing in sport, as well as exponential modelling and the application of modelling to the Bloodhound SSC land speed record. The day was attended by 17 A level students and 2 A level teachers from three schools and colleges.

5.1.5 Feedback from the A level students who attended the university outreach days

Students were asked to give feedback on the university outreach days; 90 feedback forms were received. The vast majority of students rated the overall content of the day as good or excellent.

Student rating	Frequency
Excellent	11
Good	65
Adequate	10
Poor	1
No rating given	3

Student ratings of the content of the outreach day as a whole

Students were asked what they had learnt from the outreach day. This was a free response question; most commonly mentioned were the importance of mathematical modelling and the application of mathematics to real-life situations. Of the 90 feedback forms, 50 mentioned applications of mathematics and/or mathematical modelling. This was most eloquently expressed by the student who wrote:

> Mathematics can generate beautiful solutions to real life problems.

Students were also asked whether the outreach day had changed their views of the importance and usefulness of mathematics. This was a free response question with the responses falling naturally into one of the categories in the table below.

Student response	Frequency
Increased understanding of the variety of uses of mathematics	34
Increased understanding of connections in mathematics	6
Increased awareness of possible future areas for own study	2
Increased liking of mathematics	2
Increased understanding of modelling	2
Increase in specific knowledge	1
Non-specific change	4
No change – I always thought mathematics was important	16
No change	11
No response	12

Student response to the question

'How has the day changed your views of the importance and usefulness of mathematics?'

6 Teachers' professional development

Professional development days and sessions as part of other professional development events were developed and delivered through the IMPS project.

6.1 Professional development days

Two professional development days were planned and delivered as part of the IMPS Project. One was designed for teachers of STEM A levels and took place at the National STEM Centre. The other was designed for teachers of non-STEM A levels and was hosted by the University of Leeds.

6.1.1 Aims of the professional development days

Both professional development days were designed with the following aims in mind.

- To provide teachers with information about the importance of mathematical and statistical techniques for the further study of other subjects
- To introduce teachers to some useful resources and sources of further information about the use of mathematical and statistical techniques
- To provide some ideas for supporting students working to achieve the Extended Project Qualification (EPQ)

6.1.2 Attendance at the professional development days

Despite the days being free to teachers, attendance was lower than initially hoped. Instead of having 20 teachers at each day, there were 6 teachers on the STEM day and 13 on the non-STEM day. There seemed to be two reasons for the low attendance.

- Schools' and colleges' reluctance to allow teachers to take time out of the classroom for professional development that was not focused specifically on the examination curriculum.
- Some teachers found it difficult to access the information about the days.

6.1.3 Feedback from the professional development days

Feedback from teachers who were able to attend suggests that there is a strong need for further professional development about quantitative aspects of A level subjects.

Teachers were asked to rate the following aspects of the professional development days.

- How well the aims were achieved
- How appropriate the aims were
- The clarity and delivery of the sessions
- The suitability of the venue

Feedback forms were received from 15 teachers; all the above aspects were rated as either good or excellent by all teachers except for one lower rating for clarity and delivery on the non-STEM day. This lower rating may have resulted from a desire to have all sessions more

strongly related to the current A level curriculum. One of the sessions on that day was delivered by an academic who talked about the uses and importance of quantitative methods in social sciences and how such research informs national planning. Oral feedback suggested that some teachers did not see the relevance of this session.

Teachers were also asked to reflect on what they saw as the opportunities for and barriers to cross-curricular work incorporating mathematical and statistical techniques into their subject or other subjects at A level. Teachers' responses to this question from both professional development days are reproduced in full in the next two sections.

Feedback from the STEM day 'The Language of the Universe'

- Try to talk more with/to colleges in order to make the connections. The EPQ could be introduced to my school for the students to extend their learning.
- Lack of cross-curricular planning time combined with my own lack of knowledge in the science subjects.
- Opportunities for discussing with staff/students about what techniques we used and applications of them.
- Science and Maths communicating with the same language and delivering a combined maths-focused learning experience to student.
- Time (particularly time availability to work with colleagues in other subjects) and that's why these cross-curricular resources are so helpful.

Feedback from the non-STEM day 'Everybody Counts'

- Student confidence is the main barrier – Maths is a weak area in my school and many students feel very negative about it. They also struggle with transferring knowledge and understanding between subjects.
- Lack of awareness of what other subjects' A level courses consist of. Lack of cross-curricular work in KS3 and KS4.
- Opportunities for cooperation between departments i.e. the maths department on delivering specialist training on data analysis. Perhaps developed from GCSE level. Barriers – teachers' understanding of and ability to portray data analysis and effectively integrate it into specific subjects.
- Lack of maths on the A level spec and also lack of free specialist software to use well with [the IBM statistics software] SPSS – massive jump from A level psychology to degree. Perhaps some sort of maths undergrad outreach to make maths and psychology more linked – I'd be happy to pilot this idea at my school.
- Aim to target non-maths subjects via exam boards. A level teacher sites such as AQA geography.
- Opportunities – building statistical knowledge and confidence in this area with use of test of statistical significance. Barriers – students often seem afraid of statistics.
- The perception that maths is difficult and that statistics is something that they won't be able to do when in fact they can be broken down and made simple and interesting as in sessions 2 and 3 today.
- Barriers – Curriculum time constraints; priority of assessment reducing risk taking in lesson content, particularly in practical exercises. Opportunities – First presentation demonstrated the links between sociology and geography. I will take this back with me and discuss with the geography department.

- Opportunities: Students – provides them with employability skills to prepare for university. Teachers – methods of embedding maths across subjects, develops critical thinking. Barriers: Accuracy of assessment; confidence of staff leading stats, need variations and new, more creative and enjoyable ways of teaching statistics.
- I would have liked some more explanations of how to teach statistics, especially to students of A level achievement. Psychology already has some stats involved in the curriculum but not all students are capable.

6.2 Additional professional development

In addition to the full days of professional development, four sessions for mathematics teachers were delivered to introduce teachers to the resources and ideas associated with the IMPS project. These sessions took place at the MEI conference and as part of a training day for teachers on MEI's Teaching Advanced Mathematics course.

The project leader also used the IMPS resource about statistical modelling in business, in the context of aircraft overbooking, on three professional development days organised by the Royal Statistical Society Centre for Statistical Education (RSSCSE).

6.2.1 Feedback from the additional professional development sessions

Like the professional development days, these sessions were well received by teachers.

Feedback from sessions for mathematics teachers

Feedback forms were received from 35 of the 36 teachers present. In addition to giving general feedback, which was overwhelmingly positive, teachers were asked what they found the most useful aspect of the professional development and whether they would use any of the ideas in their teaching.

Comments about the most useful aspects of the sessions included:

- Relating mathematical principles to other subjects – economics specifically being so practical.
- Introduction to economic concepts I had never met and how mathematical ideas can be applied.
- Ability to see the interaction of different aspects when using interactive resources i.e. Geogebra files.
- Actually doing a problem – time to investigate – relevant to A level syllabus.
- I really enjoyed the airlines problem and can see it being used in stats classes.
- Ideas to use in statistical lessons in teaching Statistics 1.
- How to link the maths with other subjects.
- MEI resources that can be taken to the classroom.

Mathematics teachers' responses to the question about whether they would use any of the resources in their teaching are given in the table below.

Teacher response	Frequency
Yes	19
Maybe	9
No opportunity at present	6
No	1

Mathematics teachers' responses to the question 'Do you think you will use any of the ideas in your teaching?'

Feedback from sessions at the RSSCSE days

These sessions, on the theme of statistical modelling in business, were attended by 63 A level Mathematics teachers. The sessions incorporated the aircraft-overbooking example from the IMPS resources and this was very well received, leading to some requests from teachers for access to the resources; an example is given below.

> I attended the stats day in Plymouth and found your aircraft-overbooking session inspirational! I would love to try this with my students: please would you send me details of how to access the materials you used?

6.3 Conclusions from the professional development aspect of the IMPS project

Feedback from the professional development associated with the IMPS project and subsequent emails from teachers suggest that there is some enthusiasm from teachers for integrating mathematical and statistical problem solving into their teaching and for using resources such as those produced for the IMPS project. However, there has been little subsequent feedback about the use of the resources from teachers who asked to have access to them. Teachers are very busy and it is possible that they have used resources but then not made the time to feedback on them. It is also possible that the resources they want to use were available too late for use in the academic year 2011/12 but will be used in the year 2012/13. Working with specific groups of teachers to refine resources over a longer period of time might provide a greater amount of useful feedback.

7 Work with other organisations

Meetings took place with professional organisations, learned societies and awarding bodies. Details of these meetings are given in Appendix 4.

7.1 Meetings with professional bodies and learned societies

Meetings with professional bodies and learned societies provided useful information about the mathematics and statistics that are used in other subjects and the kinds of resources which are already available to enable students to understand these mathematical and statistical topics.

These meetings also highlighted that some subjects, such as physics, require a great deal of mathematics in higher education. For such subjects, students would need to study A level Mathematics in order to progress to higher education; it is important that students and teachers understand this.

By contrast, during a meeting at the Society of Biology, a strong preference was expressed for teaching the required mathematical and statistical techniques within biology, to ensure that they were taught in context and when needed within biology.

The importance of ensuring that the curriculum makes students aware that the mathematical techniques which they learn can be used in different contexts was also highlighted in the 2011 Institute of Physics report *Mind the Gap*[22]. The report contained the following quote from a physics academic.

> 'If you learn technique A in a unit, let's say, in a module on technique A, you are never ever expected to use it in a course or module in something different. So there's absolutely no transferable skill that they are learning. It's amazing, you can actually ask somebody a question, and they say, "I have no clue how to answer that", and we say, "Haven't you seen this before?" "Yes. You mean I can use this piece of maths to solve this piece of physics? Wow." They can do it, potentially, when prompted, most of them. They shouldn't need to be prompted. They've never seen maths as a toolbox for physics before. It is purely down to compartmentalisation.'

7.2 Work with awarding organisations

We were able to meet with representatives from the AQA, OCR and WJEC awarding organisations and had correspondence from representatives of Cambridge International Examinations and Edexcel.

Meetings with awarding organisations were mainly focused on how to enable students to use mathematics and statistics in EPQs.

7.3 Work with industry

There has not been enough time to make many direct links with industry during the course of this one-year project. However, the Finance Baccalaureate being piloted at King Edward VI College, Stourbridge, (see Appendix 5) has led to contact with Mike Maddick, the head of global recruitment at RBS, who expressed approval of the Mathematics of Business and Finance resources that form part of the IMPS resources.

We have contacted the Sector Skills Council for Science, Engineering and Manufacturing Technologies and a meeting is planned to explore how the resources of the IMPS project and the resources which MEI has produced to support the use of mathematics in engineering contexts could improve students' readiness for careers in industry.

8 Curriculum development work

A range of curriculum development work has been carried out through the IMPS project. Ideas have emerged for the development of new courses and assessments with greater validity that can help students to develop mathematical and statistical problem-solving skills.

8.1 Mathematics and finance

The work on the mathematics of finance, which was done for the Finance Baccalaureate (see Appendix 5), led to an opportunity to contribute to the report of the All-Party Parliamentary Group on Financial Education for Young People, outlining a curriculum for financial education for secondary schools (see Appendix 7).

This work on financial education led to further development of a financial problem-solving cycle; this fed into MEI's response to the draft Primary National Curriculum in Mathematics.[23]

8.2 Development of a new level 3 statistics qualification

MEI has provided advice to WJEC on the development of their level 3 QCF qualification *Statistical Problem Solving using Software Pathways*. This qualification[24] focuses on the solution of a real statistical problem and has been informed by the work of the IMPS project. Some of the project's resources will be useful for teachers delivering the QCF qualification.

8.3 A level reform

The possibility of lower level options within A level Mathematics, in addition to AS Mathematics, would support the achievement of the national goal of having more students studying mathematics post 16. This will require the continuation of a modular system for mathematics post 16 so that the possible pathways are all within A level Mathematics, as suggested in the Ofqual report, *International Comparisons in Senior Secondary Assessment* (see section 1.1.6).

8.3.1 A new course suitable for students with grade C or above in GCSE Mathematics

Following discussion of a suitable course for students with GCSE grade C at MEI's Core Mathematics Development Group, initial development work has been done to produce a suitable unit. This work has been informed by the approach of IMPS resources and develops the mathematics in association with consideration of appropriate contexts. An outline of the draft content is given in Appendix 8.

8.4 The importance of valid assessment

Ofqual's *Fit for Purpose? The view of the higher education sector, teachers and employers on the suitability of A levels*[25] highlights the tension between validity and reliability of assessment.

> A STEM learned body thought that striving for both validity and reliability can have negative impacts on preparing the student for higher education as they felt it limits the types of questions that can be asked and the types of assessments that can be used. Their overall perception is students can be under-prepared for higher education because they learn how to respond to a narrow range of assessments which is not a reflection of academic life.

The 2012 Royal Statistical Society and Actuarial Profession report, *The Future of Statistics in our Schools and Colleges*[26] recommends using the experience of other subjects to improve assessment in mathematics.

> The report includes examples of assessment styles used in a number of other subjects. Many of these are designed to influence classroom practice, ensuring that the work students do gives them experience of the nature of statistics. Adopting such methods would require a change of culture among some mathematics examiners, moving away from answers being either right or wrong.
> Recommendation 12: The assessment of statistics within mathematics should be informed by good practice in other subjects.

The possibility of being informed by good practice extends beyond the assessment practices of other A level subjects. The Mapping University Mathematics Assessment Practices research project (MU-MAP)[27] has exemplified different assessment methods in mathematics at universities.

Doctor Paul Hernandez-Martinez of Loughborough University, working on mathematical modelling and problem solving in materials engineering, says the following in his interim report[28].

> The assessment of modelling projects cannot be subject to a traditional marking scheme. If we are to encourage creativity and personalised answers to modelling projects, then we cannot expect to have 'similar' answers that can fit into one pre-determined answer. In this way, students will be able to explore answers, and will have to justify their choices, evaluate their answers and try to put an argument across, all of which constitute modelling skills and which we would like to be able to assess. Students should also be able to compare their work with that of other colleagues and judge the better one.
> This is why, in order to assess these projects we intend to use Adaptive Comparative Judgement (ACJ), which is a method for assessing evidence of student learning that is based on expert judgement rather than mark schemes. In this method, assessors are presented with pairs of students' work and asked to decide, for each pair, which student has demonstrated the greater proficiency in the domain of interest, in this case, modelling skills. The outcomes of many pairings are then used to construct a scaled rank order of students.

8.5 Professor Timothy Gowers's ideas

Professor Gowers has posted a weblog with ideas on how to teach mathematics to non-mathematicians[29]. The underlying theme is explained in this quote from the weblog.

> … there is (or could be) a place for questions *that start with the real world* rather than starting with mathematics. In other words, when coming up with such a question, you would not ask yourself, 'I wonder what real world problem I could ask that would require people to use this piece of mathematics,' but rather, 'Here's a situation that cries out to be analysed mathematically — but how?'

We met with Professor Gowers to discuss these ideas and are interested in building on them to develop a new post-16 mathematics curriculum suitable for students with grade C or above in GCSE Mathematics.

8.6 Ongoing curriculum development work

Informed by the work of the IMPS project, MEI intends to play a leading role in curriculum development in level 3 mathematics, working with other organisations to help embed mathematical and statistical problem solving across the level 3 curriculum.

This will involve the development of qualifications, assessment methodologies and teaching and learning resources. It will also require the provision of extensive professional development, both for specialist mathematics teachers and for teachers of other subjects.

8.6.1 Developing A level Mathematics

In June 2012 Ofqual published a consultation on A level reform[30]. MEI's response to that consultation[31] and position paper on A level reform[32] have both been heavily influenced by the work of the IMPS project.

MEI has begun development work on a new A level Mathematics specification. Key themes of this work are:

- to attempt to develop a specification that encourages students to develop mathematical and statistical problem-solving skills
- to attempt to devise valid assessment methodologies to assess mathematical and statistical problem solving.

8.6.2 Developing new level 3 mathematics courses for students with grade C or above in GCSE Mathematics

MEI hopes to pursue development work on new post-16 mathematics courses that could engage far more students in learning level 3 mathematics and would encourage the development of mathematical and statistical problem-solving skills.

MEI is now working with the OCR awarding body to develop the draft curriculum content outlined in Appendix 8 into a qualification.

We are also interested in building on Professor Gowers's ideas, to develop them into a viable curriculum for post-16 students. This would require a radically different approach to traditional teaching and learning practices in mathematics. Considerable work will be needed to develop effective teaching and learning resources and professional development courses for teachers, and to devise appropriate assessment methodologies.

8.6.3 Work with the Royal Statistical Society

Following on from *The Future of Statistics in our Schools and Colleges* report (see section 2.1.6), there will be further curriculum development work to identify teaching opportunities for statistics in relevant A level subjects and to ensure that appropriate post-16 pathways, which provide appropriate preparation for higher education, are available to students. MEI will be working with the RSS to undertake this development work.

8.6.4 Supporting the mathematics embedded within different subjects at level 3

Building on the review of the mathematics within other A level subjects at level 3, MEI hopes to work with experts from different subjects to develop a coherent approach to the teaching, learning and assessment of the mathematics embedded within different subjects. These subjects include biology, business studies, chemistry, computing, economics, engineering, geography, physics, psychology and sociology.

8.6.5 Vocational courses in mathematics and statistics

MEI is developing a course for business: *Statistics for Management*. The approach taken in the IMPS resources, where mathematical and statistical understanding are developed from a context, seems a suitable approach to use for resources for this course.

MEI is also keen to support the mathematics within vocational courses at level 3. MEI's work in this area will also be informed by the IMPS project.

9 Conclusions

1 Mathematics teachers who have worked with the IMPS resources and engaged in professional development through the project have appreciated the opportunity to improve their understanding of how level 3 mathematics is used in context.

2 Teachers across a variety of A level subjects who have worked with the IMPS resources and engaged in professional development through the project have fed back positively about the importance of enabling students to understand mathematical and statistical concepts.

3 Students who have taken part in university outreach days through the project have remarked on their improved understanding of the value and uses of mathematics.

4 The Extended Project Qualification offers an opportunity for students to demonstrate their ability to apply mathematical and statistical techniques in an extended investigation, but it is not sufficient on its own to develop students' mathematical and statistical problem-solving skills. To use mathematics and statistics effectively within EPQs, students need access to appropriate teaching in mathematics beyond GCSE level.

5 National curriculum development in mathematics at level 3, including the proposed new A levels, must ensure that new qualifications provide an effective preparation for students to progress to higher education or employment. In order to achieve this students require

- an understanding of the importance of mathematical and statistical techniques across a wide range of subjects
- the ability to apply mathematical and statistical techniques to analyse and solve problems in context.

6 To encourage the teaching and learning of mathematical and statistical problem-solving skills, valid ways must be found to assess them.

7 In order to meet national needs, more students need to study mathematics post-16, including students with grade C or B in GCSE Mathematics. MEI's ideas for how this might be achieved are outlined in MEI's *Mathematics for all post-16*[33] position paper. This will require coordinated curriculum development between mathematics and other subjects.

8 To embed mathematical and statistical problem solving across the level 3 curriculum, extensive professional development will be needed to support teachers of other subjects to teach mathematics more confidently within their own subjects and to teach level 3 mathematics to students not specialising in mathematics.

9 Professional development is needed to support mathematics teachers to understand how mathematics is used in other subjects and in higher education and careers.

10 If other mathematics courses are made available within the A level suite, as suggested by Ofqual, transferability between possible pathways can be maintained by working in a modular system.

10 Recommendations

Some of the following recommendations reflect those in the MEI position paper *Mathematics for all Post-16*[33] and the discussion paper *How might A level Mathematics be improved?*[34] These ideas were informed by the work of the IMPS project.

1 Level 3 mathematics pathways should be developed to meet the needs of all students with grade C or above in GCSE Mathematics.

Alongside the development of new A level Mathematics and Further Mathematics specifications, new level 3 mathematics and statistics qualifications should be developed and trialled. This should ensure that coherent mathematical pathways are in place for all students with grade C or above in GCSE Mathematics whose aspirations mean they do not wish to study AS/A level Mathematics. Such students should be able transfer to AS Mathematics and beyond should their aspirations change.

2 All level 3 students should take a separate level 3 mathematics course that can support any use of mathematics and statistics within their other level 3 subjects and prepare them for the mathematical and statistical demands of higher education and employment.

3 Valid ways to assess mathematical and statistical problem solving should be developed.

Level 3 Mathematics courses must be assessed appropriately to ensure that the skills we want students to develop are the skills that are rewarded by the assessment. In order to achieve this, work is required to develop valid ways to assess mathematical and statistical problem-solving skills. As part of this work, assessment methodologies used in higher education and in other subjects should be reviewed, to inform developments in mathematics assessment.

4 Academics and other experts from a variety of subjects should be involved in the development of level 3 mathematics curricula.

A level Mathematics and other level 3 mathematics courses support many different future courses of study and careers, so academics and other experts from disciplines outside mathematics should be involved in their development.

5 Curriculum development in mathematics should be coordinated with curriculum development in other subjects that use mathematics or statistics.

Mathematics and statistics are used in many other subjects and it is important to ensure that students experience a coherent mathematics curriculum across subject boundaries.

6 University outreach events across all numerate disciplines should emphasise the importance of mathematical and statistical problem solving.

7 Students should be encouraged to use mathematical and statistical analysis and problem solving in EPQs, wherever appropriate.

8 Professional development courses and resources should be developed to support teachers of other subjects to teach mathematics more confidently within their own subjects and to teach level 3 mathematics to students not specialising in mathematics.

This can improve the teaching of the mathematics and statistics embedded within other subjects and increase the pool of teachers able to teach level 3 mathematics courses, which is vital for increasing student participation.

9 Professional development courses and resources should be developed to support mathematics specialists to understand how mathematics and statistics are used in other subjects and in higher education and careers, and to enable them to support students undertaking EPQs.

10 School and college mathematics departments should work with other departments to support the teaching of mathematics and statistics within other subjects and through EPQs.

Appendix 1: Outcomes of the Integrating Mathematical Problem Solving Project

Phase 1 (September to December 2011)			
	Outcome	**What success will look like**	**How it could be evaluated**
1	University partners are established	3–5 universities agree to take part	Agreements with the universities
2	Schools and colleges agree to work with MEI and the university partners	6–10 schools and/or colleges agree to work with the pilot	Agreements with the schools and colleges involved
3	Links are developed with awarding bodies, professional bodies, learned societies and industry across different subject areas	Working links are established with the AQA, Edexcel, OCR and WJEC awarding bodies, and with professional bodies, learned societies and industry in 5 or more subject areas	Details of links and contact with the establishments involved
4	A review of current curriculum development in level 3 mathematics is produced, including how mathematics is used within level 3 courses in other subjects, and initial curriculum development work is undertaken	The outcomes of the review will be available online and will identify key areas where curriculum development is needed	The quality and completeness of the review will be assessed; initial curriculum development work will be available for review
5	Appropriate existing materials are gathered and initial contexts and example investigation questions with supporting materials are developed	5 or more initial contexts and example investigation questions with supporting materials are produced	Reviewing the materials
6	A professional development course for teachers is developed	A course is planned and suitable supporting materials produced	Reviewing the plans and materials

Phase 2 (January to August 2012)			
	Outcome	**What success will look like**	**How it could be evaluated**
7	Contexts and materials are trialled in schools and colleges	Resources are being trialled in 6–10 pilot schools and colleges	Feedback from the pilot schools and colleges
8	Initial trialling of outreach activity is initiated between the university partners and local schools and colleges	3–5 universities are working with a total of 10–15 schools and colleges and 100 or more students are undertaking enrichment courses and/or GCE Extended Project Qualifications (EPQs) involving the application of mathematics and statistics	Gathering feedback from the universities, and the school and college teachers and students involved
9	Links with awarding bodies, professional bodies, learned societies and industry are consolidated	Awarding bodies, professional bodies, learned societies and industry are engaging with and promoting the programme	Feedback from awarding bodies, professional bodies, learned societies and industry
10	Further contexts and example investigation questions with supporting materials are developed	A total of 10 or more contexts and example investigation questions with supporting materials are available	Reviewing the materials
11	Professional development for the teachers involved is managed	At least 40 teachers have attended one of the project's professional development courses.	Feedback from the teachers concerned
12	Students' ability to apply mathematics and statistics in context is improved	Students are better able to apply mathematics and statistics in context	Feedback from teachers and students
13	The resources developed by the project to support the use of applied mathematics and statistics across a wide range of disciplines are widely available online	The resources are available through the national STEM centre and the MEI website and are promoted by the awarding bodies, subject associations, professional bodies and learned societies	Feedback from users of the resources
14	A project report is produced	The report will include a review of current level 3 mathematics across the curriculum, recommendations for new curriculum development to embed mathematics across the level 3 curriculum and case studies of engagement between schools, colleges and universities	Review of the project report

Appendix 2: Work accomplished by the end of the Integrating Mathematical Problem Solving Project

Outcome	What success will look like	What has been accomplished by end August 2012
A review of current curriculum development in level 3 mathematics is produced, including how mathematics is used within level 3 courses in other subjects	The outcomes of the review will be available online and will identify key areas where curriculum development is needed	The review has been completed and is available on the MEI website[35] (see Appendix 3).
Contexts and materials are trialled in schools and colleges	Resources are being trialled in 6–10 pilot schools and colleges	Staff from 27 schools and colleges agreed to trial resources associated with the pilot. Feedback has only been received from a minority of these but it has been all been positive.
Initial trialling of outreach activity is initiated between the university partners and local schools and colleges	3–5 universities are working with a total of 10–15 schools and colleges and 100 or more students are undertaking enrichment courses and/or GCE Extended Project Qualifications (EPQs) involving the application of mathematics and statistics	Outreach days organised by the Universities of Keele, Leeds, Manchester and UWE took place in June 2012. Over 120 students from over 12 schools and colleges attended. The emphasis of the days was a mixture of enrichment and preparation for EPQs.
Links with awarding bodies, professional bodies, learned societies and industry are consolidated	Awarding bodies, professional bodies, learned societies and industry are engaging with and promoting the programme	Contacts were made with AQA, Edexcel, OCR and WJEC leading to discussion concerning how best to encourage EPQs using mathematics and statistics. Professional bodies and learned societies have made helpful contributions to the review of level 3 qualifications and have helped promote the professional development courses associated with the project.
Further contexts and example investigation questions with supporting materials are developed	A total of 10 or more contexts and example investigation questions with supporting materials are available	16 contexts with supporting teaching materials have been developed. This has merely scratched the surface of what is possible.

Outcome	What success will look like	What has been accomplished by end August 2012
Professional development for the teachers involved is managed	At least 40 teachers have attended one of the project's professional development courses.	Two full-day professional development courses were planned and delivered. However, total attendance at these two days was only 19 teachers. Further professional development sessions based on the project materials were incorporated into other teacher professional development days, taking the total number of teachers reached to over 60. Overall feedback has been very positive.
Students' ability to apply mathematics and statistics in context is improved	Students are better able to apply mathematics and statistics in context	Feedback from students following the university outreach days shows increased awareness of the usefulness of mathematical and statistical techniques in a variety of contexts. Feedback was also received from students who used all the Mathematics of Business and Finance materials in their college. Students were engaged by the materials and could see the relevance to future careers in finance.
The resources developed by the project to support the use of applied mathematics and statistics across a wide range of disciplines are widely available online	The resources are available through the national STEM centre and the MEI website and are promoted by the awarding bodies, subject associations, professional bodies and learned societies	Until August 2012, the resources were freely available to teachers who requested a password. In September 2012, the resources were made freely available to all through the MEI website[36] and the national STEM centre; at that point the professional bodies who had engaged with the project were asked to publicise the resources.
A project report is produced	The report will include a review of current level 3 mathematics across the curriculum, recommendations for new curriculum development to embed mathematics across the level 3 curriculum and case studies of engagement between schools, colleges and universities	The review of level 3 mathematics across the curriculum is available on the MEI website. Appendix 3 of this report contains an extract from the review.

Appendix 3: Extract from *MEI Review of Mathematics in Other Subjects at Level 3*

The full review is available at www.mei.org.uk/files/pdf/Mathematics_in_Other_Subjects.pdf and includes details of the mathematics included in different subjects' level 3 curriculum specifications.

5. Is the mathematical content of other A levels sufficient to prepare students for further study?

The Institute of Physics report, *Mind the Gap* (2011)[37] reported as follows.

> A large proportion (92%) of academics felt that a lack of fluency in mathematics was an obstacle to students achieving their full potential in the long term, and more than four in five (85%) agreed that a lack of fluency affected their department's ability to deliver an optimal programme of study.

It should be noted that these findings referred to students who had both A level Physics and A level Mathematics. The report recommended changes to A level Mathematics to ensure that students' contextual understanding was better and an increase in crossover between A level Mathematics and A level Physics to ensure that students understand the connectedness of these subjects.

The UK Centre for Bioscience report, *A survey of the mathematics landscape within bioscience undergraduate and postgraduate UK higher education* (2011)[38], reported as follows.

> Students enter bioscience undergraduate degrees with a very wide variety of mathematics qualifications from A at A2 Maths to less than C at GCSE. This wide variation causes difficulty in designing appropriate courses.

University staff noted that students' fear of mathematics was a more serious problem than their lack of knowledge.

> 3.2.3 Students' attitudes (as reported by academic staff)
>
> A 'fear of maths' or 'maths-phobia' was commonly reported (12 out of 37 respondents) and it was noted that mature students in particular are more likely to lack confidence.
>
> *"But by far the biggest problem is the* fear *of maths. There is a culture amongst students, which is perhaps encouraged at school, in which it acceptable (almost fashionable) to treat maths as some kind of mystical dark art, sent to terrorise biologists. I am sure a more positive attitude would allow them to overcome most of the issues we encounter with the kind of basic maths we ask them to use/understand."*

> *"The key difficulty is not so much their lack of knowledge as their lack of confidence – an unwillingness to dig in and use number to solve problems and better understand biological systems."*
>
> Academic staff report that students often do not expect to need any maths within a biology degree and the requirement for mathematical skills comes as a surprise.
>
> *"However, there are a significant proportion of students attracted to Biology that are quite poor in their maths skills, having not done post-GCSE maths. The maths content of a Biology degree comes as quite a shock to these students. I believe there should be more maths in both GCSE and A level Biology to help secondary students understand that it is part of modern biology."*

The RSA report, *Solving the maths problem: international perspectives on mathematics education* (2012)[39] cited an earlier report from the University of Edinburgh: *Proposals to support and improve the teaching of quantitative research methods at undergraduate level in the UK*.

> Improving the mathematics curriculum and assessment could also drive up standards in English universities. Some universities do not advertise the level of maths needed to comfortably study particular subjects for fear of hindering applications. Furthermore, recent research suggests that universities are marginalising mathematical content in the delivery of degree courses because English students are not capable of studying it or sometimes because the limited mathematical facility of teachers renders it difficult for them to teach advanced mathematical content (MacInnes, 2009).[40]

Even if the mathematics specified within other A levels is appropriate, these reports suggest the mathematics which students learn pre-university often leaves them poorly prepared for the mathematical demands of degree courses in a wide range of subjects.

6. Areas for curriculum development

For students who do not choose to study AS/A level Mathematics, the following approaches represent possible ways to improve their knowledge of the mathematics and statistics required to support further study and employment.

A. New mathematics courses for post-16 students

B. Increasing the mathematical content of other A levels

C. Encouraging students to do Extended Project Qualifications which include mathematical or statistical problem solving

These are considered further below.

A. New mathematics courses for post-16 students

There are two obvious possible approaches in developing mathematics courses for students who do not wish to study AS/A level Mathematics as a subject in its own right, but who need to develop their mathematical and statistical understanding in order to be prepared for further study and employment in their chosen fields:

1. Courses tailored to the needs of specific kinds of student could be developed.

2. More general courses could be developed, suitable for a wide range of students who do not wish to specialise in mathematics.

Both these approaches have been tried; courses which are currently available are outlined below.

A1. Tailored mathematics courses

The Advanced Diploma in Engineering is a level 3 course, roughly equivalent in size to 3.5 A levels. The Diploma includes the following components:

- 540 guided learning hours (glh) Principal Learning
- 360 glh Additional or Specialist Learning
- 120 glh Extended Project
- 60 glh Functional Skills and personal learning and thinking skills
- 10 days work experience

Statistics available from www.jcq.org.uk show that the total number of students completing the Advanced Diploma in Engineering is as follows:

Year	Total number of students
2010	146
2011	177

The principal learning for the Advanced Diploma in Engineering includes a 60 guided learning hour compulsory mathematics unit. This is equal in size to one third of an AS in Mathematics.

Students can choose to take additional mathematics as part of the Additional or Specialist Learning in the Diploma. One option is to take the Level 3 Certificate in Mathematics for Engineering offered by OCR. This takes 180 guided learning hours and has been designed to enable students without A level Mathematics to progress to an Engineering degree.

The Level 3 Certificate in Mathematics for Engineering is one of two level 3 certificates offered by OCR.

Statistics from the OCR website show that the total number of entries for level 3 Certificates is as follows:

Session	Total number of entries
June 2010	12
Jan 2011	0
June 2011	18

Although it is possible for students to take an A level in Mathematics as part of the diploma, it is not known how many students have done this. It is clear that there have been very few entries for the tailored mathematics course. This may indicate that highly tailored courses that are not compulsory are unlikely to have much take-up.

A serious practical problem with having a variety of tailored post-16 mathematics courses is that many sixth forms have relatively small numbers of students* and so would find it difficult or impossible to offer a choice of mathematics courses to their students because of staffing and financial restrictions.

* In 2011 45% of schools and colleges offering A level Mathematics had fewer than 15 students taking it (DfE).

A2. General courses for students who do not wish to specialise in mathematics

AS and A level Statistics qualifications can provide useful preparation for the large number of students whose further study and employment includes the use of statistics. However, only a relatively small number of students currently take these qualifications.

AS Statistics is offered by AQA and MEI and the full A level is offered by AQA only. The table below shows the total number of entries.

Year	AS Statistics	A level Statistics
2010	1304	817
2011	1500	898

AQA's Level 3 Free Standing Mathematics Qualifications (FSMQs) were developed to enable students who do not take AS/A level Mathematics, but who are following a path that requires mathematics beyond GCSE level, to study some level 3 mathematics. They are designed to be accessible to students with a grade C or above at GCSE, and aim to support the mathematics needed in other disciplines. Each FSMQ is equal in size to one third of an AS level and it is possible to combine them into AS Use of Mathematics. An A level Use of Mathematics qualification is currently being piloted.

The table below shows the numbers of students taking FSMQs in the Use of Mathematics suite.

AQA level 3 FSMQs available to all centres, June 2011 entry	
FSMQ Algebraic and Graphical Techniques	850
FSMQ Modelling with Calculus	207
FSMQ Using and Applying Decision Maths	208
FSMQ Using and Applying Statistics	540
AS Use of Mathematics (based on FSMQs)	647
Pilot AQA level 3 FSMQs, June 2011 entry	
FSMQ Calculus	523
FSMQ Data Analysis	1947
FSMQ Decision Mathematics	1693
FSMQ Dynamics	171
FSMQ Hypothesis Testing	110
FSMQ Maths Principles for Personal Finance	132
AS Use of Mathematics (based on FSMQs)	1927
A level Use of Mathematics (based on FSMQs)	510

Compared to the number of students taking A level Mathematics each year (82,995 in 2011) the numbers taking Statistics and Use of Mathematics are very small. However, making such courses more widely available could result in increased numbers of students taking mathematics post 16. The Use of Mathematics pilot qualifications (and associated FSMQs) came under the scope of the *Evaluating Mathematics Pathways* project. Their May 2009 report[41] stated the following:

> One success criteria for the Pathways projects would be a substantial increase in the number of students studying mathematics post-16. Our evaluation suggests that FSMQs and AS/A2 Use of Mathematics qualifications provide a means of achieving this.

B. Increasing the mathematical content of other A levels

It is a reasonable expectation of students, universities and employers that A level courses should provide a suitable preparation for further study. For subjects where the mathematical and statistical needs are not extensive, it may be possible to include them within the A level specification of the relevant subject. This has the advantage of encouraging the required methods to be taught in relevant contexts. For this strategy to be successful, the following conditions need to be satisfied.

- Teachers need suitable resources and guidance
- Professional development needs to be available for teachers to enable them to teach the relevant mathematics and statistics with confidence
- Assessments must adequately reflect and assess the mathematical and statistical understanding that students should acquire for further study in the subject

For subjects with extensive mathematical and statistical requirements, additional courses will be required. For these subjects, it is essential that students, teachers and parents, as well as universities and employers, know that additional mathematics qualifications are needed to enable further progression.

C. Encouraging Extended Project Qualifications (EPQs)

It is encouraging to see the increasing take-up of the level 3 EPQ. (In 2009, 5094 students took the level 3 EPQ; by 2011 this figure had risen to 24,099 students (www.jcq.org.uk).) The EPQ seems an effective way to broaden and extend students' education and encourage the development of independent thinking and learning skills. It can also provide an important vehicle for students to demonstrate significant use of mathematical and/or statistical analysis in context. However, it would not be realistic or desirable to envisage a future where all students undertook a project involving mathematical/statistical analysis. Even if they did so, this would not ensure that they developed all the mathematical and statistical understanding necessary for further study. Consequently, other ways of enabling students to study more mathematics need to be adopted.

Although EPQs on their own are not the answer to ensuring that students are properly prepared for the mathematical demands of higher education and/or employment, it would be beneficial to encourage the use of mathematical and statistical analysis within EPQs across a wide range of subject areas.

Appendix 4: Contact with professional bodies, learned societies and awarding bodies

Professional bodies and learned societies	
Organisation	**Outcome**
The Association for Science Education (ASE)	Stella Dudzic presented a 15-minute slot in the open conference section of the ASE conference looking at one of the science IMPS resources. This was attended by PGCE students who expressed interest in the resources.
Association for Teachers of Psychology	A request for a session at their annual conference was received. Unfortunately, this clashed with another conference commitment so it was not possible to do this. However, information about the *Everybody Counts* professional development was sent to Helen Kitching at the Association.
BCS The Chartered Institute for IT	A meeting with BCS representatives Neil Brown, Roger Boyle and Adrian Oldknow on 25th April 2012 led to suggestions of resources which might be developed and a more general fruitful exchange of emails about promoting computing in education.
British Psychological Society (BPS)	No reply was received in spite of a follow up email.
Economics, Business and Enterprise Association (EBEA)	It was hoped to run a session at the EBEA teacher conference in 2012 about the Finance Baccalaureate, which uses some of the IMPS resources. This is now planned for 2013.
Geographical Association	It was too late to offer a session at their 2012 annual conference.
Institute of Physics	A meeting with Peter Main and Charles Tracy on 13th January 2012 to discuss the importance of mathematics in physics and the Advancing Physics A level led to ideas for resources about the mathematics in A level Physics. They advertised the professional development associated with the project.
Royal Society of Chemistry (RSC)	A meeting with Nicole Morgan on 27th April 2012 provided a useful overview of the mathematical resources provided by the RSC.
Royal Geographical Society	They sent helpful feedback on the draft curriculum review; a subsequent meeting with Steve Brace on 21st March 2012 included discussion of some useful ideas for contexts.
Royal Economic Society	Access to the IMPS resources was provided to the Royal Economic Society.
Royal Statistical Society (RSS)	A meeting with Roeland Beerten and David Walker took place on 16th August 2011 to discuss the importance of statistics and how MEI's work fitted in with the work the RSS are doing. This meeting also led to attendance at meetings with university social scientists and scientists to discuss the use of statistics in their subjects.

Professional bodies and learned societies	
Organisation	**Outcome**
Society of Biology	A meeting with Rachel Lambert-Forsyth to discuss the place of mathematics and statistics in biology and possibilities for future cooperation led to ideas for resources and advertising the professional development associated with the project in the Society's newsletter.
The OR Society	A meeting with Louise Orpin led to some ideas for resources and links for the EPQ page on the MEI website.

Awarding bodies	
Organisation	**Outcome**
Cambridge International Examinations (CIE)	Hilary Taunton at CIE sent Physics pre-U papers and mark schemes to allow us to see how mathematics is included in the assessments; this was helpful for the curriculum review.
AQA	A meeting on 3rd November 2011 provided useful information about EPQs. Charlotte Christie, who is in charge of the EPQ at AQA has stayed in touch and provided helpful feedback on the draft EPQ page; she was also involved in the outreach day in Manchester. AQA have passed on information about the professional development associated with the project to teachers.
Edexcel	Following contact with Jo Cleary, Edexcel have linked to the MEI page with information about EPQs.
OCR	There was some discussion at an OCR mathematics forum meeting of the possibility of a specialist EPQ in mathematics.
WJEC	A meeting on 16th January 2012 with Caroline Morgan and Tessa Gabriel-Davies provided some useful ideas for inspiration for extended projects, which are now on the EPQ webpage on the MEI website. There is further work to be done on supporting the use of statistics in EPQs, following up ideas from WJEC.

Appendix 5: The Finance Baccalaureate

The pilot Finance Baccalaureate, being developed and piloted at King Edward VI College, Stourbridge, from September 2011 involves students taking:

- A level Maths
- an A level in one of Business Studies, Economics or Accountancy
- a third A level
- a level 4 element in mathematics or finance
- an EPQ in finance or mathematics
- relevant work experience

This work is supported by RBS. The report of the first year's piloting is available at http://mei.org.uk/files/pdf/Finance_Baccalaureate_Report_for_External_Parties_May_2012.pdf

Some of the resources for the level 4 work are the Mathematics of Business and Finance resources from the Integrating Mathematical Problem Solving project.

Feedback from students using these resources showed that:

- students gained greater insight into A level economic concepts
- students gained greater awareness of the mathematical techniques used to calculate measures of income inequality
- students gained insights into the application of mathematical concepts to financial issues
- difficulty arose where students were unfamiliar with some of the software packages
- students were able to use online material if they could not attend the timetabled lessons, but it presented greater difficulties compared to students who attended the timetabled lessons
- students enjoyed the MEI material and would recommend the course to other students.

Appendix 6: A sample resource from the Integrating Mathematical Problem Solving Project

To assist the reader, the teacher notes and resources have been integrated here.

The Mathematics of Business and Finance – Multipliers

Mathematics ideas	Economics ideas
Infinite geometric series Reverse percentages	Marginal propensity to consume Marginal propensity to save The multiplier

A starter question (optional)

When you have left school and have a job, what will you do with the money?

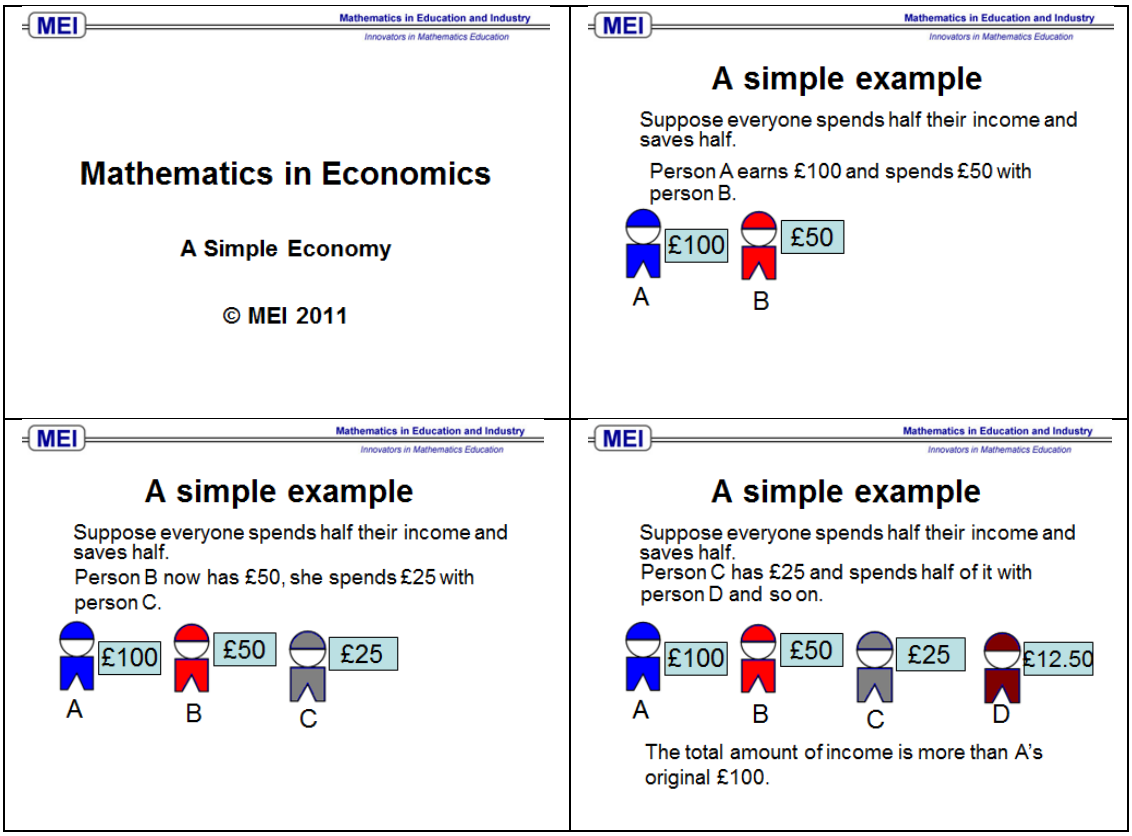
Once you have some suggestions, suggest the following categories:

- Spending on essentials (this cannot be reduced)
- Additional spending (this can be cut down if needed)
- Taxes
- Saving

Does that cover all the suggested uses of the money? Are there any which students did not think of?

A model of a simple economy

Consider a simple economy where everyone spends half their income and saves the other half. This is not realistic but reality is complicated and we need to simplify it to help us understand it. We will make things gradually more complicated.



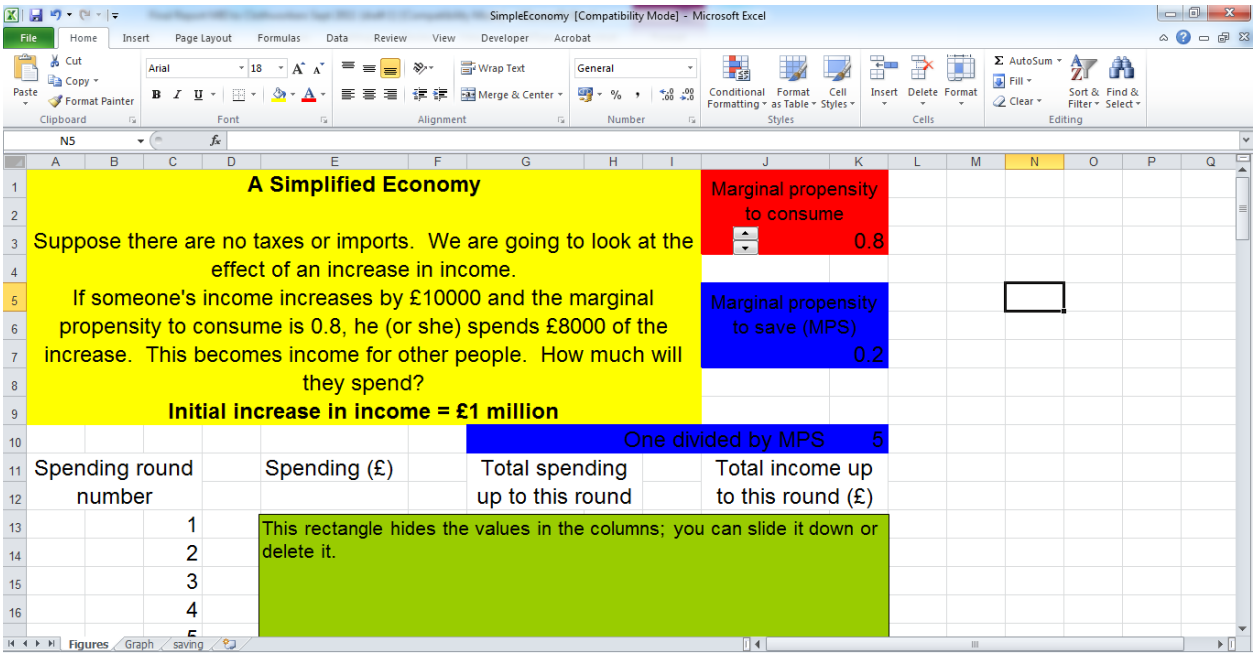
Simple Economy powerpoint

The powerpoint continues to develop the theme geometrically.

Concluding ideas from the Simple Economy powerpoint

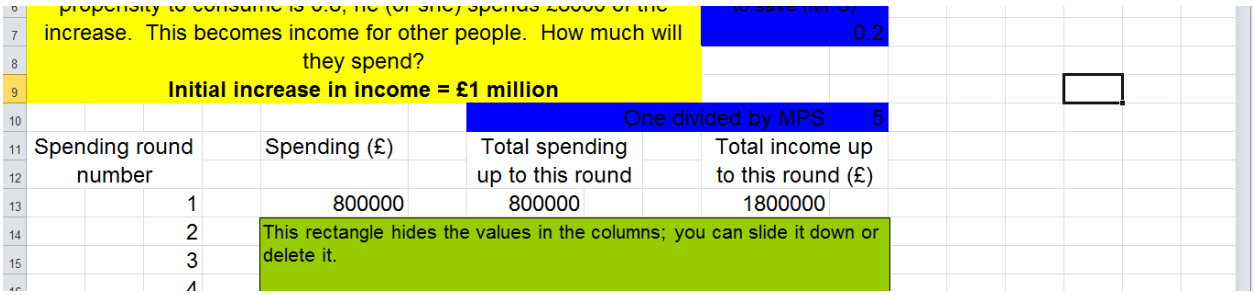
- Money circulates in an economy.
- If the amount spent in the economy increases, some of the money spent will be paid to other people and then be spent by them. So the total increase in spending is more than the initial increase in spending.
- The injection is the initial increase in spending. Multiplying the injection by the multiplier gives the total increase in spending.

Simple Economy spreadsheet



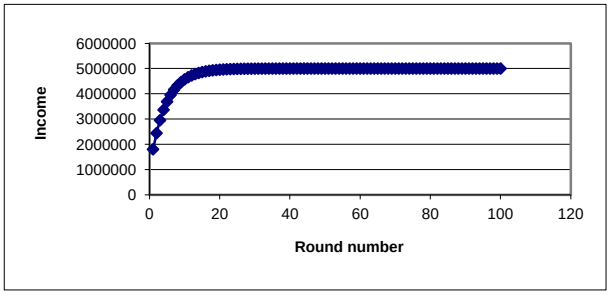
Questions to ask students:

- If the total increase in spending is £1 million and the marginal propensity to consume is 0.8, how much will the new spending be?
- This new spending is income for other people, how much will they spend? (You can slide down the green rectangle to reveal the figures a line at a time).



There is a graph on the second page of the spreadsheet. After only a few 'rounds' the total income settles at a value which can be read off this graph. Ensure students realise that the total income is the multiplier times £1million.

You can change the marginal propensity to consume by using the slider.



Geometric series and the economic multiplier

A geometric series is a sum where each number added on is the same multiple of the one before. The multiplier effect is an example of a geometric series.

Examples

3 + 6 + 12 + 24 (4 terms, common ratio 2)

1.2 + 12 + 120 + 1200 + 12,000 + 120,000 (6 terms, common ratio 10)

3 – 6 + 12 – 24 + 48 (5 terms, common ratio –2)

Write three more examples below and state the number of terms and the common ratio.

1.

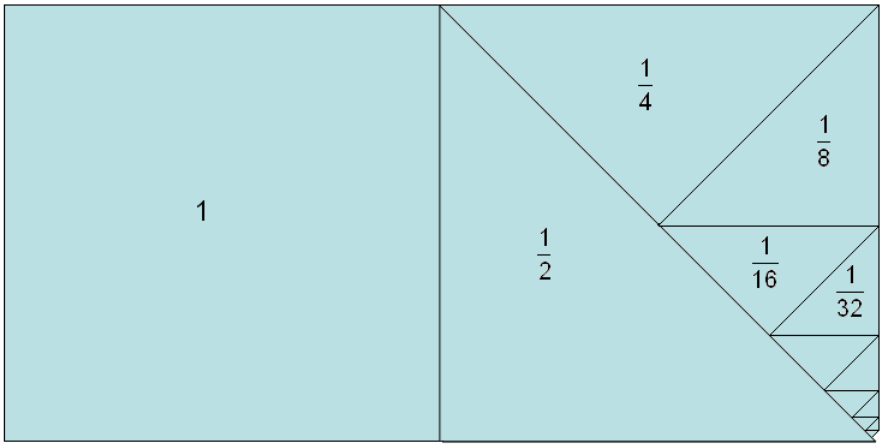
2.

3.

Infinite series

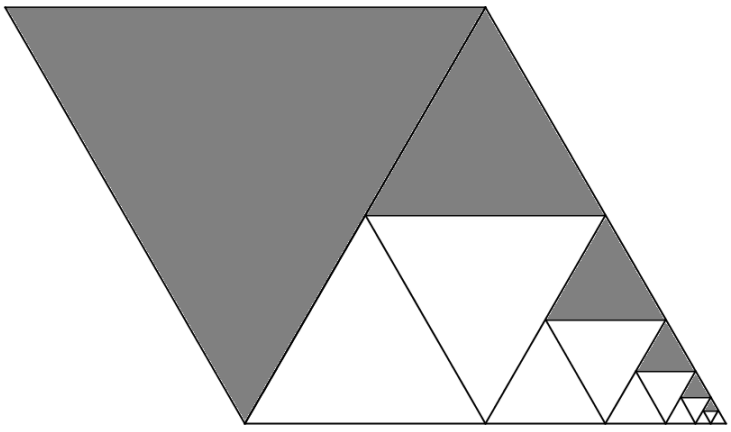
If the common ratio is between –1 and 1, it is possible to find a sum for an infinite geometric series.

Sometimes, it is possible to show this visually.



$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots=2$$

What infinite sum does the following diagram illustrate?



A formula for an infinite geometric series

Suppose the first term is a and the common ratio is r with $0 < r < 1$.

An algebraic approach

$S = a + ar + ar^2 + ar^3 + ar^4 + ar^5 + \ldots$

Multiply both sides by r

$Sr = ar + ar^2 + ar^3 + ar^4 + ar^5 + ar^6 \ldots$

Note that the sum goes on forever but the terms keep getting smaller and smaller.

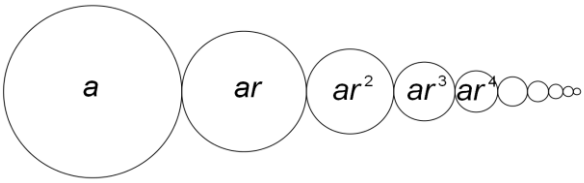
What is $S - Sr$?

How can you use this to get a formula for S?

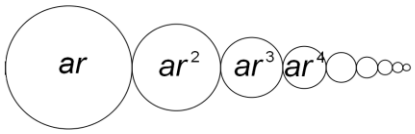
A visual approach

The area of the first circle is a. The area of each of the other circles is r times the area of the circle to the left of it.

The total area of all the circles is S.



Remove the first circle.



What area is left? How does it compare to the area that was there before?

How can you use this to get a formula for S?

The resources go on to explore how the multiplier can be calculated in a more complicated economy which includes taxation and exports.

Suggested further reading

'Much ado about multipliers' from *The Economist*, www.economist.com/node/14505361

Appendix 7: Teaching finance in mathematics at ages 11–18

In 2012, OECD PISA offered a financial literacy framework[42]. The report of the All-Party Parliamentary Group on Financial Education for Young People[43] then provided an example of how this framework could be incorporated into the UK curriculum; this was drafted with the assistance of Stella Dudzic, informed by the work done on the Integrating Mathematical Problem Solving project. This is summarised in section 1 below. In section 2, the financial literacy already present in the UK curriculum is summarised.

1 How financial literacy can be incorporated into the UK curriculum

Knowledge and understanding

Delivered in mathematics	Delivered in mathematics and reinforced in PSHE education	Introduced in mathematics and developed in PSHE education
Money and transactions Everyday payments, spending, value for money, bank cards, cheques, bank accounts and currencies	**Planning and managing finances** Measures of income, taxes and benefits, budgeting, planning ahead, benefits of medium and long term savings and investments, building human capital, smoothing spending through saving or borrowing **Risk and reward** Understanding why some financial products are riskier than others, limiting risk, the role of insurance products, credit, default, interest rates, exchange rates and market volatility	**Financial landscape** Understanding the choices available and the consequences of financial decisions, rights and responsibilities, redress, getting advice, consumer protection, effects of advertising and peer pressure on financial decisions, scams and financial crime

Key Stage 3 as part of mathematics

Competence statements *Students will:*	Notes
Money and transactions	
Be able to use simple percentages to work out sale prices mentally	Percentage reductions such as 10%, 20%, 25%, 50%, 70%
Be able to use estimation to work out a rough equivalent in pounds to a price in foreign currency, given the exchange rate	Students should be able to interpret an exchange rate given as, for example, 'We sell at 1.54, we buy at 1.69' to mean that it takes a little over 1.5 of the foreign currency to make £1

Key Stage 3 as part of mathematics and PHSE education

Competence statements *Students will:*	**Notes**
Money and transactions	
Be able to work out which item is the 'best buy'	Recognising that a 'special offer' does not always give the best value Recognising that buying more than is needed can be wasteful, even if price per unit is less The importance of personal factors when making decisions e.g. getting 200 free texts a month is not a good deal for people who rarely text
Planning and managing finances	
Be able to use a spreadsheet to draw up a simple budget	Use of simple functions only: addition, subtraction, multiplication and division Including awareness of fixed costs, such as rent or mortgage, and the importance of saving

Key Stage 4 as part of Applications of Mathematics

(Key Stage 3 work should also be assessed here, as appropriate)

Competence statements *Students will:*	**Notes**
Money and transactions	
Be able to work out VAT with the aid of a calculator	Reverse percentage calculations are not included
Be able to check a bill	Including taking account of special offers, discount and VAT
Be able to compare different offers for loans and savings.	Including taking account of charges and terms and conditions
Planning and managing finances	
Be able to work out the new income following a percentage change to income	Including wage rises and deductions from wages, such as National Insurance
Risk and reward	
Be able to use percentages when comparing investments	Including a comparison of percentages given for savings plans and percentage profit for a business deal

For students at KS4 who master the content above (Higher Tier)

Competence statements *Students will:*	**Notes**
Money and transactions	
Be able to do compound interest calculations with a calculator or spreadsheet	Contexts include savings and loans
Be able to set up a spreadsheet to do calculations involving percentages	Contexts include VAT calculations on invoices
Be able to use foreign exchange rate information to make calculations with the aid of a calculator or spreadsheet	Students should be able to interpret an exchange rate given as, for example, 'We sell at 1.54, we buy at 1.69' to mean that when converting from pounds to the currency, a customer gets 1.54 of the currency for £1, but when changing the currency to pounds, 1.69 of the currency is needed for £1
Risk and reward	
Be able to use the geometric mean to find an annual percentage increase for an investment	e.g. A house is bought for £100,000 and sold for £120,000 five years later. What is the average annual percentage increase in price?
Use percentages to work with appreciation or depreciation	Including comparison of an annual percentage depreciation (or appreciation) model with actual values
Financial landscape	
Be able to do reverse percentage calculations with the aid of a calculator	Include contexts such as charities claiming income tax back on donations
Be able to work out an inflation rate for a given time period from information such as values of RPI or CPI	Using a calculator or spreadsheet

Key Stage 4 as part of Applications of Mathematics and PHSE education

(Key Stage 3 work should also be assessed here, as appropriate)

Competence statements *Students will:*	**Notes**
Money and transactions	
Know that interest is often charged on loans and paid on savings	Including the implications of compounding interest at high percentages Students will not be expected to do the associated calculations but should be able to use paper based or IT based ready reckoners
Planning and managing finances	
Be able to calculate income tax, making use of a personal tax-free allowance and a percentage tax rate	Include awareness of current allowances and tax rates
Risk and reward	
Be able to make sensible decisions about buying insurance	Including use of 'best buy' comparisons and understanding different levels of cover
Understand the terms appreciation and depreciation and make related calculations when given a simple formula	Include the idea that some things which are bought are likely to go up in value and others down in value and how this can impact on decisions about purchase or insurance

Possible GCSE questions

1. Lorna wants to buy 4 peppers. She will not use more than 4.

   The prices for peppers are as follows.

Pack of 3 peppers £1.40

Special offer:

Buy one pack of 3 peppers, get a second pack half price.

1 pepper 59p

What is the most cost effective option for Lorna?

You must show working to support your answer.

2. This is Jason's credit card statement.

Previous balance	£420.15	Estimated interest assumes that the balance does not increase, you pay the minimum payment and the way we calculate interest stays the same. If you pay off the balance in full, you will not be charged any interest.
Payments received	£420.15	
New transactions and interest	£566.26	
Your closing balance	**£566.26**	
Next month's estimated interest	£6.71	
Minimum payment due	**£5.66**	

a) Assume that next month's estimated interest is correct. Jason decides not to use his credit card any more. He decides to make the minimum payment.

How much will Jason owe next month?

b) What advice would you give to Jason? You should use your answer to part a) in answering this question.

2 Financial literacy already present in the UK curriculum

Finance-related content of current Mathematics GCSEs

- Select and apply mathematical techniques and methods in mathematical, everyday and real-world situations
- Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions
- Use percentage and repeated proportional change
- Extract data from printed tables and lists
- Interpret a wide range of graphs and diagrams and draw conclusions

Finance-related content of pilot Applications of Mathematics GCSE

As for GCSE Mathematics, with the addition of the following content.

- Carry out calculations relating to enterprise, saving and borrowing, appreciation and depreciation and understand AER
- Use mathematics in the context of personal and domestic finance including loan repayments, budgeting, RPI and CPI, exchange rates and commissions
- Use spreadsheets to model financial, statistical and other numerical situations
- Construct and use flow charts

Appendix 8: Post-16 mathematics for all – a possible outline draft course

Competence statements ***Students will:***	**Notes**
Finance	
Be able to set up a spreadsheet to do calculations involving percentages	Contexts include VAT calculations on invoices
Be able to use foreign exchange rate information to make calculations with the aid of a calculator or spreadsheet	Students should be able to interpret an exchange rate given as, for example, 'We sell at 1.54, we buy at 1.69' to mean that when converting from pounds to the currency, a customer gets 1.54 of the currency for £1, but when changing the currency to pounds, 1.69 of the currency is needed for £1
Use percentages to work with appreciation or depreciation	Including comparison of an annual percentage depreciation (or appreciation) model with actual values
Be able to do compound interest calculations with a calculator or spreadsheet	Contexts include savings and loans
Be able to work out a monthly cost for a large financial decision such as buying and running a car or renting and running a home	Students should be aware of sources of relevant information and be able to use them They should be able to find relevant information from tables
Be able to use a spreadsheet to cost a project or business proposal, including allocation of costings to different contributors	Including monitoring whether the budget is being followed over a period of weeks or months
Estimation	
Be able to make a rough estimate without the use of a calculator	Examples include estimating the area of a room without the use of measuring instruments; having a reasonable idea of how much the total of a shopping bill will be; estimating how much an amount of foreign currency is worth in pounds
Make and justify upper and lower bounds for an estimate	
Understand that sometimes an estimate is all that is required and use estimates when checking exact calculations	

Competence statements *Students will:*	Notes
Working with large and small numbers	
Understand large or small numbers written in standard form	Standard form is sometimes called scientific notation
Interpret numbers in standard form in a spreadsheet or on a calculator	
Be able to write large or small numbers in standard form	
Use a calculator to calculate with numbers given in standard form	Including working in context with astronomy calculations, wavelengths and calculations involving atoms or cells
Use a spreadsheet to explore exponential growth and decay and represent it in a graph	Contexts include borrowing and saving money, bacterial growth and radioactive decay
Understand that a logarithmic scale uses multiplying to get to the next number	Include awareness of common logarithmic scales such as decibels, Richter and pH
Plot a graph using logarithmic scales on the axes and understand why these are sometimes preferable to linear scales	Using logarithmic graph paper or ICT
Working with formulae	
Use a spreadsheet to enter formulae using cell references	Include financial contexts but also exploring number patterns such as arithmetic and geometric sequences with a spreadsheet
Be able to interpret a formula given in terms of cell references	Students should be able to work out the result of a simple formula given in terms of cell references
Be able to use a spreadsheet to calculate a table of values for a polynomial function	
Be able to substitute numbers into a formula given in words or in algebraic form	
Be able to solve an equation arising from substituting numbers into a formula	
Be able to change the subject of a formula	

Competence statements *Students will:*	Notes
Introduction to calculus	
Be able to find the gradient of a straight line	
Interpret the gradient of a straight-line graph in context as a rate	
Know that the rate of change at a point on a curved graph is given by the gradient of the tangent	
Estimate gradients of curves by drawing tangents and interpret the meaning of the gradient	Contexts such as distance–time graphs to find the speed and graphs with no context
Use ICT to explore gradient functions for polynomials	
Be able to sketch the graph of the gradient function for a given curve	Include contexts such as sketching the speed–time graph for a given distance–time graph
Mathematical modelling	
Understand the modelling cycle and know that it allows mathematical methods to be used in many different areas	
Compare a given model with actual data using a graph	For example, compare an exponential growth model with actual population figures
Understand that a simplified model can give useful answers	
Use a simple model to make predictions	For example, use a straight-line demand curve to predict the change in revenue following a given change in price

Competence statements *Students will:*	Notes
Statistics	
Use the statistical problem-solving cycle	
Interpret a range of statistical diagrams and draw simple conclusions from them	Appropriate diagrams include the following: box and whisker diagrams dot plots scatter diagrams bar charts pie charts histograms
Be able to identify when a statistical diagram is misleading or inadequately labelled	
Be able to identify skew from a histogram or box and whisker diagram	Appropriate contexts include wage distributions and distributions of examination marks
Be able to relate statistical measures to statistical diagrams	For example, be able to put scatter diagrams in order of correlation, be able to find a median from a dot plot
Know that the Normal distribution is a model which can be used for real data	Know that the histogram is symmetrical for the population but understand that samples will differ
Be able to interpret a Normal probability plot printout from statistical software	
Measuring risk	
Understand risk given as either a probability or as 1 in *n*	Include moving between these forms
Interpret two-way tables	Include finding probabilities from two-way tables
Understand the difference between dependent and independent events	Contexts include gambling and risk of suffering from diseases
Understand that risk assessment involves measures of likelihood and impact	
Use tree diagrams to work with conditional probability	Including the problem of the false positive Students should work with frequencies in tree diagrams; some may extend to working with probabilities

References

1 ACME (2011) *Mathematical Needs: Mathematics in the Workplace and in Higher Education* and *Mathematical Needs: The Mathematical Needs of Learners*; available at www.acme-uk.org/media/7624/acme_theme_a_final (2).pdf and www.acme-uk.org/media/7627/acme_theme_b_final.pdf

2 Hodgen J. *et al* (2010) *Is the UK an outlier?*, The Nuffield Foundation; available at www.nuffieldfoundation.org/sites/default/files/files/Is the UK an Outlier_Nuffield Foundation_v_FINAL.pdf

3 Savage M. and Stripp, C. (2009) *Newton's Mechanics: Who Needs It?*, The Maths, Stats and OR Network; available at www.mathstore.ac.uk/repository/NewtonMechReportFinal.pdf

4 Smith, A. (2004) *Making Mathematics Count: The Report of Professor Adrian Smith's Inquiry into Post-14 Mathematics Education* (*The Smith Report*) The Stationery Office Ltd; available at www.education.gov.uk/publications/standard/publicationdetail/page1/MMC

5 ACME (2011) *Mathematical Needs: Mathematics in the Workplace and in Higher Education* and *Mathematical Needs: The Mathematical Needs of Learners*; available at www.acme-uk.org/media/7624/acme_theme_a_final (2).pdf and www.acme-uk.org/media/7627/acme_theme_b_final.pdf

6 Gove, M. (2011) Speech to the Royal Society on maths and science; available at www.education.gov.uk/inthenews/speeches/a00191729/michael-gove-speaks-to-the-royal-society-on-maths-and-science

7 Vorderman, C. *et al* (2011) *A world-class mathematics education for all our young people*; available at http://bit.ly/VordermanMaths

8 Ofqual (2012) *International Comparisons in Senior Secondary Assessment*; available at www.ofqual.gov.uk/downloads/category/96-international-comparability

9 Institute of Physics (2011) *Mind the Gap*; available at www.iop.org/publications/iop/2011/page_51934.html

10 SCORE (2012) *Mathematics within A-Level Science 2010 Examinations*; available at www.score-education.org/policy/qualifications-and-assessment/mathematics-in-science

11 Nuffield Foundation (2012) *Mathematics in A level assessments*; available at www.nuffieldfoundation.org/sites/default/files/files/Maths_in_A_level_Assessments_Nuffield Foundation_WEB.pdf

12 Ofqual (2012) *Fit for Purpose? The view of the higher education sector, teachers and employers on the suitability of A levels*; available at www.ofqual.gov.uk/files/2012-04-03-fit-for-purpose-a-levels.pdf

13 Ofqual (2012) *International Comparisons in Senior Secondary Assessment*; available at www.ofqual.gov.uk/downloads/category/96-international-comparability

14 Porkess, R (2012) *The Future of Statistics in our Schools and Colleges*, Royal Statistical Society and the Actuarial profession; available at www.rss.org.uk/uploadedfiles/userfiles/files/The%20Future%20of%20Statistics%20in%20our%20Schools%20and%20Colleges.pdf

15 MEI (2012) *MEI Review of Mathematics in Other Subjects at Level 3*; available at www.mei.org.uk/files/pdf/Mathematics_in_Other_Subjects.pdf

16 www.mei.org.uk/index.php?page=IMPS&section=curriculum

17 For further information see www.mei.org.uk/index.php?page=extendedprojects&section=curriculum

18 www.manchester.ac.uk/undergraduate/schoolsandcolleges/post-16/epq/

19 www.mei.org.uk/index.php?page=extendedprojects&section=curriculum

20 For further information see www.hestem.ac.uk/activity/higher-level-skills-he-stem-students-mathematical-modelling-and-problem-solving

21 For further information see http://web.aqa.org.uk/qual/projects/extended-project-epq.php

22 Institute of Physics (2011) *Mind the Gap*; available at www.iop.org/publications/iop/2011/page_51934.html

23 MEI (2012) *Fluency and Problem Solving in Primary Mathematics: MEI's response to the draft primary national curriculum*; available at www.mei.org.uk/files/pdf/Primary_NC_Jun_12_MEI_response_(final).pdf

24 For more information see www.wjec.co.uk/index.php?subject=211&level=15

25 Ofqual (2012) *Fit for Purpose? The view of the higher education sector, teachers and employers on the suitability of A levels*; available at www.ofqual.gov.uk/files/2012-04-03-fit-for-purpose-a-levels.pdf

26 Porkess, R (2012) *The Future of Statistics in our Schools and Colleges*, Royal Statistical Society and the Actuarial profession; available at www.rss.org.uk/uploadedfiles/userfiles/files/The%20Future%20of%20Statistics%20in%20our%20Schools%20and%20Colleges.pdf

27 For more information see www.uea.ac.uk/edu/mumap/home

28 Hernandez-Martinez, P. *Mathematical Modelling and Problem-Solving in Materials Engineering*; available at http://www.esrc.ac.uk/my-esrc/grants/RES-189-25-0235/outputs/read/c9d45b45-adcb-4e35-b105-05945d7da1f4

29 http://gowers.wordpress.com/2012/06/08/how-should-mathematics-be-taught-to-non-mathematicians/

30 Ofqual (2012) *A Level Reform Consultation*; available at www2.ofqual.gov.uk/files/2012-06-18-a-level-reform-consultation.pdf

31 MEI (2012) *Ofqual A level reform consultation: Response from MEI*; available at www.mei.org.uk/files/pdf/A_level_reform_consultation_MEI_final_response_11_September_2012.pdf

32 MEI (2012) *Mathematics and A level Reform, an MEI position paper*; available at www.mei.org.uk/files/pdf/Mathematics_and_A_level_Reform_an_MEI_position_paper_September_2012.pdf

33 MEI (2012) *Mathematics for all Post-16: An MEI position paper*; available at http://www.mei.org.uk/files/pdf/Mathematics_for_all_post_16.pdf

34 MEI (2012) *How might A level Mathematics be improved? An MEI position paper*; available at http://www.mei.org.uk/files/pdf/Improving_A_Level_Mathematics_final.pdf

35 MEI (2012) *MEI Review of Mathematics in Other Subjects at Level 3*; available at www.mei.org.uk/files/pdf/Mathematics_in_Other_Subjects.pdf

36 www.mei.org.uk/index.php?page=IMPS&section=curriculum

37 Institute of Physics (2011) *Mind the Gap*; available at www.iop.org/publications/iop/2011/page_51934.html

38 Koenig, J. (2011) *A survey of the mathematics landscape within bioscience undergraduate and postgraduate UK higher education*; available at www.bioscience.heacademy.ac.uk/ftp/reports/biomaths_landscape.pdf

39 Norris, E. (2012) *Solving the maths problem: international perspectives on mathematics education*, RSA ; available at www.thersa.org/action-research-centre/education/solving-the-maths-problem-international-perspectives-on-mathematics-education

40 MacInnes, J. (2009) *Proposals to support and improve the teaching of quantitative research methods at undergraduate level in the UK*, University of Edinburgh; available at http://aqmen.ac.uk/sites/default/files/Final_Report__Strategic_Adviser_for_Quantitative_Methods_tcm6-35465.pdf.

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41 Noyes, A. *et al* (2009) *Evaluating Mathematics Pathways: Interim Report – Executive Summary*; available at www.nottingham.ac.uk/emp/summaries/qcareportapr09execsum.pdf

42 OECD (2012) *PISA 2012 Financial Literacy Assessment Framework*; available at www.oecd.org/pisa/pisaproducts/pisa2012draftframeworks-mathematicsproblemsolvingandfinancialliteracy.htm

43 All-Party Parliamentary Group on Financial Education for Young People (2012) *Financial Education & the Curriculum*; available at www.pfeg.org/sites/default/files/Doc_downloads/APPG/Financial%20Education%20%26%20the%20curriculum%20-%20Final%20report%20-%20APPG%20on%20fin%20ed%20for%20YP%20-%20Dec%2011.pdf