Autumn 2020

Additional mathematics papers for entry to English universities

Their role, purpose and impact

A REPORT FOR THE JOINT MATHEMATICAL COUNCIL OF THE UNITED KINGDOM published version

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Contents

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The findings reported here are of course entirely dependent on the academics, teachers and students who participated in the underlying study, and who gave very generously of their time and reflections.

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Executive Summary

This report is of a small study funded by the Joint Mathematical Council of the UK, JMC, to investigate the impact and use of additional mathematics papers for entry to English universities. By drawing on the voices of academics, providers of support, teachers and students, the study provides evidence of the experience of these complementary users. In particular, it sought evidence from a ‘telling’ sample of state-funded ‘strong’ mathematics departments, so as to better understand gains and costs to students and teachers, and any inequity of access.

The study was undertaken following concern raised by some users at the increasing number of such papers, a concern which exposed wider reservations about their use and impact. In this executive summary we outline the background and main findings, embed those findings in slightly broader themes, and raise some questions for consideration.

Background

Additional mathematics papers now (in 2020) comprise ‘MAT’, ‘TMUA’ (both taken in October of year 13), ‘STEP’ papers 1[1], 2 and 3, and the Advanced Extension Award AEA (all taken in the summer of year 13). These papers currently have a mandatory or optional role in mathematics-dependent offers made by at least fourteen English universities. At least 7,000 such papers are taken in a typical recent year. Local support for preparation is complemented by a range of highly-regarded teacher/student support available from the DfE-funded Advanced Mathematics Support Programme AMSP, as well as online from the Cambridge STEP Support Programme, ‘nrich’, and other websites.

Summary of findings

- It was widely recognised by academics, teachers and students that the most competitive few university mathematics courses need recourse to selection mechanisms that go beyond A Levels and interviews – and that reliable interviews need considerable investment.
- There was also a widespread, though not universal, view articulated that current use of additional papers goes well beyond selection needs.
- Academics from departments which mandate, or encourage, the use of additional papers, claimed a variety of reasons. Market share, department status and numbers control were often at the forefront of responses. Interviewees also talked about the development of more robust mathematical problem-solving skills, of enrichment, challenge and independence, and often cited a mixture of mathematical and strategic reasons.
- Most teachers felt engagement with such papers was appropriate only to those easily mastering A Level work. They concurred with the above range of potential mathematical and study skill (and so, transition), benefits for such students, if there was local preparatory support available.
- Both students and teachers were concerned about significant stress associated with such aspirational goals, particularly with papers sat at the end of year 13, and especially if local support and information were limited. They talked about loss of confidence, and sometimes, unnecessary loss to mathematics as a result of ill-judged engagement.
- There is some concern from academics and teachers about the non-regulated nature of papers other than AEA, in an HEI system that has large public funding.

[1] In October 2020 it was announced STEP 1 will no longer run.

- ‘Strong’ mathematics departments[2] in state schools/colleges vary enormously in their willingness and ability to support locally: some such discourage any such engagement from students, with many having no, or just one, teacher able to do so directly. Others, including some grammar schools, are offer a coherent and sustained enrichment/support programme throughout students’ time in the school. Often, such work is additional to full teaching workloads, and would be at the expense of other interventions.
- Respondents commonly recognised that equity of opportunity for university access resulted from a complex set of intersecting factors including: the quality of mathematics teaching available, teacher workload and priorities, socio-economic background, gender, financial costs (including for widespread private tuition for these papers), regulation for only AEA, and recent proliferation of use of additional papers.

Note also that:

- Academics recommend engagement with these papers by late year 12 at the latest, though many teachers and students seem unaware of that.
- Academics sometimes had comparatively low expectations, e.g. accepting a mediocre performance in TMUA in lieu of an A grade or higher in A Level Mathematics.
- Academics were more sanguine about the effectiveness of online support as a single approach, than teachers or students were.

It is clear this is a complex field, with potential considerable benefits for well-supported, mathematically aspirational students, but also some apparent and significant inequities.

Main themes

The high-level concerns of this report are about mathematical learning, access and equity, transition, etc. related to additional mathematics papers. There are, however, some other emergent points that are not directly focused on additional papers but that are nevertheless important.

In schools and colleges

For the most mathematically aspirational students, engagement with additional entrance papers can support transition to university – but so can independent and collaborative work with other, maybe less demanding problems, and for a wider range of students. Students with mathematical aptitude benefit from early, inclusive exposure to wider and deeper mathematics than in common curriculum enactments (and from early and continuing conversations about university pathways). Widespread use of such approaches depends on valued, planned and widely-available curriculum enrichment and subject-knowledgeable, skilled teachers. However, non subject-specialist *observers* may not be able to discriminate between form and substance in such teaching.

Qualifications and education policy

The proliferation of additional papers has happened in part because A level qualifications were perceived to be insufficient to support preparation and selection for many of the most competitive mathematics-based university courses. Currently, there is little evidence that student experiences of new mathematics A Levels are more effective for these purposes, with widespread concern that deep and confident engagement with the range of expected content and processes is prohibitive for a large proportion of the target cohort of students. However, there is concern about the

[2] In this report, selected as including at least one (primarily) mathematics graduate, teaching Further Mathematics, and enjoying a strong Ofsted rating: such departments might reasonably be expected to be well-placed to support preparation for additional papers should they choose to do so.

marketisation, regulation and pressures of additional papers, and also about the capacity of much of the mathematics teaching force to prepare students effectively for university mathematics.

Support mechanisms

There are copious and praiseworthy attempts to provide freely-available external support for additional papers. These inevitably only begin to even the playing field, and there are large 'cold spots' where frequent face to face support and a good depth of intellectual peer co-working is not easily accessible. However, many teachers, even in centres which habitually enter students for these additional papers, are unaware of the range of remote support freely available – or of the difference that sustained and deliberately ramped and supported preparation can make. At a younger age, preparation and inclination for post-16 and further engagement with mathematics would be enhanced by availability of a structured enrichment programme that could be accommodated within standard classroom provision, as well as occasional use of nrich (www.nrich.maths.org), UKMT (UK Mathematics Trust) and similar questions.

Entry to mathematics-intense courses in universities

Full and balanced engagement with the range of intentions of the new mathematics A Levels is not yet being achieved, and would support a wider range of students in successfully transitioning to mathematical work at university. Comparative studies of approaches to university mathematics preparation and selection in jurisdictions without structural selection, or well-resourced independent education, might illuminate approaches to current inequities in access.

Additional entry requirements can narrow the pre-university experience, as well as deter potential mathematics applicants. Promotion of related ways of working, without participation in the related examinations, would reduce perceived pressures, and also widen valued mathematical activity.

There are inbuilt equity issues in the university applications process, that go beyond the mathematics made available to students or preparation for additional papers, e.g. variable degree of support for writing personal statements, variable grasp of the far-from-transparent relationship between indicative offers and grades achieved, unknown valuing of extracurricular activities or of independent additional mathematical activity. There is also a need for better analysis and understanding of inequity of access in relation to university admissions policies. For example, further disaggregation of entry statistics by source institution, separating out e.g. selective and non-selective state schools and colleges, might be illuminating.

Questions for consideration

- How can we ensure that every secondary mathematics department has at least one deeply mathematically confident and competent teacher?
- How can mathematics teachers in schools and colleges be encouraged and supported to teach for the full breadth and depth of intentions of national curricula and A Levels, and to include structured and progressive enrichment opportunities within their teaching?
- How could universities, and well-placed schools and colleges, more effectively support wider access to mathematically demanding face to face activities and a good depth of intellectual peer co-working, for students with unusual mathematical aptitude?
- Should universities be encouraged to co-operate on developing an alternative, regulated assessment to replace STEP1, TMUA and AEA, where such an assessment is needed?
- If students are required to take additional papers, or it is advantageous to the offer they receive, should universities be required to pay related entry fees, as is done for MAT?

- Does our current extent of pre-university specialisation exacerbate challenges in equity of access to the most mathematically demanding courses? What would be the implications of changing that early specialisation?
- How do other jurisdictions provide for equitable development of particular mathematical potential, and from an early stage?

1. Introduction

For many years, the most competitive English universities have based at least part of their mathematics student selection on the outcomes of additional paper(s) taken in addition to A Level or equivalent papers, and sometimes well before (or historically, well after) those. Until the 1980s only Oxford and Cambridge used such papers, but in parallel with greater participation in A Levels (now just over 50% of school students in England take at least one), other universities have begun to make either mandatory or optional use of such papers. In this document, we largely refer to A Levels as these comprise by far the most common route to university for most undergraduates in England. Universities also consider entry via other routes such as Scottish Highers, International Baccalaureate, etc, and generally treat those students in broadly comparable ways to those reported here for A Level students. Although input to the study was offered via a range of professional and social networks described below, no contribution came from beyond England.

The report is structured so as to first outline the range of such papers, as of Spring 2020, and indicate some ways in which English universities currently use those to inform admissions. It summarises what is already known about their impact from the published literature, and so, the outstanding research questions, around which the study was organised. Chapter 3 outlines the methodology adopted, and includes a discussion of the limitations of the study, and then chapters 5, 7-10 focus on findings in relation to each of the research questions in turn, drawing on specifics from data. The exception to this pattern is chapter 5, which presents a brief pen-portrait of organisation, experience and response to work around these papers in each of four state-funded centres apparently well-placed to support students in preparation. Finally, chapter 11 draws together some conclusions and possible points for reflection. It is important to stress that the object of the study was to garner, and to some extent draw together, a variety of lenses on the use and impact of additional mathematics papers used for English university entrance. There is no value judgment intended of any practices or perspectives reported.

Notes on reporting:

- In the chapters focused on findings, different lenses on research questions are usually addressed in the order HEIs, then support providers, then teachers, then students, since support providers are often an interface between HEIs and schools/colleges, and teachers often mediate student interaction with additional papers.
- When the report makes direct quotations, ‘CS’ refers to a college student, ‘ST’ to a school teacher, ‘Supp3’ to a support provider and ‘HEI17’ to an academic. The prefix ‘G’ is used when the response arises from a Googlesurvey. An indicative participant list is given in Appendix 1. Anonymity is further supported by arbitrary use of gendered language (he/she, etc). Occasional pseudonyms are used for animation purposes, instead of such codes.
- In several places there is a need to refer to teachers’ mathematical backgrounds, and given the range of degrees now available, a ‘mathematics degree’ should be taken to mean ‘a degree in the mathematical sciences’ whereas e.g. ‘a mathematics-intense degree’ indicates one where mathematics is a pervasive and critical aspect of the degree, as in Physics, Engineering. These terms are used since the DfE in England recognises as eligible for ‘mathematics specialist teacher’ training any graduate for whom it can be argued that their degree contained at least 50% mathematics, and such arguments are commonly made for degrees such as Psychology, Geography, Biology etc.

In parallel with this technical report, there is a summary report of a more easily digestible length. Analysis and interpretation were supported by Sanelf Hussein, UCL postgraduate student.

2. Background

We first give some background information about each of the additional papers considered, and indicate some of the ways in which they are used for admission to English universities to read mathematics. We then outline what is already known about their use and impact. That enables us to identify the gaps in knowledge, and so, the research questions addressed by this study

The papers

STEP papers ('Sixth Term Entry Paper') are administered by Cambridge Assessment. STEP papers 1[3], 2 and 3 are taken at the end of June of year 13 and are based A Level Mathematics, AS Further Mathematics and A Level Mathematics, and A Level Further Mathematics respectively. They are graded, in descending order, S,1,2,3,U, when credit is given to the six best responses to 11 or 12 questions. A typical offer to read mathematics at Cambridge, following an intensive interview and inter-college discussions, will require top grades in 3 A Levels plus grade 1s (or sometimes 2, depending on context) in STEPs 2 and 3:

> *For maths we interview well over 80% of students who apply….We have 5/6 applications per place. We've thought long and hard about subjectivity in interviews….Our interviewers are all trained very carefully …and the type of question, the way the interview is done, is uniform among colleges. We also have diversity and equality training etc. There are always two interviewers: we try to make the process as free from bias as possible. So we have multiple sources of information. We invariably want the student to get stuck - dealing with that is so important. And how they think mathematically and deal with hints. We feel it is particularly effective with experienced and well-trained interviewers. That is an enormous investment - it's the training, preparation, interviews, hours of discussing afterwards to try to make the best decision* (HEI9).

The ratio of such offers made to places available is more than 2:1, so these examinations are very high stakes. STEP I is used in offers from Cambridge to read Economics or Engineering, and is also used as one of a set of offer options by at least ten other universities, largely for courses in mathematics. In 2019 there were ~2000 entries for STEP1 (Supp4), with fewer[4] for STEPs 2,3. Cambridge typically make about double the number of STEP-dependent offers as they have places, and rather fewer than half of such offer-holders will meet the requested grades.

MAT (the 'Mathematics Admissions Test') is again administered by Cambridge Assessment, and is based on a typical (but not universal) content of the first four terms of A Level Mathematics. It is taken near the beginning of November of year 13. The first ten questions are multiple choice, and those are followed by 4 extended questions, in parallel sections for those wanting to study mathematics, or mathematics and computing. MAT is also normally required for entry to Imperial College to read mathematics, and is now available as an option associated with modified offers, by a small number of other universities. Scores are not normalised, but are reported to authorised users as a percentage along with contextualising summary statistics:

> *All Oxford applicants for mathematics or maths with computer science take MAT before interview, and performance on that is just one aspect of the information colleges use. They go to*

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[3] In October 2020 it was announced STEP 1 will no longer run.

[4] *in 2019 there were 2137 candidates for STEP 1. There were 4665 papers sat altogether (1549 for STEP 2 and 979 for STEP 3). This represents 3046 candidates (of whom 528 had conditional offers for Cambridge, 493 of them for mathematics).*

some lengths to select students who would benefit from the course, and whom they'd like to work with – it's an enormous investment, with lots of backup processes to ensure everyone with the potential to succeed here is seen. I securely share maths scores of Durham and Bath's candidates, without revealing details. This year all students can request their MAT score as part of our feedback, so I suppose they could take that score to any other institutions (HEI4).

TMUA ('Test of Mathematics for University Admission') was first set in 2017, and is scheduled for the same day as MAT papers, near early November students' year 13. It consists of two 75-minute papers based on mathematical thinking and mathematical reasoning respectively, each drawing on a slightly expanded AS Mathematics content, and each consisting of 20 multiple choice questions. A variety of universities (at least ten: Cambridge, Warwick, Bath, Durham, LSE, Sheffield, Lancaster, Southampton, Nottingham, Cardiff) use TMUA as either a compulsory or, more commonly, optional part of entry offers associated with slightly relaxed other demands, for courses in mathematics, economics, and/or computer science. Results are reported on a scale to 10, and students can sometimes choose whether to share their entry and results with the institutions they are applying to, so it is often billed as a 'no-risk' option.

AEA ('Advanced Extension Award'), is taken at the end of June in year 13, and is based on the A Level Mathematics common core. AEA is the (only surviving) successor to historical 'Scholarship' or S-level papers associated with earlier A Levels. Results are reported on a Distinction/Merit/Pass/Ungraded basis, and are typically used as one optional path to a slightly reduced A Level requirement, although, with STEP I, AEA is sometimes also used as enrichment/challenge for high-attaining year 12 students.

We note that in this system, Autumn papers clearly lead to more secure university offers during the course of year 13, and so possibly a less stressful summer season; Summer papers can accommodate those who make later decisions about their preferred university pathways, and take account of the learning accumulated throughout year 13.

English university admission use

As indicated above, English 'selecting' university mathematics offers, typically made before students have taken their A Level examinations, can be couched very variably. Other universities, typically but not exclusively less competitive, do not include performance in additional papers in their offers. For students wanting to read mathematics, typical entry requirements for Autumn 2020 as cited on their websites include:

- *At King's College London:* A*AA at A Level. Must include a grade A* in Mathematics and Grade A in Further Mathematics. We will accept Further Mathematics AS-level grade A only if you additionally achieve a 3 in any STEP paper or a Merit in AEA Mathematics.
- *At Durham:* A*A*A or A*AA Mathematics/Further Mathematics/ano including a score over 6.5 in TMUA or equivalent in MAT or getting a 1 in any STEP. Or A* in Mathematics, A in Further Mathematics AS Level, and two further A Levels at grade A plus above TMUA/MAT/STEP as above.
- *At UCL:* A*A*A in Mathematics/Further Mathematics/ano or A*AA + 2 in any STEP paper or Distinction in AEA.

- *At Bristol:* A*A*A with A* in Mathematics and A in another mathematics-related subject, or A*AA with A* in Mathematics and A in Further Mathematics. No mention of additional papers.
- *At Lancaster:* AAA including Mathematics or AAB including Mathematics and Further Mathematics. One grade lower for a 3+ in any STEP paper, or 4.5+ in TMUA.

Note that these are only nominal requirements, which are often lowered once results are published in August, especially in less competitive years.

Trustees of the Joint Mathematical Council of the UK ('JMC') have a particular interest in related issues since the variety of additional papers, together with a recent proliferation in their use, bring with them obvious questions around equity of access to competitive universities to read mathematics, and of opportunity for young people to optimise their mathematical capabilities, both at school and in Higher Education. There is also potentially longer-term impact of differential opportunity at this stage, together with a concern that additional hurdles beyond A Levels might discourage some students from applying for mathematics at all. This is of particular concern when in 2019 A Level Mathematics entries were down by 9% compared with 2018, and A Level Further Mathematics entries were down by 10%[5]. UCAS applications to read mathematical sciences were also down by 18% for home students and 10% for overseas students, these despite fairly stable A Level entries overall, and rising UCAS applications[6].

What do we already know?

Since universities in England are autonomous bodies, there has been comparatively little scrutiny of admissions practices in relation to degrees in mathematics that require, or offer advantage to, students taking additional papers prior to university entrance, often alongside their A Level examinations. MAT, STEP and TMUA area all unregulated qualifications, administered through Cambridge Assessment, so the range of entry and performance statistics in the public domain is very much reduced compared with regulated qualifications. Ellie Darlington of Cambridge Assessment has pursued some related issues, exposing the different mathematical demands of MAT, AEA and STEP papers[7] in Darlington (2015). To this suite has since been added the Cambridge Assessment TMUA examination. Related published work includes accounts of impact on undergraduates (e.g. Darlington and Bowyer, 2018), and a MATH taxonomy (Smith *et al.*, 1996) that can be used to analyse the demand profiles of different papers, including (in Darlington, 2014) for comparison of undergraduate and A Level papers.

Darlington (2014) argues that there was evidence of a significant 'gap' between the mathematical preparation offered by participation in A Levels in mathematics, and undergraduate mathematics, although with new, more mathematically demanding A Levels for first mass examination Summer 2019, and designed to be more aligned with universities' mathematical requirements (ALCAB, 2014), any current gap is barely yet evidenced. Globally, though, we have considerable evidence suggesting that transition to university mathematics-intensive study involves

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[5] https://www.jcq.org.uk/examination-results/a-levels/2019/main-results-tables
[6] https://www.ucas.com/data-and-analysis/undergraduate-statistics-and-reports/ucas-undergraduate-releases/applicant-releases-2019-cycle/2019-cycle-applicant-figures-30-june-deadline
[7] The MAT challenges students by requiring them to solve unfamiliar problems which do not have an obvious method towards a solution. AEA Mathematics challenges students by requiring in-depth use of the content of A-level Mathematics in order to solve more complex versions of A-level Mathematics style questions. STEPs, on the other hand, combine the use of more advanced mathematics with some questions more familiar to A-level Mathematics students, as well as questions which require deeper mathematical thinking. In doing so, STEPs give students an insight into what undergraduate mathematics assessment (editor: at the target institutions) might be like. (Darlington and Bowyer, 2015, p.126).

shifts in epistemological rigour, formalization and abstractness, in forms and purposes of organization, and in personal capacities (e.g. Gueudet, 2008), so there are obvious questions about how and where that is managed. Golding (2020) identifies ways in which it is possible to being to support that transition at school level. University mathematics departments also differ in what they actually value: Darlington (2014) evidences some university assessments where it is possible to achieve a First class award purely through reproduction of lecture notes.

Darlington and Bowyer also write (2018, p125) ‘Students of the mathematical sciences are more likely to achieve top grades at A-level than students in other subject areas, with 8.1% of them achieving three or more A grades at A-level (Vidal Rodeiro & Zanini, 2015), a figure much higher than in other degree subjects. Furthermore, the number of mathematics undergraduates has been steadily increasing over recent years, from 13,188 in 1996/7 to 30,340 in 2014/15 (Higher Education Statistics Agency, 1998, 2016). Consequently, there is a need for admissions tutors to find additional measures of differentiating between well-qualified candidates, given the A grade reflects a high degree of accuracy as well as mathematical competency’. While overall applications for mathematical sciences are no longer increasing monotonically, this is perhaps true for the most competitive universities (say Cambridge and Warwick, who either require or prefer STEP papers, for Oxford and Imperial, usually requiring MAT papers, and perhaps a small number of others), although again, the new A Level assessments should be more aligned with what is typically valued in university mathematics or applications of mathematics. Of the focus additional papers, use suggests MAT and STEP papers 2 and 3 are generally reckoned to be most demanding, and AEA least so. Note that TMUA makes no demand on mathematical communication, since it is purely multiple choice; MAT and STEP papers, in contrast, make considerable demand on students’ ability to communicate their mathematical thinking, and AEA some demand in addition to that typically required by A Level, although the gap is (and was intended to) lessen when compared with the new (first large scale examination in 2019) A Level papers.

Darlington and Bowyer (2018) largely address the medium-term perceived benefits to undergraduates who engage in at least moderately successful ways with these papers, via a sample of 430 undergraduates, 361 of whom were studying mathematics and the rest for an honours degree in which mathematics was one component. All had taken (not just studied for) at least one of AEA, MAT and STEP papers, including 322 (75%) who had taken STEP. Their sample was therefore highly skewed: STEP papers are designed to be accessible to at most the top 5% of A-Level Mathematics candidates (University of Cambridge Faculty of Mathematics, 2015). Student responses confirmed Darlington’s (2015) analysis of the nature of the mathematical demands of the different papers and its relation to the nature of undergraduate mathematics at the destination universities (at the time of the survey, largely Cambridge, sometimes Warwick, for STEP). Darlington and Bowyer conclude that ‘Participants were positive about STEP, commending its similarity to undergraduate-style assessment and its challenging questions’. However, their abstract also says ‘The students’ views suggested that those wishing to be well prepared for tertiary mathematics should take one of these papers, preferably STEP. However, whilst universities may not necessarily wish to require applicants to pass extension papers, it may be beneficial for universities to recommend students to take them, in order to improve their mathematical thinking and expectations of undergraduate mathematics study.’

These conclusions seem perhaps over-stated given the sample, and to ignore the fact that any of these papers requires a deep and confident grasp of the related mathematics (which for e.g. STEP papers II or III, includes, or at least significantly benefits from, familiarity with much of a Further Mathematics AS and/or A Level specification, together with additional knowledge). Golding, Redmond and Grima (2019) suggest the vast majority of A Level candidates do not have that grasp even when they come to A Level examinations, and certainly not well in advance of those, when they would need to be preparing for additional papers. Indeed, Golding (2019) shows that even for mathematically very selective cohorts such as those at one of the national Mathematics Specialist Schools, preparation for such papers can be a distraction from acquiring robust grasp of A Level requirements in mathematics, for students who are not exceptionally advanced in their mathematical grasp.

It remains the case that we have little systematic evidence of the impact on school students, school teachers, or students 'unsuccessful' with such papers. Nor do we have studies focused on equity of access or 'widening participation' issues. We know little about the reasons for proliferating university use of these papers, nor about university academics' perceptions of the impact on students of their engagement with them.

A number of research questions of interest to the mathematics and mathematics education communities, and beyond, then arise:

RQ1. How and why are the papers being used by universities?
RQ2. What support for these papers is available to students in different settings?
RQ3. What is the impact on students of engaging with these papers? (What are the transferable benefits of having engaged with such questions at different levels of intensity, for example the benefits for university progression? What are the benefits (or costs) to those who enter for them but do not 'succeed'?)
RQ4. What is the range of students who would benefit from preparing for such papers, and what support would they require? (Is it beneficial for a wider range of A-level students to embed consideration of the questions in STEP, MAT, AEA and TMUA in their regular classwork?)
RQ5. Are there any related issues around equity of access to preparation for these papers or more competitive mathematics-intensive courses in English universities?

In Autumn 2019 JMC launched a call for proposals to undertake a small study of related issues, in an attempt to begin to answer such questions. This report is the outcome.

3. Methodology

Use is clearly varied, with different approaches and perceptions adopted by different university academics, different school/college teachers, different university applicants… Given the dearth of evidence in the area, establishing representativeness is prohibitive, and we also felt that it was important that participants in the system at all levels were given opportunities to make some input. More targeted data collection, though, was aimed at achieving a potentially 'telling' sample (Mitchell, 1984) that would 'punch above its weight' in terms of implications.
Five main approaches to data collection were therefore adopted:

1. A broad synthesis of related 'grey' data (websites, reports, responses to email requests for information…), with existing specific and wider literature;

2. Interviews with a variety of providers of external support for paper preparation (representatives of the Advanced Mathematics Support Programme, the STEP Support Programme…….);
3. Theoretical sampling from students in year 13 who have applied for reasonably competitive mathematics-intensive university courses, and their teachers, in at least four *state* 11-18 schools, and at least four Further Education or Sixth Form colleges, and their teachers. Focus groups or interviews with students and interviews with teachers. Sampling is discussed further below;
4. Interviews with HoDoMs and/or mathematics admissions tutors and/or academics teaching first year mathematics at a small purposive sample of at least four Russell Group institutions *mandating, or strongly encouraging via reduced offers, additional papers*, plus two others comparable with some of these who do not;
5. Three googlesurveys, one for any year 13 students applying for such courses, one for their teachers, one for any HE mathematics academics, promoted via social media, HoDoMs and teacher conferences and professional networks.

The research questions targeted by each data collection method are tabulated below in Table 1; in Table 2 we outline the scale of data collection achieved. All interviews and focus groups were audio recorded and transcribed.

Table 1: Constributions to answering research questions by data source

	Grey lit review	Teacher interview	Teacher survey	Student interview	Student survey	Support interview	HEI interview	HEI survey
RQ1: Use by universities	√	√	√	√	√	√	√	√
RQ2: Support available	√	√	√	√	√	√		
RQ3: Impact on students		√	√	√	√	√	√	√
RQ4: Benefits of wider use		√	√	√	√		√	√

Table 2: Data collected

What?	Source	n=? and reference in text
Information about entry and support, including past reports of effectiveness of support programmes	University and support provider websites, email enquiries	
Year 13 teacher interview transcription	11-18 state funded school	11 (from 8 schools): ST1-11
	FE or sixth form college	11 (from 7 colleges): CT1-11
Year 13 student focus group or interview transcription	11-18 state funded school	19 (from 5 schools): SS1-19
	FE or sixth form college	25 (from 5 colleges): CS1-25
Interview transcription	Provider of preparation support	4 (STEP support programme, Advanced Mathematics Support programme): Supp1-4
Mathematics admissions or similar academic interview transcription	University	9 (from 7 universities): HEI1-9
Year 13 student g-survey	17 schools or colleges	82 from 17 centres: GS1-82

Year 12/13 teacher g-survey	63 schools or colleges	66 from 63 centres: GT1-63
Academics g-survey	4 HEIs	4, 3 of them useable: GHEI1-3

Further details of participants are given in Appendix 1.

Sampling achieved

For purposes of achieving a state-funded school/college sample with the potential to be 'telling' (Michell, 1984), mathematically strong departments were targeted so that any emerging issues were, if anything, under-stated and could reasonably be expected to be more pressing in mathematically less strong departments. Here, we approached Heads of Departments in centres known to teach Further Mathematics, and to include at least one mathematics graduate in their number, and wanted to include at most one school and at most one college known to be academically selective. We looked also for centres with strong 'Ofsted' evaluations. From 30 approaches made, we secured agreement to participation from 8 schools, including one state grammar school, and from 7 FE/sixth form colleges, including one mathematics specialist school. In other respects the sample was fairly 'convenient' rather than e.g. geographically representative, given the limitations of resource. In fact, planned student interviews at two of these could not be completed because of coronavirus lockdown, though we were able to go ahead with teacher interviews remotely. Related students were asked instead to complete the student survey, but the response rate was understandably low in the unsettled circumstances (a total of 3 students from those two centres, of a planned 9).

Inclusive Google surveys of teachers and students were promoted through professional associations, professional contacts and social media, in an effort to open up opportunity to contribute to the debate. Responding teachers came from 29 independent schools, 19 selective state schools, and 15, non-selective state schools. All responding students came from within those state-funded institutions, since access was via teachers: about 70% of the responding students came from just 4 post-16 colleges, with the rest from 6 grammar schools and 7 11-18 comprehensives. No replies to any surveys came from beyond England. All independent and grammar school teachers, and most comprehensive teachers responding were themselves providing active support for additional papers, and almost all students themselves had taken or were intending to take one or more additional paper, although the invitation was for wider response. Survey participants are therefore quite unrepresentative of post-16 schools and colleges in England; nevertheless, they offer perspectives of interest. Indicative characteristics of all participants are given in Appendix 1.

University participation was by direct approach via published mathematics admissions or personal contact. Responses were received from 7 of the 13 institutions approached, including Oxford and Cambridge. Telephone interviews were held with either a mathematics-specific admissions officer, or another academic with a current or recent involvement in admissions and/or transition to university. In one university three such academics were keen to contribute.

'Grey' literature searches suggested that apart from resources available from paper websites, the most influential support programmes for these papers are the DfE-funded Advanced Mathematics Support Programme (AMSP), the Cambridge University STEP Support Programme (SSP), and nrich (www.nrich.maths.org), which is also funded by Cambridge University. Within the AMSP, there are a variety of roles and we conducted interviews with those responsible at 3 different levels. We also interviewed a senior provider in SSP/nrich support.

Interview and focus group schedules were developed from the research questions, and iteratively edited in the light of initial analysis of responses. Surveys were launched rather more than half way through the more targeted data collection, so as to build on refined questions, and with a similar but more inclusive structure, since the target population was wider. Examples of tools are given in Appendix 2. All tools also solicited core characteristics of the respondent, so as to contextualise responses. Interview and focus groups were audio recorded and recordings (sometimes selectively) transcribed. All data were then analysed by research question and sub-question in a grounded way, leading to the analysis reported below.

Interpretation

The genesis and scale of the study mean the findings are exploratory rather than exhaustive or necessarily representative: this issue is discussed further in the following sections. The study is of a small sample of academic, school/college teacher and pre-university student participants and draws on almost exclusively qualitative data, and often, respondents' personal views. The analysis and interpretation of those data are therefore subjective, although presented with some attention to trustworthiness. Steps taken to increase that trustworthiness of the findings reported are reported in Appendix 3.

Ethics and research integrity

The study was conducted under UCL Research Ethics Committee ethical approval reference REC 1296. In most cases the institutions are not identifiable from the details given in Appendix 1, although Oxford and Cambridge recruitment provision is distinctive. However, all interview and survey data represent individual viewpoints rather than necessarily an approved institutional positioning. The study reveals a range of different approaches and attitudes to the focus papers, their preparation and use, from university, school and college professionals. These were solicited, analysed, and are reported here with the intention, so far as is possible, of avoiding any researcher judgment of them. Inevitably, though, the researcher brings her own past experiences of university, of school teaching and of preparing students for these papers, to the work. Without that, it would be difficult to begin to understand the field, but that background also brings threats to the research integrity. Steps taken to address issues of research integrity are also included in Appendix 3.

4. Findings: How and why are the papers being used?

Use by universities

Not all entries for additional papers are associated with university offers: 'Our most able year 12s take STEP 1 for fun' (ST7), and many students in year 13 take more than one additional paper. However, in 2019, there were ~3000 MAT entries (Oxford University website), 300 AEA (JCQ website), and ~2000 STEP1 papers (Supp4), although STEP and TMUA participation statistics are, strictly, not in the public domain. In 2020 STEP1 and AEA will not take place, because of pandemic restrictions, although AEA grades will be awarded.

Responses suggest the papers are, predictably, used very variably by different universities: for selection, for market share, numbers control, for their intrinsic enrichment and challenge, or, very commonly, for a mixture of such reasons.

Selection:

It was unusual that responses stopped at the need to select from a large number of applicants, each of whom was predicted top grades at A Level. However, our participants widely considered that 'the most competitive universities', which they usually expanded to mean Oxford, Cambridge, Imperial and Warwick, need selection criteria that go beyond even A Level Further Mathematics top grades:

> *I can see why Cambridge need a STEP: they assume a very enhanced mathematical awareness and ability to struggle. That probably extends to Imperial. But for unis like us, we need students who can do Mathematics and Further Mathematics A Level well. I don't think we need people who can do STEP - it's a bonus. We're not set up to need that, we don't assume that level…... There's nothing in the first year degree programmes that requires that….For fluency in calculus you need Further Maths.* (HEI1).

The Cambridge University website goes further than selection needs, saying:

> *'Cambridge Colleges like to make offers involving STEP for the following main reasons:*
> 1. *STEP is a far better predictor of success in the Mathematical Tripos than A-levels. One reason for this is that the questions are less standard and less structured, which helps to distinguish between ability (or potential) and good teaching.*
> 2. *Preparation for STEP serves as useful preparation for our course.*
> 3. *The STEP marks and the scripts themselves are available for inspection by college staff. This means that it is possible to make allowances for a near miss and to make judgements on the actual work rather than on just the marks or grades.*
> 4. *The meaning of A-level grades may differ significantly between the different boards, so STEP provides a fairer 'across the board' comparison'.*

Some teacher, and one academic, participants wondered about the evidence for STEP distinguishing between potential and teaching, since they considered STEP 'highly teachable' to a range of high-attaining students. One also pointed to Ofqual's efforts to ensure that the meaning of A Level grades is comparable across different Awarding Organisations. Cambridge, have, though (HEI9) undertaken statistical work that demonstrates a good correlation between STEP performance and performance in the Tripos.

> *'(The discounted offer) helps us deal with the almost overwhelming number of applications we have'* (GHEI1)

There was comment about the relative timings of the different papers, and the implications for selection:

> *I am aware of all the figures and of the work that goes into the formulation of MAT and TMUA papers and I understand the argument that their use helps manage the admissions process, but I do not believe it is right to test students, with a view to making a decision on whether to reject an application, or make a discounted offer, on the basis of a test taken at the start of year 13. Far better is to give the schools concerned the full two years to develop the students' skills and then test them at the end of the process. This why I prefer STEP to TMUA/MAT'* (GHEI2).

Other respondents felt there was advantage to the timing of MAT (and TMUA) papers:

We do encourage them to apply for Oxford rather than Cambridge - it's a less stressful process, starting something rather than end point, and if the outcome's not what they want, they have time to recover (CT3).

The same teacher, though, had reservations about the recent pitch of MAT questions, particularly for students who have come from 11-16 schools to post-16 study:

*The last two years (of MAT) have been unusually difficult… we try to persuade them to make it easier. We keep being told that if it's rather hard then they'll just pitch who they invite to interview differently, but of course an overly ambitious paper affects the ranking, and students' confidence…(*CT3)

Where competitive departments do not formally require additional papers, their use might influence offers, either at 'clearing' stage or during the initial offer round:

We don't formally consider it, it wouldn't be part of the offer, but it might help someone gain a place if they miss the A Level offer. Typically, we do take people who've just missed that, in which case we look for the most promising students. If they can do it without having a detrimental impact on their 3 A Levels, then a decent job of one might get them in. But that situation arises fairly rarely: in a typical, year Further Maths gives us the additional information we need. I'd always recommend they prioritise those 3 A Levels, plus if necessary Further Maths AS (HEI6);

'*We don't use 'extension papers' in our offers, but students who are taking them anyway are encouraged to tell us as it helps us to decide whether we will give them an offer or not'* (GHEI2).

However, some more informed students reported gaming of such approaches: *'I'd got it on my UCAS form, but I don't need it for the offers I've got, so I'm not going to take it'* (CS6).

Market and numbers control use

There was indication from several universities of these papers being used for market rather than selection purposes:

*We have STEP in our alternative offers. Two reasons. We never felt the need to ask for STEP: Further Maths grades are a very good indicator, but when rivals started incorporating STEP into offers, we became a second tier choice – it affects student perceptions. That was the main reason - market value. Nowadays most of our candidates have taken STEP1. We think it is a good thing because they are a bit prepared to start course, they've engaged with more sophisticated maths, with writing solutions more sophisticated than A Level, you touch on how to write a proof. So we're happy with that. Commonly, entrants have not been successful with STEP1. Very few got a place via the STEP offer (grade 2+), which is a harder offer than 2A*s. Some have, but they got A*A*A so didn't use that. But that extra effort pays off when engaging with uni maths (*Supp1).

We let people in if they've missed their offer by a couple of grades, so sometimes they don't have that A in Maths. …. We've never seriously considered requiring Further Maths, or an AS… Uni policy is to use a fair number of unconditional offers…We also drop a grade for TMUA 4.5+ or for a STEP pass, any grade. About 50 of our intake would have tried STEP, and about half of them pass. That's us trying to get in as a favoured insurance choice, and it works for us….. But STEP is incredibly difficult, so you can't say this is a threshold to succeed with maths here. Whereas for TMUA, we can say 'this is the right sort of maths'. We make a big play of 'why not have a go and don't declare it if you don't do well'. …. We don't make many reject decisions… it's less effective as more people do TMUA. We do make use of the results if they miss the offer (AAB at A Level) by more than a couple of grades (HEI5).

The departments of HEIs 6,7 have offer profiles similar to, or sometimes stricter than, that of HEI5, but choose not to make explicit use of additional papers in offers:

> *The aspect of maths we're most keen on pushing is the Core, or the pure, parts of maths, e.g extra Further Maths Pure if they haven't done it. …In terms of thinking, I'd say it's getting used to writing and reading proofs. The new A Level seems to have taken some steps in that direction. We shan't see the impact for several years, especially given hiccoughs like this year. (I think) questions pulling together different strands are now more common. So there's potential in the new A Level. The strategic advantage of asking for additional papers is not a serious discussion here. This is the sort of role Further Maths plays for us, so we make a differentiated offer…..I don't think we'd want to go two grades below our standard offer* (HEI6);
> *If you do well in an interview you can get a lower offer, typically that the non-maths component comes down a bit. Even in clearing, they try to keep A in Maths. We don't want to set them up to fail….they need to know the A Level, especially the pure stuff, and prioritise that, for succeeding at university, but especially if we interview we can be confident about offers* (HEI7).

Enrichment, challenge and independence

Going beyond selection purposes, there was among sample academics, teachers, and most of the students familiar with these papers, wide recognition that engagement with these papers can provide differential enrichment and challenge reasonably aligned with the changes in approach to mathematics valued at university: there is also an acceptance that among the most mathematically-capable students, successful engagement with such questions in addition to A Level work is likely to support transition. Note that the first respondent assumes an access to STEP questions throughout years 12 and 13:

> *A development of a greater level of problem solving ability, in preparation for a mathematics degree is best achieved through a consistent exposure to STEP papers, throughout year 12 and 13. Ideally all schools would have the capacity to offer this, but I appreciate it is an unlikely development. AEA goes some, little, way towards these outcomes also. I do not have much time for the view that STEP papers are biased towards a preparation for study at Cambridge, and the structure of the Tripos examinations - a good preparation in problem solving study will benefit anyone, wherever they study (GHEI1);*
>
> *It is always good to see A-level material being used in more challenging contexts. Able students benefit from being stretched and, considering STEP in particular, studying for these papers can enhance your understanding and aptitude. In the short term, this will help them with their A-level studies and, in the long term, it will enable them to adapt more rapidly when the material becomes more challenging at university. (GHEI2).*

Many respondents from all constituencies valued problem solving skills, deeper mathematical thinking, getting stuck, development of resilience, and thought that questions from these papers promote such experiences; some, particularly academics, teachers and students who had engaged most thoroughly with MAT and STEP, mentioned the promotion of communication skills in MAT and STEP. Additionally, some academics referred to the development of independent study skills they thought were promoted by engagement with these papers:

> *It sets them up for university: they have to engage with new ideas independently, and get through that barrier of understanding independently, which is just what we want* (HEI2).

However, some teachers were sceptical about the degree to which such demands on independence were experienced uniformly:

We're in the fortunate position of being able to support students for these additional papers, as part of our timetabled allocation, and to underpin that from when they first come to us. It's win-win, since we both enjoy that and for the exceptionally able, learn from it. For less than brilliant students, those who are just very strong, these papers are eminently teachable, and it's not honest to pretend otherwise. I think it's a myth that students like ours are gaining some sort of extraordinary independence from such engagement – even our Olympiad students need that encouragement and interest in what they're doing, even if on occasion we don't understand that. In most state schools, they just don't have comparable experiences, teachers often are scared of any involvement since they're out of their depth, and getting there does indeed require dedication to the cause (GT73).

Mixed reasons for encouraging use

though, were often given by academics: selection, status, numbers control and reputational edge were often offered before any mention of enhanced mathematics awareness or experience, for example better aligned with that expected in HE. Many assertions of benefits were made in terms of 'it's obvious that' or 'we think', though few of the academics interviewed was aware of any empirical work carried out in their departments to verify the claimed links. One said: *'Students with grades 2 in STEP 1 are about the middle of the performance spectrum in our first year mid-year tests. Those with higher grades, or with STEP 2 and 3, are above average. Those without STEP are lower than average'* (GHEI1). The same academic quoted in turn advantages of preparation, competitive edge, strategic potential, and selection:

'Our primary reason is to increase the level of preparation of students for the degree that we offer - the primary reason. We already insist on Further Mathematics but, even with the new structure, STEP is a better preparation 2) To help us stand out against some of our potential competition in the UCAS process. 3) To keep the issue of STEP visible to those who formulate College-wide admissions policy so as to possibly make it easier to switch to insisting on STEP if support becomes more widely available, as a mechanism to making it easier to deal with the almost overwhelming number of applications we receive…..'(GHEI1)

Other academics had reservations about some aspects of the system, while still valuing the mathematical potential of these papers, though note also the assumption about A Level experience below: across HEI respondents, there was little spontaneous mention of the expectations of the new mathematics A Levels, which were designed to support more effective transition to HEI mathematics-intense courses (ALCAB, 2014).

I want them to get used to being stuck. We value sheer bloody-mindedness, and you can't test that in a standard school class where you get single approach and instant gratification. It's a real difficulty that A Levels don't provide that. …We were reluctant to include TMUA since they won't provide stats on gender. but we had to because of the (volume of) applications issue (HEI2).

Several academics talked about a changing situation in terms of recruitment, and about the importance of realising that university level mathematics might not align with students' previous experiences:

*Some of us believe we could do away with these at (HEI), in the current situation: we don't need them for selection purposes. Previously we were having to select, now we're only too glad to get students.. .We're having to make provision for weaker students now, with extra training session (for e.g. A*B students). Most don't take the offer of using STEP, but concentrate on A Level grades….But we're also having to rejig our syllabus in year 1 so we can address their understanding. The number of people in that position might increase, or else we reduce our* *numbers. Too many people get to uni and find mathematics is not what they thought it was. I make a big point in open days: I want you to come here if this is the right place for you and maths is the right course for you. Sometimes it's the parents who get the message. It's much easier to get messages across to a human being. Of course, engagement with extra papers has mileage in having a negative impact if they're on the wrong pathway for them* (HEI2).

Despite early claims that TMUA was needed because of the near-demise of AS Levels, so that HEIs have little objective information on which to base selection, that reason for use was only mentioned once by responding academics:

With the new 'linear' A-level, we have very little concrete information about their post-16 performance at the offers stage and so things like TMUA results can allow us to make more informed decisions. From an admissions perspective, TMUA seems best as the results are released in time to help with offers whereas, from a mathematical perspective, STEP is probably the best at stretching the students (GHEI2).

It is perhaps worth noting that some students feel that universities are not always rational in their use of additional papers: '*Chemistry, a subject requiring mathematical skills, asked for the TSA section 1 (for Oxford entry), a paper which didn't contain mathematical problems. A more maths heavy paper should be used, if they want to know how well you'll succeed in their course'* (GS15).

Who do teachers encourage to engage with these papers?

Teachers at schools or colleges with a track record of sending several students to competitive universities to read mathematics were typically confident about encouraging participation in these papers, though still discriminating. They widely suggested that they would only actively encourage those they felt were aiming for top A Level grades:

Any student who ought to get A in A level maths or A/A* in A level further maths. Extensive experience with such papers indicate that those not at this level will not succeed* (GT19);
Any top student who can take on the additional revision load without impact upon their other studies. Students who show flair, interest and genuine passion for problem-solving (GT35);
I would encourage top grade students to apply to universities that require additional entrance tests, but I would not encourage them to apply to Oxford: in the past our students have not done well in applying there (0 students in 7 years vs about 1 a year into Cambridge) (GT6);
Encourage exceptional, highly motivated students aiming at the top 5 universities/colleges, Discourage able but non-exceptional students (GT43).
*We would encourage all students who are interested to give a paper a go. None discouraged. We would though promote the additional papers to our FM students and those aiming for top grades who hope to study mathematically-based degrees. As the timing is separate from the usual A-levels and a negative outcome has no impact, then there are fewer barriers to take up. However we wouldn't teach for it but rather signpost and offer help (*GT52).

On the other hand, some 11-18 comprehensive teachers were dismissive: '*None* '(GT7); '*We leave it up to student choice. We have very little facility to support them, so it's up them'* (GT32).
Note that some in passing indicated a high degree of selectivity for their Further Mathematics A Level course:

If a Further Maths student is not looking to apply to a top institution for such a course, we feel that they probably should not really be studying the subject. A-Level Maths students are generally aware of their ability by the time they apply so won't need additional papers (GT60).

What we see, then, is a variety of reasons put forward to support the use of these papers, ranging from promotion of more deeply challenging mathematical ways of working than are often achieved in A Levels, through to marketing and numbers control use – or a combination of these. Teachers vary in their approaches to promotion of engagement, ranging from an approach inclusive of most high attaining mathematics students, to an absence of encouragement for any student.

5. Interlude: Brief portraits of provision in four post-16 settings

This section outlines the support for additional papers offered by four (state funded) settings visited, each of which is graded 'Outstanding' by Ofsted. They are offered in this section without individual commentary, but serve to indicate the range of well-informed teacher input and guidance available even within such well-placed institutions. There was a feeling among some respondents that selection that draws on additional papers is about students' independent development of deep mathematical thinking, but these examples show very clearly that only some have to draw on their independent initiative, organisation for learning, persistence, and learning, whereas others receive highly focused and structured teaching for them.

Case Study 1: ***Inner city comprehensive school in a socially deprived area, Ofsted rating 'Outstanding'.***

Three teachers in the department are confident to teach A Levels in mathematics although only Eddie, the new HoD, has a degree with a major mathematics content. Jack is just finishing his first teaching of A Level Further Mathematics, and draws heavily on the 'Integral' resources as a result of the (highly rated) AMSP professional development course he did. Until recently, few students from the school went away to university, most going locally, and application advice is rudimentary. The previous HoD says she doesn't remember previously having had students offered places at Oxford or Cambridge, although this year they have two in year 13:

Wilma took the MAT and has a place at Oxford to read mathematics if she gets A*A*A in Ma/FM/Physics, but says she's not confident she'll get those. She knows Jack struggles with teaching Further Mathematics A Level in depth, so does a lot of work on her own. She feels she needs greater access to harder questions to be confident of those A*s, and isn't sure where to find those.

Tamsin has a mathematics place at Cambridge if she gets a 1 in STEP 2, and A*A*A in A Levels. This is a (more accessible) 'contextual offer'. She is doing an AMSP Saturday course she funded herself (The school is reported to have said they couldn't support her as there were several such students in the next year). She enjoys it and says she's 'beginning to understand what they are talking about'. She also says she's 'looking at' the STEP Support Programme materials. But she's 'not yet' (mid-March) doing past papers in the week, and says she will once they've 'completed the A Level courses'. She seems to have little idea of the investment needed if she's to make the offer.

The department is keen to encourage participation in UKMT challenges throughout the school, and both girls regularly reached follow-on rounds although they didn't progress any further. Jack supported them one hour a week extra to his teaching timetable from September 2019, but says he found it too time-consuming to maintain, since he's unfamiliar, and unconfident with, MAT or STEP questions, and additionally, has a demanding year 10 tutor group. Eddie now works with Tamsin during Friday 'detention time' when he's not needed by other students in the room, but he says it's mostly responding to work she does, and he feels very stretched by the range of other expectations placed on the department, e.g. in boosting grace 4+ attainment at GCSE, or making additional provision for the weakest in every year group. The teachers aren't confident with the Oxbridge system: they have little grasp of the time and depth of investment most successful students need to make in STEP preparation, and they're not familiar with the Siklos book or materials other than the SSP modules (see Chapter 6), nor with the Cambridge offers : places ratio of ~2:1.

Case Study 2: ***Highly selective suburban boys' grammar school, Ofsted rating 'Outstanding'***.

All teachers in the department are graduates in mathematics-intense areas, over half of them Oxbridge mathematics graduates

At Key Stages 3,4 mathematics teaching concentrates on algebra and problem solving: entry to A Level courses is by performance in algebra-based weekly tests through years 10 and 11, not via GCSE results (so it's basically a subset of those with grades 8, 9 doing A Level – the old A* -, with a few grade 7s in there if their algebra is really strong. There are 4/5 Mathematics A Level groups per year, and 2/3 separate Mathematics+Further Mathematics groups, each of ~20 students from a 6-form (180 student) entry in year 7.

There are open, 'Maths Clubs' for years 7 and 8, and a 'Maths clinic' for all ages, which anyone can turn up to, but some are directed to. Weekly 'Elite maths' clubs are by invitation but need 3 rooms for year 8, 3 for year9, 4 for years 10/11 and 2 for year 12 students. They are broad scope, preparing for later rounds of the UKMT challenges, so Kangaroos, BMOs, Olympiads, team challenges, and later, for MAT and TMUA questions. These are targeted by year group but each is supported by older students who work with the younger, with teachers overseeing.

Teachers use STEP and MAT questions regularly as fillers or enrichment for quick finishers at A Level; teachers and students also work with questions sourced from https://madasmaths.com, https://crashmaths.com/.

The 'STEP Society' meets one lunchtime a week from September for year 13, for all students who might get an offer involving STEP - and others wanting to dig deeper. This tackles STEP 1 in Autumn term, then moves to STEP 2/3 in January. It tackles one question a session, worked individually or in pairs on whiteboards round room, with an emphasis on elegant solutions and communication of those. The lesson I saw was by an extremely experienced 'top end specialist' (sic) teacher and included seven students, reduced from 'a much greater number' earlier in the year: two needing 1,1, on STEP 2,3 (both for Cambridge mathematics); three needing a grade on STEP 1 including one needing a 1 (for Cambridge computer science); and two who might take STEP papers to access slightly easier offers, but might not. The teacher serves to prompt/challenge re alternative and more elegant methods. This session drew from the Siklos book (see Chapter 6). During the week, those *needing* STEP qualifications work from the website on past papers, or from the Meikleriggs website. Some also use the nrich community, and two doing STEP 1 are going to UCL's AMSP-supported STEP classes.

Teachers in Case Study 2 were clear that for them, preparation for entrance to the most competitive universities to read mathematics-intense subjects was a 7-year project, building up to engagement with highly demanding questions for the strongest mathematicians, perhaps from BMO or similar, by the time they start year 12. They are careful to limit exposure to whole STEP papers to year 13, so retain a range of those for students to practise on as whole papers, but regularly use STEP and MAT questions in ordinary lessons. It is interesting to compare the Cambridge preparation and offer cited above with that for SS1, a student at an inner city 11-18 comprehensive. He needs a 1,1 on STEPs 2,3 plus A*,A*,A or a 1,2 plus A*A*A*, so a less 'contextualised' offer than Tamsin's in Case Study 1. He uses the Siklos book and STEP Support programme (see below) for independent study, has access to a STEP Access tutor for an hour every 3 weeks, and does one of each paper each week. In earlier years he took part in UKMT competitions, but has had no other targeted support outside lessons.

Case study 3: ***Large General FE college in a city, Ofsted rating 'Outstanding'.***

Large, 'successful' mathematics department, about 160 A Level mathematics students each year and about 40 Further Mathematics students. Those teaching A Level are almost all graduates in mathematics-intense subjects.

Teachers used to provide lunchtime or twilight sessions for those doing additional papers, but say those are very labour-intensive and can't really be justified given the ever-increasing other calls on teachers' time and energy from students with special needs or at risk of not reaching 'target grades'. They say engagement with STEP questions in particular is very time-consuming, and recognise they're lucky to have anyone (in fact, two teachers) with the mathematics to potentially do that. Currently they just point students to the STEP Support programme and offer them the opportunity to join the local specialist maths school provision. In-house, they provide core A Level teaching and catch-up/intervention sessions, and engagement with UKMT individual and team challenges. Students are not discouraged from applying for ambitious courses that require additional papers, but teachers say they don't try to keep up with current available support, so are not in a position to point students to it, or advise them on productive timelines, etc.

There are, as is typical in the college, two groups each of ~20 Further Mathematics students. I spoke with five students from one of those groups who will have engaged with additional papers – one, for engineering, MAP. Two will do STEP 1, both for mathematics courses. They have been invited to local specialist mathematics school STEP sessions but say those are way 'out of their league', and the teaching inaccessible since it's at a different depth from anything they've seen before. Both 'have had a look' at the STEP Support Programme, and have done some work with it. Two did the MAT for an Oxford application, but neither had an offer. One of those says he 'looked at STEP' and decided it wasn't for them. The other also did TMUA, but performed mediocrely. None of them seems aware of the sustained depth and breadth of additional work often undertaken by students elsewhere.

Case Study 4: ***Large sixth form college in urban area, Ofsted rating 'Outstanding'.*** Large mathematics cohort – nearly 500 A Level Mathematics, 140 Further Mathematics students in year 12, typically some 20% fewer in year 13. All teachers of A Levels are graduates in mathematics-intense course, with several being Oxbridge graduates or similar high-flyers from abroad. Two of these (CT3,4) provide support for working towards additional papers. From January I year 12 there's an open 'problem solving class' that's timetabled, and all A Level mathematicians are encouraged to try that, and persist as long as they choose. (*'Some students with high GCSE grades cannot think'* (CT4)). The course begins with UKMT questions, then TMUA and STEP1 questions, working up to MAT and STEP questions by mid-Autumn of year 13, and STEP 2/3 questions for January of year 13. Students work on a sheet of problems a week, individually or in groups at their discretion. This class aims to promote persistence and mathematical confidence in grappling with harder problems, as well as communication and evaluation of solutions. From the beginning of year 13 it's complemented by a weekly 'interview preparation' opportunity in small groups, where there is also a wide range of mathematical talking and problem solving supported.

Students in year 12 are well supported to explore what courses and universities they might target, and I spoke with a group of five year 12s from one Further Mathematics group (CS1-5) who were already (March of year 12) clear they wanted to go on to study a mathematics-based degree at a competitive university. Their stories were largely of 'being good' at mathematics previously, appreciating the doors further study would open, but of being 'turned on' in year 12: '*I spent most of year 11 doing nothing much'*(CS4) (others nod or otherwise agree), *'but this year…it's intrinsic, I probably would want to continue with it even if it didn't get me anywhere, because it's so satisfying'* (CS1). *'There are nice solutions. I was turned on by videos (my teacher recommended) on YouTube, to see how people do complex problems'*(CS2); *'It's the best subject, I like the feel of struggling and then finally getting to the solution'*(CS5).

Within the two groups of year 13 students (CS 6-15), several had decided to go for Science rather than mathematics, or for a mathematics course that didn't involve additional papers: *'I talked with someone who said maths was about proof, so I chose Science'*(CS7); *'Science felt less competitive, and more different things, and more fun'*(CS10); *'I like the range of study in a NatSci course, and you don't have to choose what you specialise in until you know more about what it's like at uni'* (CS 11); *'I thought I might try for Oxford or Imperial (for mathematics), but I'm hoping now to go to Nottingham: I'm predicted 4 A*s, but I didn't want the pressure…. But the (additional course) gave me a greater understanding of a uni view of maths, and of problem solving, so that's what got me hooked'*(CS6). These students talked about UKMT questions building up problem-solving skills more effectively than A Level questions in their textbooks: *'Questions are more of a stretch…. They made me step back and think what the question is asking'* (CS8). Their additional classes stressed communication of UKMT solutions, even though answers only are required in official 'challenges'. *'I think you get better at explaining why you're doing the stuff, and better at understanding other people's arguments, or explaining your argument'*(CS10); *'better at working with maths collaboratively* (CS11)'; *'It's not about the content, it's about the ways of thinking and communicating'(*CS13).

Two students holding offers for mathematics at Cambridge said *'STEP gives you a more in-depth understanding of problem solving: first time I look I don't have a clue, sometimes you have to think before you get an idea that might work'*(CS14); *'We do two extra sessions a week for STEP, with tips and tricks. They're a lot more difficult than normal maths questions - and sessions help with that. I'm getting better ways of doing questions, simplifying questions, forcing your way in, ways of thinking about them. I do lots of questions out of class, about two whole paper's worth a week, then I'll do entire past papers closer to the time. I use stuff on the website for mark schemes…. I used extra resources last year – my teacher pointed me to some videos, and a couple of other websites where there are lots of questions and solutions, and a student forum, so I still use that a lot'*(CS15).

6. What support is available to students in different settings?

The case studies above show that in-school/college provision for support varies widely across state-funded education, with some provision comparable with that provided in many highly academic independent schools. Informal or formal support in school/college ranges from 'we discourage our students from making any applications that would need additional papers'(ST10), to the twice-weekly expert, increasingly selective and focused support from age 11 seen in Case Study 2. Many mathematics departments do not have a teacher confident to teach even Further Mathematics AS, and many more do not have a mathematics graduate. When they do, support for these papers is competing with other additional calls, as seen in Case Study 3, or is offered with insufficient time to make a deep impact, and/or insufficient knowledge of the system to understand what is needed, as in Case Study 1:

> *The issue is teacher time and priorities - the opportunity cost, given it takes an experience and knowledge to devote 40 minutes a week to just find those two questions. there are probably only two teachers even here who can do that, and the other is more than fully occupied being Head of Department. It interests me to do that and I'm feeding on those four pupils who seem hungry to do it - I get the qualitative payback of them meeting nice maths and appreciating that. A less experienced teacher wouldn't be able to think about such things – for me the 'day job' looks after itself. When I leave…we don't have an in-house maths geek, which is what you need. It's the accessibility issue: are we super-serving a small minority by offering this? Is this a nice hobby for me as a teacher? Is that justifiable? Most kids don't have access to such a teacher - there are not enough of them to go round* (ST9).

Some academics appreciate the pressures and issues:

> *We like to see active engagement in our applicants, particularly what you've done independently... I happened to interview two candidates from the same independent school on the same day, and you can see how their UCAS forms were similar, how coached they'd been, lunchtime seminars from local uni, etc - actually it's often the school that is making that effort. Smaller teaching timetables, smaller classes in some schools, there's more capacity to do such things. In a comprehensive, you're weighing up all the time, who has the greater need? Schools don't worry which uni, whereas they do worry about C/D measures – that's what they're evaluated by. When students actually take themselves to a conference in London, that's different - and you can sometimes tell that from the teacher's statement. There are some schools who won't pay for papers, it's down to parents. I don't like that. And it's the same with some online courses. But some schools simply don't have a single teacher who can engage with them, in any meaningful way. The entrance papers don't sit easy with me, especially if just to make us look fancy uni* (HEI7).

The range of teachers and students who have engaged with these papers talk about their 'teachability' and 'learnability', with support, for students coping well with A Levels – particularly for STEP and TMUA papers, less so for MAT. AEA papers were not high in the profile of responses, which is unsurprising given the total of ~300 entries in 2019. But such 'teachability' inevitably discriminates in favour of those with personal, informed teaching available – despite universities' claims to value the papers in part because of the independent thinking developed. Teacher and student respondents commonly mentioned the use of private tuition for such purposes, at a cost of a typical £350 per day for a small group course, or up to £160 per hour for individual tuition or application support.

However, the DfE funded Advanced Mathematics Support Programme (AMSP) at https://amsp.org.uk/ provides a range of opportunities for both teachers and students at state funded schools and colleges in England. There is a similar, but different, support available in Wales from the Welsh 'Further Mathematics Support Programme'. The most competitive universities, and some individuals, also provide an extensive range of free online support materials available to students globally, and we consider each of these sources of support in turn.

Information provided by Supp3 suggests AMSP courses vary in intensity, and are often free for state school students, especially those from low participation areas, with independent school students eligible to pay (more) to attend if there is space. Their Problem Solving Matters problem-solving courses, at six HEI venues, each take place on Saturday mornings, two before the summer of year 12 and one at the end of September in year 13, at Oxford, Durham, Warwick, Bristol, Imperial, and now Manchester. They are intensive, with students or young teaching fellow mentors from the university. There are also regionally organised regular problem-solving classes, 52 of them in the last financial year, taking place on Saturdays or twilight. About 30 of those are year 13 classes, focusing towards the end on STEP:

> *(In my region) we start with year 12 just before the October half term, for those standing out as wanting a bit beyond what the curriculum offers, and studying Further Maths. We provide according to need, expanding classes as necessary. In year 13, we start with two groups, some drift off after the Oxford interviews. We usually drop to under 15. This year we used guided self-selection to split STEP2/3 preparation from the rest. As a result, some changed to wanting to read maths rather than for example computer science – they 'caught' maths. The year 12 group in theory follow resources on Integral linked to MAT questions, a sheet of questions each week. We hope Oxbridge hopefuls actually engage between sessions. TMUA questions are largely avoided at present since not too many - time is an issue with TMUA, so you need to practise whole papers. The big advantage with face to face groups is you work with others actively through the session, sitting down doing maths together, and some of the students don't ever work with their intellectual peers in the rest of their lives. Compared with online courses, that's the most valuable aspect of a good workshop* (Supp2).

The reference to 'Integral' above is to a large resource of highly-regarded online materials developed for teacher and student use by MEI and available via AMSP. These focus on A Level and beyond.

For students in 'cold spots', there is live online support provided by MEI, that AMSP buy into. This is targeted especially at provision for anyone without other support, and supports preparation for MAT and TMUA. Several teachers, students, and one academic, pointed to the advantage for development of mathematical communication and identity of working on these papers live in a group, rather than online, either in a remote group or by oneself:

> *It gives the brightest a peer group. It makes sure they're all getting stuck. And the lazy boys have to learn to communicate their thinking, for MAT or STEP: we make them do that for TMUA questions too, even though in the exam it's only multiple choice'* (HEI1).

AMSP also provides online and face to face short teacher CPD for those wanting to work with problem-solving or prepare their students for any of the additional papers. The AMSP's predecessor organisation FMSP had a formal evaluation in 2016/17 (e.g. Boylan et al., 2016), in which support for students via problem solving classes and STEP/MAT/AEA preparation, and teacher CPD for both, was evaluated in passing and shown to be valued and appreciated. However, in both cases, participants were largely from centres already more secure in offering Further Mathematics, and this remains a challenge. Supp3 expanded on AMSP provision for teachers, and the need for that:

As the years have gone on, there are fewer and fewer people in state schools who can do that provision at all - they might not have a maths specialist at all, in some schools, so how are their students going to catch maths? We provide quite a lot for teachers to support them in really mathematical provision, but often they don't turn up, even if they register: 'Introducing problem solving in year 12' course, a STEP course, a ' getting students ready for admission tests' – and often have to cancel them since there are none or one booked. AMSP provision is all state schools only, but in general if there's space we let others come, though in this case we don't since it includes cover. It's a luxury for teachers, low down on the priority list - there's plenty else to think about, for example just getting their head around the new A Level. Our 'Next Steps' course focuses on the transition process, UCAS processes, what different HEIs are looking for, as well as tests. We tell them what support is available for their students. Online PD asynchronous courses are better attended, also free, but with high non-engagement rate, so there are no easy answers. ....The MAT questions are HARD, and they're all intimidating to non-specialist teachers already struggling with A Level (Supp3).

The Welsh FMSPW http://furthermaths.wales/ similarly provides a series of events to support teachers and students to engage with STEP and AEA mathematics, and general problem solving; it additionally provides live online sessions for students preparing for STEP/MAT work from June of year 12. Supporting unfamiliar mathematics problems)/MAT focus. These are free to teachers and students in state schools/colleges. We were unable to find any comparable provision in Scotland or Northern Ireland, and neither did we have any response from any teachers or students in those jurisdictions.

The other big structured preparation programme is the online STEP Support Programme (SSP), run by the Cambridge mathematics department in conjunction with nrich (www.nrich.maths.org), and open to all. In March 2020 there were around 2000 students demonstrating sustained engagement with the programme. As outlined by Supp4, materials include topic notes, hints, and full solutions for every question, including some along the lines of 'first I did this'. There are 25 'Foundation' assignments that draw only on GCSE maths, then they build up through STEP 1 to STEPs 2 and 3. These give an introduction to A Level topics, since students might meet those very late, especially if they're taught Mathematics A Level in parallel with Further Mathematics. There are also notes re small topics not now included in A Level, e.g. triangle of forces. Foundation units support a greater confidence and interest in mathematics degrees, drawing only on GCSE or AS mathematics. They are intended to be accessible to able A Level students during year 12 (Supp4). STEP 2/3 work consists of 8/10 modules each containing 4 STEP questions and arranged around a single STEP area, plus an additional module of 10 questions that didn't fit into a particular category – around 150 questions in all. There is a student room where collaboration and help are offered, and advertisement via nrich.maths.org and social media. Some students only meet the idea of STEP at interview, which is late – the guidance says 'suitable from year 12' and 'that's when they really need to start engaging'(Supp4). SSP also offers a day for all state school Cambridge offer holders.

As additional support, the Cambridge Assessment Admissions Testing Website offers Advanced Problems in Mathematics by Dr Stephen Siklos, as a downloadable book, and past papers from 1998 with many solutions, plus other resources for STEP, MAT and TMUA, all of which are administered by Cambridge Assessment. Cambridge university is also instrumental in supporting Underground Mathematics, which as one strand includes STEP questions with solutions, and nrich (www.nrich.maths.org/), which in its 5-18+ provision includes nrich Advanced Problem Solving modules.

As indicated above, there appears to be a thriving market for private tuition in preparation for additional papers, and there was some concern from two very separate teacher participants that not all those involved in development of each year's tests are always meticulous about keeping a distance from direct student preparation. This is no doubt a small-scale issue, so on-going vigilance is key for maintaining integrity of use.

Oxford university itself, according to HEI4, does not offer direct preparation for MAT[8], but funds a variety of outreach work, for example It All Adds Up for year 9-12 girls, their UNIQ Summer Schools, and 'Opportunity Oxford'. The DfE-funded KCL and Exeter Mathematics Specialist schools, and the emerging schools joining those, are obliged to offer a variety of enrichment opportunities to younger students in their area, and some independent schools also offer opportunities open to other students.

Additionally, some other universities also offer local support for the most aspirational young mathematicians:

> *We offer additional enrichment opportunities to sixth formers, and they're very popular. We don't usually get any direct benefit, but we do think they're worth doing as part of our if you like, moral purpose* (HEI7).

As indicated, only mathematics departments in state schools or colleges were approached for live teacher and student voices, and among those, only those with good Ofsted reports, known to teach Further Mathematics at A Level, and with at least one teacher who was a graduate in the mathematical sciences. Information from interviews, taken together with student and teacher survey responses, underlines the high level of generosity of teachers: responses to questions around availability of teachers for support almost always suggested availability outside lesson times at any breaks, lunchtimes, or before/after school slots, as well as by email. Encouragement for the focus of that support, though differs by teacher and by department, as in Case Study 3 above.

> *From June in year 12 we put on maths enrichment class two lunchtimes a week, and we have about 50 students come regularly, a lot more to start with - cream of the Further Mathematicians, largely. We prepare them for additional papers, they do an extra paper for their Further Maths A Level, to give them a better change of that A*, we give them a few things they wouldn't otherwise see. We do extra Pure, all really good foundation for uni maths, a bit more abstract, e.g. group theory, so they get more confidence and more abstraction before they apply for maths degrees. It's largely me and another teacher. Not many could run enrichment (CT9);*

> *We occasionally have a student who applies to a top university and needs to do an additional paper, but wouldn't go out of our way to encourage them to do so: we'd say find similar universities that don't need extra papers, because we really don't have a very successful record. We only have one teacher who can teach Further Maths, and she's fairly loaded so doesn't have a lot of time to support. We suggest they join an online community, and use resources on websites…. But they find it really hard and fast-moving to keep up with, … and found they were expected to do too much extra work that didn't really fit* (ST10);

> *Recently, we've found it increasingly hard to appoint A Level teachers, so some of our KS5 teaching time goes into less mathematically experienced KS5 teaching rather than talent spotting for AEA. But we're in an incredibly strong school compared with most (city) comps. Super-serving the Further Mathematicians is not our priority - they're being well served. So now it's more self-selecting and 'if you're interested please tell me' - at Christmas of year 12. Autumn of year 13 is too late for MAT, and they need a term to settle in. Earliest it's realistic is early*

[8] in April 2020, though, Oxford initiated a MAT livestream prep course, which will start again in Jan 2021. (https://www.maths.ox.ac.uk/study-here/undergraduate-study/maths-admissions-test/mat-livestream) Social media suggest this has been very well received so far.

February time in year 12. At present I'm offering MAT support to 4 students: the only paper that insists on STEP is Cambridge. They're wanting maths degrees. They're thinking of Oxford rather than Cambridge, since we've had more success in the past. We haven't pushed kids towards Cambridge for years because it's very hard to justify the investment we'd need to make to give students a realistic chance - it doesn't merit the overhead…. Kids get a post-school maths club. In the KS4/5 area there's a table for those who are keen. 'Look at these two questions, I'll give you 45 mins and come back and we'll talk about what you've found'. There will be 2 or 3 teachers staffing the room, so they'll get about 5-10 mins of my time (ST9);

In year13 give interview style questions, and make them do those at front - really puts the pressure on. We made it massively more organised….Obviously there are all the past questions, loads of websites, MEI Integral, ... They have to have the independence to take it forward themselves - we don't have any capacity to take in work and mark it. We also do mock interviews. So we have invested in a sustainable system. (CT9).

The most experienced providers of in-school support were explicit about the need for sustained investment in preparation, and also pointed to the impact of more aspirational A Levels on willingness to engage with beyond-A Level material:

Very often students or schools don't appreciate the investment needed, longitudinally. Also, students don't understand that more prestigious means more difficult. (Our) timetabled lessons are for all year 12s - all 140 Further Mathematicians. We use UKMT materials initially. There are a lot fewer now - they walk with their feet when they feel it's not longer helpful, but can come back any time. Further Maths isn't enough to make them confident for a prestigious course - they need some eyes opened….A few years ago lots of students were willing to try, but the new A Levels have brought greater pressure, so some capable students are dropping the extra since they're finding the new A Levels quite demanding. For less outstanding students, if they enjoy it OK but it's not right for weaker students to continue. …We used to have a lot of students going to AMSP courses but now they're too busy, most of them. (CT3).

Many teachers who responded to the survey recognised the added value that in-school support adds:

The support pupils need is a lot of practice at persevering on demanding problems, being guided by a knowledgeable teacher only when required – but they've got to have that teacher. External resources such as courses and worked solutions can help, though these work best when they are supplements rather than the main thing a pupil relies on (GT58);

We have many more students attend lessons in Y12/start of Y13 than actually end up taking the papers, but they've got to be heading for top grades. It improves their general mathematics. They need some teacher input and feedback, just giving them online resources is obviously better than nothing, but teacher input is very very important (GT38).

As might be expected ('It's part of what we're here for' (CT7)), the specialist maths school in our sample made extensive in-class and additional provision for additional paper preparation:

Last year I attended STEP classes every Monday, from March in year 12 onwards. There was prep to do for each session. Sometimes we worked individually, sometimes in a group, there was lots of work over the summer, then sessions every week for MAT papers in Autumn term. I also attended Maths Challenge sessions for the later rounds: that was good for having to communicate.

Preparation for the real MAT was limited by the number of past papers - only 12, and there's nothing comparable out there. STEP questions also helped in preparing for MAT. For TMUA, it's mainly looking at past or specimen papers - once you've got the idea, it's a different sort of thinking and there are techniques and different understanding, including strategies for the multiple choice nature of it. There wasn't a lot of extra content, but MAT and even TMUA needed more in-depth thinking about it, and applying it. That comes with practising questions, and understanding how the questions want you to think. The STEP1 introductions in the document support you in that.
Last year we had the lunchtime problem solving, in the first term in year 12, working with SMC type questions, then the team challenge, then in January we started using the Siklos materials where you have the warm-up. Students from the college who come to the STEP sessions say we're taught completely differently here, it's not about A Levels. And I guess we are, I just don't notice because I haven't done A Levels anywhere else (CS24,25).

Some teachers pointed to the proliferation of papers, the challenges that brings for teachers in getting to know and support a variety of different expectations, and the challenges on timing of papers:

Things seem to be getting really quite complicated with some students having to sit several extra papers - streamlining between institutions would be very helpful. So some students have to sit both MAT and TMUA on the same day, and it would be better not to have both AEA and STEP1 for reduced offers, in the summer. And to have those where they were, at the end of A Level exams, not in the middle: that's not fair, since it hits students differently, and if it's in the middle of a bunch of A Level papers they don't know where to put their efforts (GT42);

I couldn't cope with TMUA as well - how much of my teacher can I invest in getting my head round all these different papers, for a very small minority of pupils? (ST8)

The picture, then, is one of a norm of considerable generosity by many teachers, for this or other particular purposes, in institutions where there is no timetabled provision for support, though it remains the case that teacher capacity for specific mathematics support at this level is very variable across schools. Teachers and students were explicit about the need for local, as well as online or externally-provided support, although lightly-informed interest from a teacher goes a long way. The DfE-funded AMSP programmes for teacher and student support re highly regarded, and the live and face to face nature of some of these particularly valued. Many teachers and students recognise a role for online materials that are studied asynchronously, but most consider those a poor substitute for face to face support, preferably locally. Many students in mathematically well-resourced institutions complement school/college-organised support with sustained and substantial engagement with such additional resources.

7. What is the impact on students who engage with these papers?

(What are the transferable benefits of having engaged with such questions at different levels of intensity, for example the benefits for university progression? What are the benefits (or costs) to those who enter for them but do not 'succeed'?)

HEI participants, when they talked about learning benefits to students of engaging with additional papers, usually privileged problem-solving skills, deeper mathematical thinking, getting stuck, and development of resilience; there was some mention of the development of communication skills for students engaging with MAT or STEP. They typically said different papers offered very different experiences – although outside Oxford, Cambridge and Imperial, HEI use for admission purposes is fairly eclectic, and not always uncritical:

> *For students working towards AEA (a bigger group) and STEP-esque (for the top level) questions who want to think more creatively and enjoy the challenge, and with TMUA also having very interesting questions, there's a benefit to the students if it doesn't compromise other things too much. If what is meant to be achieved is achieved (with the new A Levels), then at least some of those students, say the second tier, will be sufficiently challenged if they're doing Further Maths especially. For the very top students, it gives them a different way of thinking. Though I do think the TMUA structure of multiple choice is completely wrong – I went to a course on TMUA and the guy in charge said he would 'train students in the art of multiple choice' – which of course you can do. So what's all that about? But there's some really good questions there* (HEI1).

One HEI academic described the use of a 'cut-down' TMUA with first year undergraduates, for formative purposes, and also pointed to the benefits of these additional papers for the quantity of time students have spent working with mathematics:

> *We had a cut-down version of TMUA with a first year and Ellie Darlington, in Teams. A lot of our first years did very badly. A minority did well. Largely they did well below a third of questions well. I think that is probably about right, since our target at (HEI) is those with potential but they didn't yet get the mathematical reasoning or thinking. So this helps with self-realisation and is our target in the first year: 'Don't worry if you screw up'. There's a learning curve in terms of style and balance of expectations. Even with the new AL they won't have spent the time. So a massive advantage of engaging with TMUA at school is you've spent more time working with your maths' (HEI5).*

Another academic, at a university not requiring additional papers, pointed to the beyond-content characteristics that support good transition to university mathematics. She saw the new A Level as potentially supporting those, and felt they should be promoted for all A Level students, not just the high-flyers who might engage with additional papers:

> *Second years who help with my support sessions see there is a difference between them and the first years with the new A Level - and they think the new version is better, though colleagues are not seeing a lot of difference. If they come to me (for support) they've often got imposter syndrome - at uni 70% is really good: it's about how you express things… If we're looking for them to develop more mathematically, they could perhaps do with more resilience, the confidence to know they'll get there if they keep thinking, learning how to communicate mathematical thinking better. The big difference with uni maths is, that normally at school, you've got the tools to do a question, whereas at uni, it's a proof and you've not been shown exactly how to do this. With the new A Level we might get there - but teachers at the minute don't know how to teach for that proof in new situations. So formal reasoning could perhaps be more pushed even at GCSE. And comparing quality of solutions. Those things shouldn't be restricted to the very top students* (HEI7).

Full-time support providers often 'know' the additional papers very well, and much of their work might be concentrated on working with those, so that they were able to discriminate between them for affordances and accessibility:

> *TMUA is accessible from an early stage. This year's paper has ugly questions, previously they were just challenging enough to be interesting. But there's no need to develop communication. MAT has nice questions also, its' much more difficult to 'teach for' and although the bulk of questions are multiple choice and can often be used in year 12, the big questions require clear*

communication – and actually, we require that in the use of all questions, even TMUA. Students resist, but it's right. TMUA and MAT questions support getting stuck in year 12. Both have an emphasis on combinatorics, logic, explaining things, asking different questions from A Level, they're about understanding the maths in depth. But then for MAT, you have to think very deeply with it. I also use UKMT questions -they're quirky, and good for problem solving skills and combinatorics. There's quite a lot in there to get people thinking mathematically. STEP questions are more tedious than MAT questions. STEP mechanics are pure questions, with easy mechanics. They're a bit samey, once you get used to them, but not so familiar initially. Some require a lot of time and additional techniques, and you're disadvantaged if you haven't met those techniques. Preparing for papers requires development of a good work ethic, determination, putting yourself under pressure - useful for all students and these have often cruised so far.. The all need an experience of struggling. It can be a huge shock if you haven't had it (Supp2).

One support provider pointed out the strategic benefits to students of accessing this expertise:

I'm giving TMUA a very different story - don't bother writing down integration, just do it, substitute as you go - nothing about communication. A Level also is forgiving on that. So TMUA, yes, it's about getting sharper with the basics, though part of that comes just with the extra exposure, but a lot is about learning some strategies, and being confident with those. That has little to do with preparation for university mathematics (Supp3).

Another described how students often clarified their future plans as they engaged with papers:

The process of preparation can itself persuade students they want to read maths, when they haven't found it interesting previously. But it might also tell them that uni maths isn't for them, at least not in a competitive university (Supp4).

Teachers talked about engagement with additional papers supporting depth of conceptual understanding, mathematical horizon-building, encounters with some additional material and especially new, and more sophisticated, approaches to problem solving, experience of having to engage with protracted solutions and sustained 'being stuck', and for some, development of independent learning skills.

I think they're getting unstructured, interesting questions, and if they have that opportunity, they're building up skills and appreciations that will stand them in good stead medium-term. MAT Papers are well-presented, even the stepping of those first multiple choice questions. They're attractive bits of maths, and visibly, those students are enjoying it - 'that's a really nice solution'. Relatively few A Level questions have that. STEP questions are just dry, and very hard – A Level questions on steroids. (ST9)

A few identified the development of oral and/or written communication skills as a key benefit, sometimes pointing to the archetypal 'able but lazy boy' who manages to get as far as A Level without internalising the benefits for himself, or the reader, of coherent communication of mathematical argument:

The main benefit, especially of STEP, is resilience. Many of these students have not ever got stuck before attempting STEP and they need to learn strategies to deal with this before tacking university mathematics. Lateral and synoptic thinking are also strengthened as the questions often require students to go beyond routine techniques (GT44);

They're good for having to communicate…. Uni entrance exams really just want you to understand what it is you're working with, so I think they have that in common. Thinking does develop by talking with peers, seeing how the teacher prompts, and learning over time to reflect that in a written argument (CT3).

Teachers agreed that student confidence can be impacted in either direction – and they also pointed to engagement with more abstract mathematics, and mathematical reasoning, serving to clarity students' intentions – whether affirming an intention to read mathematics, or suggesting that perhaps that wasn't after all the right path.

However, two did point to instances where possibly over-ambitious use of additional papers had led to a capable student deciding not to read mathematics, because they concluded it was 'too hard', even though in both cases the teacher felt the student might have thrived at a slightly less competitive university than those being targeted

She wasn't over-confident although she had a lot of mathematical potential: she could synthesise ideas, and make inferences several steps ahead. She often left me behind in her thinking. But she did a MAT paper a couple of years ago, and it was just too hard: Oxford think it's fine because they'll just mover the boundaries of who they invite to interview, but they then rank people differently, and confidence and risk-taking play a much bigger part: it was devastating and she decided maths wasn't for her, if it was that hard at university (ST11).

In both cases these teachers reported a 'hands off' approach to student university choice. Finally, there was a small but powerful counter-voice, from teachers in high-performing independent schools:

*(Benefits?) None. Their time would be better spent exploring mathematics without the pressure of an additional exam. For example, using the UKMT mentoring problem sets (*GT62);

These papers undermine the social fabric. State schools, in particular, can't provide the same sort of support that we do, even if they have the teacher expertise. Unless we can do better at giving access to high quality maths education to all able young mathematicians, they're simply not a fair ask (GT43).

Students quoted benefits as additional (usually perceived to be appropriate for them) challenge, enjoyment and satisfaction, reinforcement of progression intentions for some, disillusionment or change of intentions for others (which they still identified as a benefit of engagement). Note that students who had experienced additional support noted both enhanced grasp of mathematical processes, but also exposure to additional material and techniques, despite claims that these papers draw on standard A Level content: *'Development of mathematical intuition' (GS38); 'Further understanding and use of advanced reasoning' (GS3); 'More maths knowledge, techniques and ways to apply maths not taught at A level' (GS35); 'Sparks more interest, given these harder and unconventional questions'(GS680;*

Just getting to be able to think better with what you know, to see the connections and implications within it, and with other ideas. You also get to be able to explain that better (CS 25);

I really enjoyed how the questions are more about your in depth knowledge about the maths and I think it's an undervalued skill, I think that's the point of these exams, to show you can think about maths deeply and understand it rather than being knowledgeable (GS82);

Higher A-level grades, more confidence in my subject and a more in depth knowledge of maths beyond the A-level curriculum (GS43);

It's really helped me to work on my problem solving and also how to break down a question to actually see what they are asking and what bits of knowledge I need to use (GS74);

I did find the difference in how MAT questions are compared to regular A level questions refreshing because it meant I had to properly learn how to work with the math I learnt and what it "meant" rather than just learn how to use maths to pass my exams. Also because of this required a more in-depth knowledge I had to actively think and apart from that being valuable on its own, it helped me out mentally through a time I was struggling through (GS81);

Some also pointed to developing greater independent and organisational study skills:

Looking at putting together skills I already had and new skills I had to research in areas of Maths (GS34);

I improved personal discipline and time management in order to fit in TMUA preparation while maintaining homework and revision for other subjects. I was also preparing for STEP that I didn't end up taking, but the papers are very long and challenge students to focus for long periods of time. That's probably good (GS48);

It helped with scheduling revision as well. The entrance exam was something I cared massively about, so I got into a full revision schedule much like I would for A-level, only a lot earlier. I then (would have) just need to maintain this schedule going to A-Levels (GS29);

Resilience and determination, thinking more abstractly and more logical deductions (GS77);

I developed my problem solving ability, and became better at independently learning from my mistakes (GS70).

They did, though, point to stress (especially associated with STEP, because of its timing and function), and the learning cost to other A Levels for some: related costs to students are discussed below. When asked about the degree of commitment and independent working needed to engage with additional papers, some seemed unaware of what was going to be needed to meet the grades, some talked about 'school/college telling them what they needed to be doing', and yet others either had made, or were making, considerable commitment and sustained independent application:

I spend over half my time working on STEP, about a quarter on Chemistry, because that's the one vulnerable to not getting the grades, and anything else on maths and physics – I'll have to learn some physics facts at some stage. With weekend and online STEP classes, and the extra in-college STEP classes, it doesn't leave a lot for anything else, if I'm honest (CS14);

Although it's time consuming as it falls outside of school time (and I do four A levels so I already have a lot of work as it is), I do really enjoy them and they are really helping me to think in a new way and it's testing me on lots of areas at once (GS74).

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Possible costs to students

For students well able to master A Levels, engaging with well-supported work represented by these papers was, as we have shown, widely held to benefit them mathematically, and in terms of transition to university mathematics. But many teachers and students also identified risks or cost to at least some students (beyond the financial, which we consider in chapter 9), and those should be weighed against benefits. Potential costs include to A Level performance, to a broad and balanced education, and to mental wellbeing, but again, teachers had different views, including that their existence undermines wider participation in mathematics at university:

I strongly consider these additional papers ..to be artificial barriers that are simply designed to limit the numbers of students that top universities consider for maths degrees and that in so doing they provide disincentives for many people who would succeed at a maths degree but are put off by these barriers. The real loser here is mathematics because those who are put off are typically those with other interests and skills that could be hugely beneficial to mathematics (GT62);

Learning to overcome difficulties or "failures" is a necessary skill, and in my experience, failing to get an Oxbridge interview has been the first time some pupils have failed at anything significant. Developing the resilience needed to manage their disappointment and, crucially, to not let it affect their future progress is important. There is no cost if managed properly, and realistically, teachers involved in preparing pupils should have an idea of whether they are likely to "succeed" or not, and manage expectations accordingly (GT58);

Good students have no risk. Weaker ones who are wanting to do maths at uni sometimes skimp the basics because they're concentrating on STEP, or MAT. We don't let them do STEP in year 13 if they don't do the chunk of summer work we set: it costs A Level robustness. So those who continue know a bit what they're letting themselves in for (CT7);

Costs are mostly to do with stress, but also the false negatives that exist. While I think papers do a better job of discriminating based on ability than interview they are not perfect. The most promising student I have ever taught underperformed in the PAT, and I think has less belief in his own ability than he now should, especially having been published at 18 (GT40);

If they invest and don't get the offers, it's demoralising. It's a good life experience, for elite students, and after MAT/TMUA they have time to bounce back, so I don't have too many problems with that. But they can go either way – it can curtail enthusiasm. I prefer Oxford's model to Cambridge's - you can then concentrate on A Levels rather than that summer being so contingent. Weaker students are getting interviews at Cambridge, with preferential admissions. But Cambridge aren't expecting many offer holders to succeed - only about a third to a half will, and those mostly from well-drilled independent and grammar schools. That's a very high ask. I think for more disadvantaged students, that enormous pressure is unreasonable (CT9);

It does support their A Levels but also adds additional pressure and time constraints. We have had a student close to breaking point trying to prepare for these exams, especially now STEP is right in the middle of all of their other exams. The extra support is great but means students are attending extra sessions all over the place and online and that additional pressure is also hard for them – they're only eighteen months past being GCSE students (GT16);

The only benefit is entry to university. Short/medium term it distracts from A Level courses and increases stress and anxiety (GT20);

They can be quite daunting, and in my experience may put off students who may have flourished in a maths course once they had got past the initial struggles of university (GS7);

STEP in particular is time consuming to prepare for, and even exceptional students (including 7A at A Level) have found it difficult to manage* (GT24);

Costs: reduction in confidence especially if paper is harder than in previous years. Loss of time spent on A levels whilst preparing (GT31).

The word 'stress' occurred over 45 times in survey responses from 82 students, all of whom had been involved in some way with these additional papers: we did not have any responses from students who had never engaged but might reasonably have done so in terms of their mathematical potential. The survey was completed by students in March of year 13, so some time before the immediate approach to STEP papers: *'Stress'* (GS39); *'Revising for them interrupted revision time for my A Levels. The stress of an extra exam took my focus away from my lessons'* (GS34); *'Extra revision and more stress'* (GS36); '*Take a lot of time and extra pressure'* (GS38); *'Very time consuming and stressful, very energy draining'* (GS78).

Some students identified more specific costs to serious engagement with additional papers, sometimes specific to this year although 'mock' results have always been used as evidence in case of illness or trauma during live examinations:

> *Revising for these tests took away revision time from mocks resulting in lower predicted grades (in unrelated subjects – in this case Chemistry for a mathematics applicant, and now mock results are key information for grades to be awarded in Summer 2020 (GS28);*
>
> *Concentrating for long periods of time and practising outside of normal school and homework or revision time took a large toll on both mine and my friends' mental health. My life became 100% maths which is enjoyable, but prevents development as a person and in the community - for example, I didn't volunteer or do work experience, and my social life was lacking. I was also putting pressure on myself (and many others had pressure from their parents) to do well in these exams, because they determine whether you can go to a good university, which was difficult because it was earlier in the year than A levels, so the stress is maintained for longer. The pressure was placed on an individual exam rather than performance across 3 A levels and 9 exams* (GS47);
>
> *Pressure of knowing there's so much work to do, and only a small chance of even getting an interview... I didn't want that pressure on myself, so I chose five unis without that... It's a risk and you might end up underperforming in A Levels... STEP is a ridiculously high standard. The timing of the STEP papers is a turn-off and a real pressure... You need to spend half your time on STEP and only half on A Levels - that's a risk. ..When I was thinking maybe maths, it was definitely Oxford rather than Cambridge because of the timing pressure... Yes, me too (CS11-16*).

Some comments from HEIs, though, suggest that there are large numbers of students taking e.g. TMUA, who are not realistically even aiming for confident A's in Mathematics and Further Mathematics A Level, and it is not clear where the balance of benefit lies. For example, HEI5 earlier said his institution often admits students without an A in Mathematics A Level, and/or without Further Mathematics even at AS, and then went on to say, *'4.5/9: about 45th percentile, or 16/17 out of 40 - is good for our intake. There's a good element of guesswork in there, given it's multiple choice, so they're not really 'succeeding' with TMUA'.*

One support provider was very reflective about the balance of benefits and costs for students attending his Saturday STEP classes:

> *For those less successful in my classes, the benefits/costs vary. In my STEP 2/3 course, Cambridge offer holders who work hard and believe they have a chance, often still don't get grade. Most say they understand maths more, their first year became easier, the classes eased the mathematical transition. They're still operating in a secure environment in school, so usually, the challenge here is manageable – if not, they vote with their feet. If they complete the classes then once transition is complete, they don't regret the effort. That's one group. Another realise half-way through how much effort is required, and give up: they do the homework, turn up, but have internalised that Cambridge is 'not for me' and aren't prepared to put in necessary effort. I*

don't have a problem with that - it's a positive outcome - part of these classes is to inform them 'this is the level needed for this arena'. I make it clear it's OK to not go to Cambridge, when that feeling emerges - and some of my PhD helpers went through that. For STEP1 course - and a mix of the two was a disaster - within 3 lessons the STEP1 students would drop out - they felt inadequate, the gaps were widening. So I have two courses separate. Even for STEP1 in order to progress, you need to commit - not as much, but if it's not there, you don't progress, and it doesn't become any easier, they don't get a better understanding so I think it is frustrating. I think it sometimes affected the enjoyment of mathematics. Is that over-aspiration? Or is it part of your education to find out where your limits lie? It might even be that a maths degree is what they'd enjoy - perhaps it helps them find the right sort of course, or university, but I'm not always convinced (Supp1).

While there is evidence of considerable mathematical and study skill benefit to high-attaining, well-supported students who engage with the questions on such papers, and some evidence of benefits to transition into mathematical courses at university, then, there are some reservations about the need for such study to culminate in a high-stakes assessment, and widespread concern about the stresses on many students who engage with such papers, especially if such engagement is over-aspirational or under-supported. There are also concerns about the pressures on teachers to engage in such preparation in addition to standard workloads, although other teachers found such work highly fulfilling.

8. What is the range of students who would benefit from preparing for such papers?

A common view from academics was that most A Level students would benefit from engaging with the more accessible of any of these questions, although few teachers, even in the most academic institutions, thought that was so. Few academics seemed familiar with UKMT activity that is very widely used in schools. There was a limited academic grasp of the range of functioning, or current workloads, of A Level students who might progress to less competitive courses to read mathematically-intense courses such as engineering, but some awareness of teacherly constraints:

Everybody who wants to study mathematics/physics at a university (should engage with them). Ideally all schools would be able to support such study with capable staff, but that is not going to happen. The short-cut/cheap alternative of offering online materials will work if there is sufficient awareness of the benefits of such study and if schools were able to push students towards the materials in due time, rather than half way through year 13 (GHEI1).

Support providers, who in many ways often act as an interface between schools/colleges and universities, also usually gave more nuanced responses that highlighted the depth of need for prolonged and deep engagement with such questions if pre-university students are to reach their potential performance with them:

(What range of students do you think would benefit from engagement with such questions?) That's a hard question. STEP in particular, you would need to have a well planned developmental programme to use in any meaningful way. It wouldn't be helpful: STEP is a journey, it takes time, inevitably. Either you commit to that journey in order to commit, or it's

not a helpful process. Because...when you look at Polya's problem cycle, the first stage is to understand the problem That stage is not in A Level, even the new A Level, but is in the STEP questions: rarely can you grasp the question in full, what it's asking you. It takes a long time for the students to understand how to read the question. The STEP language is a very specific language, specific to STEP, not to mathematics, that you can learn how read. STEP questions are well-structured, yes complex, but they're in an unfamiliar language. They're telling us exactly what to do, but you have to learn how to read it. And you can see, as students build their experience, they come to be able to have an entry to these questions. So for enrichment sources in ordinary schools, with kiddies in year 11 or 12 who grasp maths quickly and in depth, I'd go for the UKMT questions, or if I were being ambitious, work towards the multiple choice questions in the MAT paper: a little bit different from A Level, involve problem solving, yet the barrier of understanding the problem, though important, is not there to the same extent. So they're reasonably accessible, though some of them are very challenging (Supp1).

One support provider pointed to what he felt was sometimes an expectations gap:

There's a disjunct between what HEI expects when they arrive, and what they're like at school. HEIs want students good at both knowing stuff and problem solving. New A Levels are not making much impact on that, unlike GCSE - A Level has stepped back from that quite a lot, because of the range of content there is there: you can't have everything in a short time. So HEIs in general don't really understand what students are like, and their past experiences. STEP 2/3 are aimed at people who need to know what the tripos is like - hard and horrible, unapologetically. They force students to think about what they know, more deeply, rather than 'Pavlovian maths'. So does MAT – but that tests more deeply, can they genuinely think at a sophisticated level. TMUA has a different sort of pressure - efficiency of thinking. There's usually quick elimination of most possibilities. Those might reflect some HEI needs - maybe??? But they're not the priority for most A Level students, who are fully absorbed by transitioning from GCSE maths and getting a grip on the A level spec, and that's fine (Supp3).

Among teachers, only a few of those in the most academic departments considered sustained use beyond those students seriously considering applying for HEIs requiring, or strongly recommending, such papers, and in that case, largely only in year 12. Most felt they were not appropriate – but also that students in the current performativity system would resist involvement:

I use such questions very very seldom, since they'd be so hard to access for many in an A Level class - and also because of this deep-running culture of exams as currency rather than an intrinsic virtue. By the time they get to the sixth form, it's so hard offering them things even with instrumental value but outside that framework, there's resentment about expending time on stuff that's not going to have currency in the exam, especially if they're not going on to do maths. That's where we are. Very few teachers have the clout to resist that (ST9);

We don't use these questions much in ordinary lessons- perhaps with y13 high-flyers in normal maths lessons, to stretch them. But not for year 12s. Some of the mechanics questions are pretty straightforward, in comparison. And just before Oxford interviews, I give them a choice in the starter (CT3);

Occasionally we use in normal Further Maths lessons, if the students are feeling particularly tough. But even with our classes, they can't benefit often, with 25 in the class - they're usually going to be great only for the very top end (CT9);

No (we don't use them): they are too hard and not helpful for those who are not aiming for the top grades (GT8);

No, because these questions would be largely inaccessible to most of the students in the class. We are trying to improve confidence not undermine it (GT11).

They've got to be interested, intellectually engaged, and heading for at least an A in Further Maths. I don't see a great benefit - often they'll drop out of additional provision, and that's particularly true of summer exams. It makes a difference whether it's pre- or post-decision. They sometimes jettison what they don't need - perhaps include it on their UCAS form without commitment to seeing through. As a class teacher, I had a few questions I did use, some of them from very old Oxbridge papers. I found that though they seemed fairly innocuous to me, there was something about them that made them crash even with good Further Mathematicians. I think it was that demand for a different sort of thinking, needing a desire to engage, and a commitment. Some of TMUA you could use like that, but it requires very careful selection and presentation. I'd use nrich questions…. Or I'd make up questions of my own, to stretch them (CT10);
Realistically, only the strongest mathematicians who should get a double A in Maths/Further Maths are able to confidently access the MAT/TMUA, and especially STEP, while studying for other A Levels at the same time. They require more than just a superficial understanding of when to use a technique - they have to know how and why it works, and how/why it may be better than an alternative approach. Many pupils are let down by gaps in subject knowledge from lower down the school that undermine their progress* (GT58).

Some teachers pointed to perceived additional time pressure for new A Levels:
That would be great but with the new specification we don't have the time we need to cover the problem solving in its current format with the lessons we have let alone doing more (GT16);
I think it is (important to sometimes use these questions for a wider range of students), but time constraints (8 hours a fortnight with classes as large as 28) mean we tend not to (GT66).

A handful of highly academic teachers at high-performing schools reported including use of such questions in regular A Level lessons; others widely pointed to the investment needed in 'coming to know' how such questions worked, and that use without very careful preparation could very quickly destroy mathematical confidence, even among students with high potential. Teachers in all situations were very aware of a potential cost to other A Levels, or even to mathematics A Levels, of significant engagement with additional papers, with a widespread attitude of 'only for those already very confident with standard material':
(Benefit for) A Maths, A/A* Further Maths. The online STEP support program is now very good, but students need someone well qualified to read and critique their solutions. Mathematics is always written for an audience (GT19);*
Those capable of A in Maths and at least A in Further maths who enjoy a challenge. Ideally would have teacher input each week from at least summer term in Y12 and self-study for the summer break (GT30).*
One respondent situated his answer in a broader context, addressing the issue of the 'very good' young mathematician at any stage:
Any pupil who has aspirations of a high grade in routine public examinations is benefited by being extended with harder questions. This is true in all years, not just 12/13, but the demand needs tailoring to the students. It is infinitely better to give pupils access to hard questions on material they understand than to accelerate them so that they take routine public examinations early. We would go to the stake on this! (GT4*).*

All of the student participants had engaged in additional paper work to some extent. They generally viewed such work as potentially interesting, but MAT and STEP 2 and 3 as 'very hard, and needing significant investment of time'. They widely felt that students need to be 'on top of' all their standard A Level work before sensibly thinking about adding in one of these: although the use of additional papers to give flexibility of one grade for admission purposes is widespread, there was a feeling, supported by many teachers, that serious investment in an additional paper could often cost more than that, for other than 'top students'. There are additional hurdles to accessing STEP questions before the end of year 13 where there is parallel, as opposed to serial, learning of A Level Mathematics and Further Mathematics, as necessary in many centres, so that students in such centres need spare capacity to first engage with material out of the planned sequence. A small number of teachers at highly-achieving schools said they created their own 'enrichment' questions for A Level challenge, without regularly drawing directly on questions from these additional papers, but that was unusual.

9. Are there related equity, including widening participation, issues?

(What are the disadvantages and costs for students who could benefit from preparing for such papers but are not supported to do so?)

Responses related to issues of equity often focused around availability of local support for enrichment and extension, and structural obstacles to achieving that – including from an early age; a consensus that steps to take preferential admissions further could be counterproductive; gender related issues; costs involved, including for private tuition; and concerns around both lack of regulation for STEP, MAT and TMUA, and recent proliferation in use.

Quality of mathematics education for the mathematically-inclined

A key underlying issue is that support for students with particular mathematical aptitude is variable, from early in their school career, so that many who could benefit from engaging with really challenging mathematics at an appropriate level, do not have the opportunity to do so – and might never find out that they could enjoy and excel in this area.

> *We don't want to add in barriers, but need to know are you interested enough to do this? The website doesn't talk about challenge, getting stuck, persistence, and the benefits of engaging with that pre-university – it's hard to do that without putting students off, so it's a fine line. I do that when they come to visit. Should Further Maths be included in the inequities question? If we seriously want to tackle widening participation, we need to question Further Maths, but the AMSP is making a really good impact. Where there's not Further Maths, it's usually because there's not engaging and challenging teaching lower down the school* (HEI3);
>
> *We (at a specialist mathematics school) do open our additional classes to students at (the local) College, and you can see they haven't had the day to day background or the support, every day from day one, that the students here have. Though some of them also have been part of our 'maths communities' in previous years so they've had challenge and new ideas, and pushing for rigour and thinking with what they know, in between six and ten days each year, before they come here* (CT8);
>
> *State schools are disadvantaged by lacking time, money and well qualified teachers needed to support such students. I could hardly keep up with the basic workload – it's ridiculous, so I*

moved. Good preparation often starts with extension classes or clubs for much younger students (GT19, independent school).

Those are issues that are far wider than the additional papers needed for university but are about the stretch and challenge needed for all students to reach their mathematical potential. Many, but by no means all, secondary schools engage with the UKMT challenges, but that is no substitute for day by day challenging, mathematically informed teaching. The issue is a well-established one in English education:
https://www.ukmt.org.uk/sites/default/files/ukmt/policies/2020-10-education-of-able-students.pdf
Further work in this area is reflected the ACME document “Raising the bar: developing able young mathematicians”: http://www.acme-uk.org/media/10498/raisingthebar.pdf; the Nrich page “Supporting highly able mathematicians”: http://nrich.maths.org/7741 and the Mathematical Association policy document “On Enrichment”:
http://www.ma.org.uk/resources/Policy_on_Enrichment.pdf.

However, while there are many materials published that offer ‘top up’ enrichment, that is no substitute for all mathematics lessons offering challenging, rigorous work appropriate to the student, and a ‘top set’ in a comprehensive school might well contain the highest achieving 25% of students in the cohort. There are some Key Stage 3 publications that support enriched and challenging teaching of such students within their usual class settings, but few for Key Stage 4 students. Mathematically-informed teachers of such students should be able to make such provision themselves, but need to see that as important and an issue of equity of opportunity to develop potential, as well as a priority among the competing demands made of them. Many state schools now have no teacher mathematically well-placed to make such provision either pre-16 or later, and it should be remembered that our sample contained only teachers in ‘strong’ mathematics departments, even in the survey.

Two teachers also talked about family background:

There’s also an issue about how they come to self-select (for our after-school Maths Club) - this year's all have family background and knowledge. So the kids often know they want it when they come into the sixth form – they have that cultural capital. So we could only address that by targeting students in different ways, but we don’t have the capacity to do that. So we do the best we can, with self-selection. We've had one or two others, e.g one who went to Cambridge to do Computer Science as the first of her family to stay at school beyond GCSE. That's what comprehensive schools are supposed to do. But she had so much natural ability and study skills, you almost needed to just say 'here are the resources' (ST9).

Family background makes such a difference: one frustration is that I was preparing the children of academics to become academics: the differences in informal support are important, especially if there are few of you in an institution - family talk, experience, employment. Good students who were unsuccessful, e.g first in family to university, that was often a significant factor, I felt. Extra papers exacerbate that inequity since there's more to do to try to achieve some levelling of the playing field - putting reading in front of them, giving them more time, etc, and new non-maths papers that have mathematical demands, put more pressure on the system. Sometimes there's more direct support at home, people they can talk to intelligently specifically about the questions, and so on. That's difficult to compensate for. Others with similar potential might just not believe it's for the likes of them, and that’s exacerbated when they all too commonly might not ever see a serious mathematical teacher. Even looking back, I was the only maths graduate *for six years of my career, and it took a lot of persuading SLT that the most competitive universities were for the like of our students. And the situation is ten times worse now (CT10).*

In this study, we deliberately excluded independent schools from fieldwork, though surveys were opened up to teachers and students from all sectors. One support provider talked very persuasively about the inequities inherent in the education system:

Even the grammar schools find it hard to compete with some of the big independent school, well-oiled machine for producing Cambridge graduates. Also 11-16 schools aren't even seeing demands of A Level - they have one goal in mind, which is the performance of the whole cohort at age 16. Problem solving in the GCSE is not well done, there are competing calls on time, so students get an impoverished mathematical diet….. The problem really is about the English system, with some academic independent schools preparing their students for Cambridge from the beginning, in all sorts of subtle ways, and they have a critical mass of academic teachers as well as a lot of high potential students. There's an almost insuperable effort to start preparing for STEP in February and make it. I always accept students from independent schools (on my courses) - 3/4 students, maybe 12/13 from state schools. I think the mix helps everyone. They work very differently, independent students work well, write the solutions reasonably well - whereas state students are typically less formal when they write. After the first few sessions, standing out disappears really quickly and the real players come to the fore. It's really good for all of them. They learn to interact, to respect, to place themselves socially. Some grammar schools do a really good job, with a critical mass also and links with former students, but most state school students start proper preparation in February and have to work insane hours - they really really need to want that. And I think a lot of them don't understand that - they think they're nearly there. That breaks my heart (Supp3).

Teacher capacity and priorities

Even among those mathematics departments with the teacher potential to support to the highest levels, our sample showed at least two 'strong' depts said their students 'don't do' additional papers. Several others were apparently very unfamiliar with processes, timescales, commitment needed (*'If they get a Cambridge offer, we'll look at web support with them'*(ST10)), and with e.g. Cambridge offers: places ratio. Competing priorities, and limited experience and perhaps knowledge are real issues: *'If we support these students, we can't support those struggling to get an A Level'*(CT11); '*We don't let them apply to Cambridge: STEP is too hard and we've not had any success with applications.'(*ST11). An additional issue for 'ordinary' comprehensive schools is that by definition of the target group of students, young people who would appropriately target MAT or STEP 2/3 papers in year 13 are unusual, so that typically-sized comprehensive schools do not easily build up a knowledge and expertise with the demands of, and investment needed for, these papers:

Students have a tendency to think their other A Levels are more important than STEP workshops in particular, even if they're wanting to read maths. Which I find extraordinary: they need to be able to juggle their workload so they appreciate the significance of this. They need influential others to support them in seeing that. Where do they learn that? I think there's a very big issue with teachers in school supporting application, but not understanding the system sufficiently to see there's a need to then prioritise that, and over the sustained time needed. With Cambridge that's a much bigger issue, especially if you don't prioritise STEP. Will Cambridge turn them down if they miss on Chemistry? Probably not – though of course they might need the Chemistry *if they miss Cambridge. There's an unrealistic expectation of getting into Cambridge if you have an offer* (Supp2)

Similarly, one teacher in an 11-18 comprehensive school also pointed to the ‘knowledge of the system’ she possessed as a Cambridge mathematics graduate:

I know my way around, I know they should really be building up to this from low down the school, I’m confident to wrestle with the questions and support the process from a distance, advise about choosing colleges, interpret what’s said on websites and so on. But there’s a limit to the time I can give to a single student who will have a great career if he goes to Bristol rather than Cambridge - I have 200 other students whose maths learning I’m responsible for (ST2).

There appeared to be limited appreciation by HEI participants of the realities of a 16/17 year old engaging with these papers from scratch, without reasonably, even if not mathematically, informed school support: ‘*I have no difficulty with the lack of in person support: part of the process is to select those who can study independently and show a real commitment to the subject’* (GHEI3); ‘*You’ve your small Further Maths classes, TMUA is just fun, so enter all your Further Mathematicians’* (HEI5). This seems disingenuous, including since several respondents talked about Further Maths classes of 25+, and also to conflate the benefits of using such questions, with the desirability of being entered for an additional examination. There is some small recognition in preferential offers that e.g. students in the Case Study 2 grammar school are in a different position from those in some other institutions, but the degrees of ‘independent study and commitment’ clearly vary enormously for the same mathematical potential and a similar end point of mathematical functioning. Others seemed resigned that what is needed to support preparation for these papers, is not widely available:

Small problem-solving classes are ideal, and what some schools do. This has significant cost implications: not all schools will have the resources or inclination to offer any support for such papers. Many schools don't even offer Further Mathematics! Thus, on a national level, I'd expect to see inequity (GHEI2).

Preferential admissions

However, there were considerable doubts expressed about the wisdom of offering further concessions in preferential admissions. The issues were summed up by CT3:

The issue is we're able to offer that support, and we have some mathematically strong feeder schools. I don't know how you make it fairer - will they drown when they get there, if you give more preferential admission? It's always going to favour the people who've had the support. I see Oxford is beginning to offer a foundation year. That’s great, but the reality is that students need good, probing and exciting maths teaching from much earlier, or it’ll be just luck if they happen to ‘catch’ an urge to take it further (CT3).

In terms of external ‘levelling of the playing field’, it should be emphasised that AMSP courses are aimed at state funded centres, with students and teachers from independent schools usually only included if there is space, and normally, at cost. AMSP offers an additional subsidy to teacher participants in target low participation areas. Universities and other publicly funded bodies are incentivised to ‘widen participation’, resulting for example in additional personalised support being available in the case of SS1 quoted above. However, there was some concern that there is a danger both of ‘box ticking’ students in particular socio-economic or participation groups at the expense of others 'in the middle', and of too much pressure to widen participation being counterproductive for the resulting students:

> *I worry about the students who are in the middle, those without the pushy parent and the private tuition – they're going to get there anyway. I'm thinking of the students who are good, strong, hardworking students genuinely hooked on maths who deserve to have the chance of a competitive course and will make really good use of it. They're getting squeezed out by all this use of additional papers we can't support them with, and increasingly preferential offers* (ST11);
>
> *Even the highly motivated, interested, exceptional student benefits from someone to talk to, even if they've been in the Olympiad team, for instance. We have a nice conversation and they teach me a few things. It's the less exceptional students where you make the difference. Those students at institutions where there's not gold standard provision miss out - they're not necessarily highly mature or motivated or organised, and they miss out* (CT10).
>
> *It's no good pretending students can engage with hard maths if they don't have the tools to do that – it would be a miserable experience and nobody would gain. We know that students from state schools perform slightly better when they're here than students from independent schools with the same grades, but we already take that into account* (HEI4).

There are overall admissions statistics of some sort available for most HEIs, and these often include admissions by socio-economic group, though they often do not disaggregate by course or by individual socio-economic status. In terms of mathematics entry statistics, the source institution is often only identified as independent/state-funded, whereas we have seen via the case studies that within the state sector, some grammar schools, for example, are able to offer targeted, expert support throughout a student's life with them, even given larger student:staff ratios than most independent schools. Expanded disaggregation of mathematics admissions data at the most competitive institutions might be illuminating. Further discussion of proposals for enhanced preferential admission can be found at https://www.officeforstudents.org.uk/publications/contextual-admissions-promoting-fairness-and-rethinking-merit/ and https://www.dur.ac.uk/resources/dece/ContextualisedHEadmissions.pdf.

Gender

Female focus group participants often said the MAT model of additional papers was most constructive (though not if it falls at half term!): purely multiple choice papers such as TMUA seem risky, and STEP is perceived to be highly competitive, as well as having outcomes late in the application process. We did not have any such comments from male students, and some academics recognised that common distinction:

> *Gender split: it's in our published stats. It's marginally better than the Further Maths gender split. But less bad than the STEP stats, and we think that's linked with risk-taking and behaviour on harder papers: our female students tell us they perceive MAT as a less risky route to entry, and probably associated with less competitive attitudes in the department (HEI4);*
>
> *The 'do it for a challenge' brigade is typically boys rather than girls. I suspect TMUA, in terms of those who apply and declare that to us, it's about 2:1. I think Durham aren't that bad. The more interesting qu is, is this the sort of test that boys will jump at and girls are less keen - which might be the case* (HEI5);
>
> *The reality is Cambridge is very competitive, and whereas lots of boys thrive on that - I've taught very few girls who are comfortable with that. You need a lot of confidence. From a state school,*

that often amounts to a brashness – unless you've such inner mathematical confidence you don't worry how different you are. Girls typically don't want to be seen as that (Supp3).

There is now an additional stress of STEP papers being timed in the middle of A Level examinations, a move which affects students differentially, depending on when their other examinations are, though again, there were more female than male students mentioned the risk to other A Level grades of such placement. Changes to a post-results admission system might help, though it's not clear how the current depth of interviewing could be maintained within that. One of the participant academics thought that the use of 'alternative offers', especially if they use Autumn papers, could reduce pressure in the A Level period. She suggested the combination of admissions policy and messaging adopted by her university supported relatively equitable admissions of about a third women:

We admit about 1/3 women, 2/3 men. The women do slightly better at the end, and it seems to be important to them that they know that. And that's significantly better than some of the other competitive universities. Which suggests we're not off-putting women (HEI2).

The range of groups of participants pointed to stubbornly persistent inequalities in female participation from A Level onward. Oxford admits ~30% female to their mathematics courses, in line with applications and with Further Mathematics A Level participation. STEP/TMUA entry statistics, including gender split, are not in the public domain, and the STEP Support Programme does not require participants to register. The Cambridge undergraduate mathematics (including the small Mathematics with Physics) course admitted 15% female undergraduates in 2018, the last publicly-available figures. Applications and offers to these courses were both just under 24% female. Several teacher participants with experience of supporting students with applications for Cambridge mathematics have a perception of proactive use of marginal offers, and admissions, to favour women, and felt that was justified given the significant under-representation at all stages to admission:

We've had offers for girls that are preferential, definitely, which is good. Further Maths here is usually about 3/4 male, but we're always working on that. We try outreach into schools, and have had a department review re teaching approaches re gender. If they've been strong minded enough to get as far as the additional tests, they'll be OK. Quite a lot of girls prefer MAT rather than that enormous pressure at the end - they're not such risk-takers, and the STEP paper system is perceived as fairly high-risk, including getting accommodation in your reserve choice, if you don't make the STEP offer. And some of the boys don't identify with that at all (CT9);

There have been many years (in this large high-attaining college) when we've had all male applicants. The papers reward risk-taking, including over the choice of questions. There are students who get there with an unusual degree of diligence, often male –comparable girls don't think they're good enough in the first place…. We have noticed that Cambridge manipulate offers to favour girls, but I don't think the conversion is gender-neutral. I think there has been a readiness to recognise the desirability of increasing the number of female offers (CT10).

Financial costs

MAT paper entry is free for applicants to mathematics courses at Oxford or Imperial, though students can be asked to pay additional invigilation/administration – reported to be commonly of the order of £50 in some centres. TMUA entry costs £31[9] (£55 in 2020) and is computer-marked since it's multiple choice; STEP papers normally cost £54 per paper, though each of STEP 2 and 3 will this year cost £99 (STEP1 is cancelled for 2020, and will not run thereafter). Some teachers felt the charges discriminated against able students, since their more moderately-attaining peers did not have to pay extra to access appropriate university courses. Several pointed out that Cambridge pre-interview assessments in Natural Sciences, Computer Studies, etc., are without entry charge for students, so felt it is not clear why STEP papers are treated differently. If a university requires one or more additional non-standard papers for entry, surely there is an argument that they should pay for that? In many centres, entry fees are in addition to invigilation/administration fees. Other centres pay, for at least students they consider, or who apply to be considered, 'disadvantaged'. The cost of taking additional papers is therefore an obvious source of inequity, but pales into insignificance when compared with the costs of private tuition in preparation for these papers: whole-day classes are widely available for £300, and individual tuition for one of these papers is commonly advertised at anything between £50 and £120 per hour. However, the benefits of such tuition, if not available in school/college, were recognised:

> *There's definitely a confidence part to this, where you don't know what to do, or you have to explain yourself or you have to apply work to subsequent parts. I think there are some exam technique things that students aren't always familiar with. Multiple choice questions are quite different, and benefit from particular strategic approaches. Applicants don't get benefit from their thinking, only for their answers, in mark terms, but then we often get them to talk through their thinking in interviews, and see how they deal with nudges. I'm aware of a big business in private tuition and coaching for interviews, that do help with that confidence to articulate thinking. We don't endorse that, or think they're good use of students' time. But where do you go if you get stuck on a MAT question? We're trying to work out some answers* (HEI4).

Regulation

Some academics and some teachers had concerns that these papers, except for AEA which has a very small entry, are high stakes for students but are not regulated: *'AEA is at least regulated, and you can get information about it. Why the need for secrecy, if they're sure what they're doing is fair? No one appears to care'* (HEI1). Participants usually recognise the value of autonomy for universities, but are concerned about an associated lack of transparency, and in some case, by a perception of aggressive marketing of TMUA and hard admission choices for universities:

> *They should be regulated - by someone. That arises from HEIs being very keen to protect their independence. The excuse for TMUA was the demise of AS, but that's not the case for STEP1 – you don't get results until summer. Oxbridge are so independent there's little chance. The maths community should perhaps claim ownership. I personally think multiple choice has no place in a uni environment - I don't care if you get the wrong answer. It worries me that we include it which implies approval - but it's a pragmatic decision* (HEI3).

Further concerns related to the circumstances specific to summer 2020 examinations, when there will be no results enquiries or appeals for STEP2/3, or appeals for AEA except on procedural grounds, and STEP1 is cancelled. However, opinions were divided on what would have been preferable arrangements, given coronavirus restrictions.

[9] Reimbursement is available for TMUA in some circumstances, see https://www.admissionstesting.org/for-test-takers/test-of-mathematics-for-university-admission/dates-and-costs/ , although several teachers said the scale of costs required for these papers was a significant disincentive to a wide and varied group of students.

Proliferation

There were widespread comments from participants in all groups about the undesirability of so many additional papers, and of proliferation of use, with many feeling that STEP1 and TMUA target similar groups of students, and are used similarly, and often unnecessarily, by universities:

> *Keep STEP or MAT for Oxbridge and maybe Imperial and Warwick. Scrap TMUA… I don't know what we're trying to do other than putting in an additional hurdle, it hardly says 'come and enjoy grappling with maths at university', does it? You also need clarity about covering the cost, that other university applicants don't incur. Schools might have to charge, because of their budgets, but why does it have to cost so much? And why can't Cambridge pay like Oxford do, and like Cambridge do for other courses?* (HEI1);

> *I think it would be better to make STEP easier and then drop TMUA* (HEI5);

> *I couldn't cope with TMUA as well - how much of my teacher self can I justify investing in getting my head round all these different papers, for a very small minority of pupils? So that's very distancing unless a school sees itself as a forefront elite institution* (ST9);

> *Herding universities is hard, but the thing would be less pernicious with a national system, and fewer papers. There have got to be levels, and students not take part in every level. Provision of resources and support would be more efficient and might bring more players in, since a lot is coming in through one channel* (CT10).

Teachers commonly had concerns about the additional pressure on students from STEP papers, especially now those take place in the middle of A Level examinations rather than after them, although as above, some academics valued reflection of the complete sixth form learning in such papers. Teachers were also critical of the recent timing of MAT/TMUA during school/college half term breaks, which they felt should have been easily avoidable. Two wondered about using (perhaps a renewed) TMUA as the first part of MAT, and following it with a set of longer questions as high-level discriminators. As reported above, academics reported that, confirming common teachers' suspicions, some universities use additional papers to promote market share or as a convenient control of numbers, rather than as a genuinely necessary discriminator among those with top grades in A Levels. One academic added another consideration, that he felt probably impacts inequitably:

> *It's all tied up with reputation. There are also issues about the grades asked for vs the grades actually expected, and that probably militates against less confident and informed student. UCAS could decide to cap offers at one grade above the mean entry required over the last 3 years, rather than something imaginary: at present we're all asking for more than we know we'll end up with, but we can't afford to drop that, since it will make us look less 'worthy' to potential applicants. It would reduce the stress considerably, and reduce the 'feeling thick' once they get there.* (HEI1)

Such perceptions were echoed by some participant teachers who frequently, while acknowledging a need for additional information around the top-performing students in the cohort, queried the now-widespread use of these papers as an additional hurdle for many, and were able to identify individual students who had decided to apply for a slightly different course in order to avoid additional pressure. Some also pointed to the need for schools or colleges sometimes to teach Further Mathematics A Level in parallel with A Level Mathematics, and the impact on what mathematics was learned in time to tackle these papers:

We feel doing Maths and Further Maths in parallel, with calculus not completed until deep into year 13, disadvantages them relative to students (we perceive as being mainly from private/public schools) who complete maths in Y12 and FM in Y13 (GT36).

Few students commented on the number of different papers, but those who did were not impressed:

I just don't see why they need so many different ones – they all cost time and money, even if you've got very effective support for them, as we have. And if you want to succeed, you have to mortgage the rest of your life. The extra sessions we have are really interesting, but the pressure of the papers is a big turnoff. Why can't the top universities agree on a single approach? And if they need something else between that and A Levels, just have one, at an agreed time? (CS14)

Challenges for equity of access

There are clearly, then, a range of challenges to equitable access to succeeding with these papers. In the teacher survey, variable teacher expertise, and pressure on teacher – and student – time were very frequently cited, including by teachers at high-achieving independent schools. These are quoted at length so as to give a wide view of the range of responses given, bearing in mind that of 66 teachers responding to the survey about half came from fairly academic independent schools, and more than half of the rest from selective state schools or colleges:

Provision of sufficient subject expertise, and pressure on teacher and student time. In particular there is a lot of pressure on the students who often have many commitments outside the classroom and struggle to fit everything in. The biggest problems for many schools (though not ours), and especially state schools with bigger classes, more challenges, bigger teaching loads, are (i) most learning in most classes rejecting everything not directly needed for GCSE or A-level, given the performativity messages; (ii) parents communicating to students the ideas that top universities are not for them (negative aspiration); (iii) Time for teachers to spend with students solving difficult problems, teaching workload is heavy enough (GT4);

Support for the large variety of papers offered (GT8)

Supporting these students and having the time and expertise to help. If I wasn't at my current school there would be no teachers with the level of maths needed to support (GT16);

Offering support to students but no one now has time on their timetables to do so as was once the case. So we are relying on good will of teachers (GT22);

With only occasional entry it is hard to provide structured support especially as all the tests have different styles and requirements. Support relies on teachers giving up non-contact time rather than being given a teaching allowance (GT3);

As a small state school sixth form we found it massively difficult to properly support students for these, it's hard enough to run further maths. We have no time, not enough teachers with expertise. We point them to AMSP but that is about it. I feel it disadvantages state school students (GT320;

There needs to be a plan far enough in advance to identify and support the right students to enter these exams as the main benefits come from the preparation rather than the exam itself (GT44);

Our main issue is the lack of time we have to deliver the course - every lesson is accounted for. We also have large classes in year 13 (17 and 18) in recent years and whilst this is positive it does mean we have higher proportions of "weaker" students for whom pace has to be slowed. This makes it hard to properly push most able, and is more of an issue with new spec due to higher demand and volume of content. We would do a much better job if we had the recommended 10 hours (GT46).

We need ONE exam board with a truly linear qualification, with some non calculator content and a general validity of assessment, and ONE additional paper for the brightest (GT58);

No issues for equity, IF the teachers have good subject knowledge, are passionate about maths AND are professionals who recognise the true meaning of education (GT60);

We are not able to support these students as part of the normal timetabled day which means that we have to run sessions after school. For some this means they are unable to attend the sessions as they are not able to get home after the session (GT63);

I know from talking to students after they have started university that studying Further Maths to either AS or A2 gives them a really good boost on arrival at uni. There are some great courses/days/summer school opportunities for that, and for extra papers, that our students have taken advantage of - please pass on our thanks for those (GT66);

We don't have those sorts of students very often. More worryingly, I am starting to get good Further Maths students when they start researching maths intensive course saying they are going to stop Further Maths as it is not needed for their offers: if unis want it, they need to be asking for it (GT 67);.

I'm the only one in the dept with a maths degree and in any position to support students. We are in an area of deprivation and our students don't have easy access to other students going through the same. It's too far to travel to any AMSP courses, and many of them don't have reliable internet access at home, so online courses aren't a runner, though we encourage them, and try to maintain an interest and meaningful support. (We also have issues with the selection and interview process. We are a state school with students from poor backgrounds who feel very put off by Cambridge and Oxford because they don't feel they will fit in. This isn't helped when they go for interview and end up with a load of private school students that makes them feel even more out of place) (GT16);

We provide limited support, but this is not funded by our (state) school and not timetabled within the school day. We cannot staff extra timetabled lessons although we have a number of well-qualified teachers able to provide them (GT19);

Some of our (grammar school) students choose to attend AMSP courses in London where, although the course itself is free, they have to pay for their own travel. Apart from that for our students there is no issue accessing support in school and on-line, but they do have a critical mass of academic peers here…. If we had any students who we felt were not able to access what we felt would support them or had problems with the fee, then as a school we would help financially (GT23);

We are able to offer some help but students often rely on an external tutor for extra help, and those are extortionately expensive. We usually only have one a year. It's unfair to those who can't afford it. (GT31);

As a small state school sixth form we found it massively difficult to properly support students for these, it's hard enough to run further maths. We have no time, not enough teachers with expertise. We point them to AMSP but that is about it. I feel it disadvantages state school students (GT32);

We put a lot of support in place, but I am concerned that this is not possible for many schools. It is also obvious that richer students are able to pay for private tuition, which provides a significant advantage. This is obviously also true of regular A Levels (GT40);

As staff we lack any real expertise and above all time to prepare students for these papers. The issue is possibly exacerbated as we are an 11-18 school in which A Level results are not our main accountability measure. Where external courses are available these are sometimes hard to attend due to transport issues in our rural location or clashes with timetabled lessons when time is already so tight (at our school we have only 8 hours per fortnight in years 12 and 13 when 10 is recommended as the minimum).We had a former colleague deliver a (paid for by students) STEP course on Saturday mornings three years ago but demand has since not been there. Further students are often reluctant to give up even more time for such courses on top of other studies, part time jobs and social commitments, and why should they? They don't have to for other university courses. And it means we're shutting out those who are not already 110% committed to maths – that's not very bright (GT47);

The AMSP and the Maths School make courses and resources available at no cost (except for travel). Our students have to be fairly confident and resilient to access these (GT48);

Huge issues in disparity of support. Our students are very well supported because we have the time and expertise to do so. I can't imagine what it is like in a school where those are unavailable (GT63);

The 82 students responding to the student survey had all had some interaction with the additional papers, though not all ended up entering for one or more. Most were highly appreciative of support from their institution, or elsewhere, and particularly supportive of opportunities to work face to face with their intellectual peers. As a result of such experiences, some commented on the variation in opportunity across different schools or colleges:

The college will pay for the paper if few enough people are taking it, but in all cases that I've known about we have had to pay. Obviously, this restricts access to higher universities if people don't have enough money to spare for an extra exam. Different colleges and schools provide different levels of support, and most of the questions can easily be accessed by less academically able students if they have had sufficient preparation (GS48);

Generally across schools there doesn't seem to be much guidance about how to apply and how the whole system, but very fortunately my school is very supportive in preparing us and informing us and preparing us for these examinations. I do think that accessibility is definitely an issue and in some schools that don't have resources to be able to train students for these does mean that there are a lot of issues around fairness (GS75);

There are not specific lessons for these and the questions take a lot of getting used to. I am fortunate to have a teacher who recommended starting preparation early, but many students I have met did not realise until much closer to the exams so they're already on the back foot (GS76);

MAT is extremely hard to prepare for as there is no mark scheme to tell you how many marks you should get for each part. STEP can easily be tutored for, such as integration tricks that people who just cover A level may not see. That's really not fair (GS78);

I don't have too much of a problem with some additional papers IF they are free and not at the same time as A Levels e.g. In summer before Year 13 (GS65);

If students have to contribute towards the cost of taking these exams, especially at non fee-paying schools, this could discourage students from applying to top universities (GS43);

The main issue I see is that papers like STEP are taken during the already packed and stressful A Level period. I have been attending STEP preparation sessions at my school to help achieve an A in further maths, but taking the paper itself would be pressure that I could do without over the exam period. Also why should these papers be required if the A Level is rigorous enough? Surely an A* or A in Further Maths is a strong enough indication of a student's proficiency without subjecting them to the additional paper (GS80).*

There is, then, unequivocal and widespread perception, particularly across teachers and students, that the recent proliferation in use of additional papers has inequitable repercussions – for the poorly-resourced, for many girls, and for a large number of young people without access to adequate mathematical stimulation, knowledge and challenge.

10. Conclusions and points for reflection

This study was of necessity limited in scale and scope, so that for example, evidence from the independent sector is limited to that from surveys. By focusing on 'strong' mathematics departments in state-funded schools and colleges, the picture obtained is of 'likely best possible': there are many mathematics departments which do not have a single even nominal 'mathematics specialist', many 11-16 schools where GCSE, as the prime accountability measure, is taught as an end in itself rather than in a glimpse of greater mathematical horizons, and 11-18 schools where, if Further Mathematics A Level is offered, the teacher has a poor grasp of newly-introduced content. Even before the current hiatus, recent pressures on schools have made it difficult for teachers to take advantage of laudable efforts from e.g. AMSP to address such issues.

Across participants, there was on balance a recognition of a need for Oxbridge, Imperial, Warwick (and possibly, two or three more institutions) to look beyond A Levels for selection purposes; there was some cynicism and concern, across constituencies, about other use. Academics' accounts of mathematics department motivations included both the mathematical and the strategic. Teachers, students and some academics widely felt proliferation of suggested use of such papers can be unhelpful to students and to their schools and colleges, sometimes undermining confidence to progress to very well-respected mathematics courses. They evidenced widespread and significant student stress associated with preparation for STEP papers in particular, because of their timing. Schools/colleges now face additional pressures from the proliferation of additional tests in other subjects also. Any additional mathematics papers, but especially the wide variety now used, were widely felt to discriminate against students in typical all-ability 11-18 schools and those in post-16 centres which have a high proportion of students with other intervention needs.

However, teachers and students commonly felt that in-depth *locally supported* engagement with such papers has the potential to enrich, challenge and deepen mathematical preparation for university, supporting effective transition, *provided* students were coping well with their A Level work. Most felt that mastery of A Level work should be prioritised, with the new mathematics A Levels proving demanding on all concerned, so that extensive exposure to such questions was appropriate only for the highest-attaining students, for whom the 'horizon-lifting' could prove pivotal and highly motivating. Sustained engagement for other students was felt often to be counterproductive, although some exposure could moderate aspiration in appropriate ways. Teachers were explicit that many, if not most, state-funded school/college mathematics departments do not have sufficient teacher expertise to support preparation for these papers themselves, and if they do, there might be other compelling priorities for intervention. Knowledge of 'the system', including constructive timelines for preparation, and familiarity with progression rates, was variable even among responding teachers, all of whom had been involved in such preparation in some way. Teachers and students were clear that online resources and external events could be very helpful, but were no substitute for local support and informed encouragement. Academic appreciation of students' school/college contextual variation, including in relation to the needs of applicants' peers, was sometimes limited.

There was general respect, even admiration, for AMSP provision, and also for the STEP Support Programme and nrich support, but this was usually coupled with a resignation to the perceived impossibility of 'levelling the playing field', in relation to teaching expertise, richness and depth of mathematics learning experiences, school/college priorities, and also individual home academic 'capital'. Some concerns related specifically to conditions prevailing this year, when for example some schools/colleges have chosen to prioritise non-exam classes in lockdown provision, and have not made any provision to support those preparing for STEPs 2/3; others have continued intensive, remote and now face to face, support for those papers.

In many ways, students in some highly academic grammar schools are in a fairly comparable position to those in the more academic independent schools regarding these papers, as in Case Study 2: there is not a clear independent/state divide. Some academic sixth form colleges or departments in general FE colleges, similarly, are able to, and prioritise, provision of sustained and informed support for additional papers, though only once they get to know students a little way into year 12. However, in many state schools and colleges any such provision is pro bono and in addition to teachers' normal workloads, and the corresponding pressure on teachers should be recognised. Further, within non-selective institutions, profiles of the student population, funding, class sizes and teacher expertise vary, so that inevitably, mathematics departments will make different decisions about priorities for additional untimetabled teacher effort.

The consequences of failing to fully support, or even discouraging, the range of students who might realistically aspire to and thrive in a mathematics-intensive undergraduate course, is a potential loss to individuals in terms of their personal, occupational and societal thriving – and a loss to wider society. In particular, the teaching profession is woefully short of such mathematically-knowledgeable entrants, with the vast majority of entrants to mathematics ITE, and so to secondary mathematics teaching, being casual users of mathematics, or of statistics, at best, resulting in a disappointingly vicious circle of mathematical impoverishment in many classrooms. Below, we endeavour to summarise points evidenced for possible consideration by different parties, in relation to provision for those with particular mathematical aptitude.

Areas for Reflection

Although the focus of the work on which this report is based, was on the use and impact of additional mathematics papers for university entrance, that work revealed issues going well beyond those directly pertinent to such papers.

In schools and colleges

- There remains a profound need to improve the fundamentally mathematical quality, including challenge, of the mathematics education offered from a young age, in an inclusive way that supports the emergence of potential as well as providing for those who have already been noticed as of particular mathematical bent. Engaging with additional university entrance papers is not the only way to provide challenge, and is very late in students' mathematical pathways. The need includes the recruitment, development and retention of many more mathematically-knowledgeable and confident teachers. Such a focus would lead to a virtuous circle of improved mathematical experience.
- Further overt encouragement of the use of e.g. UKMT and nrich resources for 'deepening' mathematical thinking would be helpful - but also curriculum valuing of genuine mathematical reasoning and problem-solving, of getting stuck and persisting, and of oral and written communication, noting that some mathematically-inclined students will resist that until they see a personal need.
- Such practice could be overtly valued by senior leadership teams and Ofsted, although it is difficult for non subject-specialist observers to discriminate between form and substance in such approaches.
- Mathematically-inclined students would benefit from early, inclusive exposure to wider and deeper mathematics, and early and continuing conversations about university pathways: 'hands-off' approaches to university applications can be damaging. Mathematics departments could conduct professional conversations about priorities for use of time, energy and resources of all kinds, both among themselves and with SLT and governors. But there is a need also to maintain a broad and balanced curriculum and extra-curricular experience in what is, internationally, already one of the narrowest post-16 curricula.
- For the most mathematically aspirational students, engagement with the material in additional entrance papers can ease transition to university – but so can independent and collaborative work with other less demanding problems, and for a wider range of students.

Qualifications and policy

- The new A Level is not yet having the impact hoped for in schools and universities, though we are still comparatively early in its enactment. There are some perceptions of excessive content marginalising opportunities for problem solving and proof, even when teachers are familiar with the intentions of the new A Level. In the current performativity climate, it is important A Level assessment values the fruits of the range of A Level intentions, if necessary by modifying the range of content expected.
- AEA is now irrelevant to most institutions, with ~300 entries nationally in 2019, and many of those as a 'stretch' for year 12 students who will take other additional papers in year 13.
- There is some concern from academics and teachers about the non-regulated nature of papers other than AEA, in an HEI system that has large public funding.

- The study underlines the need for more mathematically knowledgeable teachers who can teach for the emergence of, identify, encourage and respond to indications of particular mathematical aptitude, and then persist in supporting mathematical aspiration and challenge. Every secondary mathematics department needs access to at least one deeply mathematically confident and competent teacher. That is unlikely to be found in those whose first degree is in psychology or chemical engineering and who are labelled 'mathematics specialist' teachers after sometimes very short subject-specific initial teacher education, without considerable and sustained subject-specific CPD opportunities.
- This work did not address in any detail the provision offered in independent schools in England. Many jurisdictions select for academic, or sometimes, mathematics, potential during the course of schooling, and in the case of grammar schools in England, that can support the needs of the most mathematically inclined, apparently very effectively. Comparative studies of approaches to university mathematics preparation and selection in jurisdictions that do not have structural selection, or well-resourced independent education, might illuminate thinking about ways in which current inequities in access could be addressed.

Provision of support

- There are copious and praiseworthy attempts to provide freely-available support for additional papers. These inevitably only begin to even the playing field, and there are large 'cold spots' where face to face support and a good depth of intellectual peer co-working is not realistically accessible.
- Many teachers, even in centres which habitually enter students for these additional papers, are unaware of the range of remote support freely available – or of the difference that sustained and deliberately ramped preparation can make.
- Materials for KS4 enrichment: Beyond the occasional use of nrich problems, or UKMT or similar questions, teachers of KS4 students would benefit from for a more structured enrichment programme that could be accommodated within standard teaching of mathematically-inclined teenagers.

Variety of additional mathematics papers

- Engagement in the sorts of questions promoted by additional papers is supportive of student transition to university mathematics – but largely, for those already coping very well with mathematics A Levels.
- Where students are required to take additional papers, *or it is advantageous to the offer they receive*, there is a clear argument that related entry fees should be paid for by universities.In any case, universities could helpfully find ways to promote and encourage participation in related work, without participation in the related examinations. This latter course would reduce pressure on both students and teachers.
- Where universities make use of mandatory or alternative use of additional papers, there should in parallel be clear recommendations as to likely necessary length and extent of preparation. It is easy to avoid such statements on the grounds of inclusivity of late decisions, etc., but that further disadvantages students without recourse to local informed support.
- There is an appetite, across constituencies, for a rationalisation of any additional papers, e.g. it is not clear that AEA, STEP1 and TMUA target different cohorts of students. Moves to any resulting single assessment could productively consider loosening the time pressure inherent in TMUA, and incorporating widely-valued written communication of mathematical thinking in at least part of the assessment. It is widely considered that 'accommodation of late entrants' is not a strong argument for the retention of both opportunities.

- Teachers and students widely consider MAT and STEP 2/3 to be highly demanding, and to support the development of slightly different mathematical skill sets, with STEP papers more coachable. HEIs should be aware that many students find the timing and nature of STEP papers, and especially STEP 2/3, to be highly stressful, although it is recognised that STEP 2/3 reflect the highly competitive nature of mathematics study at Cambridge. The evidence from this study is that these characteristics of STEP 2/3 serve to discourage girls in particular from application to Cambridge.
- There is a small concern around MAT and TMUA being timetabled on the same day. More importantly, it is critical they are scheduled after common Autumn half term dates. Those responsible for STEP (and other additional summer examination dates) should, similarly, revisit the decision made to move STEP to within the A Level period, since this impacts students differentially and sometimes, quite severely, and instead, timetable such papers for as soon as reasonably fair after the main A Level period. Schools and colleges could support that by planning 'leaving events' or similar for after any additional paper slots.
- Additional entry requirements can serve to narrow the pre-university experience, as well as to deter potential applicants to mathematics.

Entry to university mathematically-intense courses: other issues

- University mathematics departments could helpfully encourage a full and balanced engagement with the range of intentions of the new A Levels, if necessary with a reduced content, in order to support a wider range of students in transitioning to mathematical work at university.
- Students would be better able to judge the genuine course expectations, and universities less encouraged to make unrealistic offers, if e.g. universities were required to publish the average A Level performance of the last available cohort, or for a new course, the nearest such historical course. New offers might conceivably be capped in relation to those. This would put students from less well-informed or supportive backgrounds in a better position to make realistic but aspirational choices.
- There are inbuilt equity issues in student applications, that go beyond the mathematics made available to them, or extent of local support for additional papers, e.g. a very variable degree of support for writing personal statements. Mathematics admissions information could specify the role of additional aspects to personal statements, e.g. the extent to which offers value extra-curricular activities, whether they value independent additional mathematical activity, etc.
- The range of threats to equity of access should be considered when evaluating admission policies. These go well beyond demand or opportunity for additional papers, or socio-economic, gender or ethnic status, but further analysis would be helpful.
- Widely maintaining first year university as a transition year, from which other routes are accessible, would support a more balanced pre-university experience for many.

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Appendix 1: Study participants

Data source	Participant	Institution and mathematics teachers
Year 13 student focus group transcription	SS1-3	Inner city 11-18 comprehensive, 3 mathematics graduates incl. Cambridge HoD
	SS4-6	Rural 11-18 comprehensive, HoD a mathematics graduate, 1 other in dept.
	SS7-8	Rural 11-18 comprehensive, HoD a Cambridge Natural Sciences graduate, 3 mathematics graduates in dept.
	SS9-17	Suburban grammar school entirely staffed by mathematics graduates, several of them Oxbridge graduates.
	SS18-19	Inner city 11-18 comprehensive, 2 mathematics graduates in dept.
	CS1-2	Inner city selective sixth form college, entirely staffed by mathematics graduates, several of them from Oxbridge.
	CS3-17[10]	Large urban sixth form college, A Level teaching largely staffed by mathematics graduates.
	CS18-23	Large city General FE college, A Level teaching largely by mathematics graduates.
	CS24-25	Specialist mathematics school, entirely staffed by mathematics graduates, some from Oxbridge
Year 13 student g-survey	GS1-82	67 of the responding students came from 4 post-16 institutions, 13 from 6 grammar schools and 12 from 7 11-18 comprehensives.
Year 12/13 teacher interview transcription	ST1,2	Inner city 11-18 comprehensive, 3 mathematics graduates incl. Cambridge HoD
	ST3,4	Rural 11-18 comprehensive, HoD a mathematics graduate, 1 other in dept.
	ST5	Rural 11-18 comprehensive, HoD a Cambridge Natural Sciences graduate, 3 mathematics graduates in dept.
	ST6, 7	Suburban grammar school entirely staffed by mathematics graduates, several of them Oxbridge graduates.
	ST8	Inner city 11-18 comprehensive, 2 mathematics graduates in dept.
	ST9	Urban 11-18 comprehensive, 3 mathematics graduates in dept.
	ST10	Suburban 11-18 comprehensive, one mathematics graduate in dept.
	ST11	Inner city 11-18 comprehensive, 2 mathematics graduates in dept.
	CT1,2	Inner city selective sixth form college, entirely staffed by mathematics graduates, several of them from Oxbridge.
	CT3,4	Large urban sixth form college, A Level teaching largely staffed by mathematics graduates.
	CT5,6	Large city General FE college, A Level teaching largely by mathematics graduates.
	CT7,8	Specialist mathematics school, entirely staffed by mathematics graduates, some from Oxbridge
	CT9	Large General FE college, A Level teaching largely by mathematics graduates.
	CT10	Recently retired from large city sixth form college, half staffed by mathematics graduates
	CT11	Urban General FE College, one mathematics graduate

[10] (in 3 groups, one of 4 year 12s)

Year 12/13 teacher g-survey	GT1-66	66 teachers: 31 from 29 independent, 19 from 19 selective state, 16 from 15 non-selective state schools/colleges.
Paper preparation support provider	Supp1	AMSP Area Coordinator and Russell Group, typical offer
	Supp2	AMSP senior, ex-Area Coordinator
	Supp3	AMSP National Coordinator
	Supp4	STEP Support Programme senior support
HE transition academic interview transcription (all Russell Group)	HEI1-3	typical offer A*AA incl Ma, FM. A* in Ma or FM, recommend STEP or other paper. E.g. any STEP grade 2 reduces offer by one grade.
	HEI4	Oxford: requires MAT. A*A*A incl FM, A* in Ma, or A*AA plus FM AS A. STEP or AEA recommended.
	HEI5	Typical offer AAA incl Ma, AAB if include Ma and FM, may reduce with STEP or TMUA
	HEI6	Typical offer AAA incl Ma, AAB incl A in Ma, A in FM or FM AS
	HEI7	Typical offer AAA incl Ma, AAB incl A in Ma, A in FM or FM AS
	HEI8	typical offer A*A*A + STEP grade 1 **or** A*A*A* **or** A*A*AA incl A* in Ma, FM. Reduced offer (A*A*A) with good MAT score or TMUA score >=6.5
	HEI9	Typical offer A*A*A, incl FM, plus STEP 2,3 both at >=grade 1 or 1/2
HE transition academic g-survey	GHEI1-3	Russell Group HEIs, A*A*A including Ma, FM. Reduced offer of A*AA with good performance on an additional paper.

Appendix 2: Exemplar tools used

A. Googlesurvey for year 13 mathematics students aiming to go to university to read a mathematics-intensive course (mathematics, engineering, physics, economics, etc)

(for y13 students any time before end of April 2020)

Thank you for completing this anonymous survey, which is part of a small study funded by the Joint Mathematical Council of the UK. The resulting report will be available on the JMC website from Summer 2020. The resulting report will be available on the JMC website from Summer 2020: if you would like to be sent a link to that, please give your email address at the end. We think the survey will take about 15 minutes. It has been approved by the UCL Research Ethics Committee (REC number 1286). We very much appreciate your time and thoughts: those help us understand better how university entry requirements, including any additional 'extension' papers for entry to mathematics and similar courses at English universities, are impacting students' experience and learning.

This survey has 3 sections: the first (longest!) asks about you and your academic plans and experiences; the second asks about your perceptions of any additional entrance papers you or your year group have, or are, studying for; the third asks about the different sorts of support you have access to in relation to meeting your university offers.

Section 1: Background information

1. Which is your school or college? (This information will not be quoted but enables us to contextualise your answers)
2. I am happy for you to use what I say in this survey, keeping both me and my school/college anonymous, for purposes of research, academic papers and marketing. Y/N
3. I understand you might keep the data securely for up to ten years for such purposes. Y/N
4. My first 2 choices of university and course are:
5. To access my first choice course, I need to achieve:
6. By the end of year 13, I am predicted to have the following qualifications and grades:

Section 2: Your involvement in such papers

1. Some competitive university courses in mathematics-related subjects require, or recommend, that students take additional university entrance papers. Which of the following additional mathematics or Physics papers are you or some of your year group sitting this year? Please tick all that apply.

AEA (Advanced Extension Award)/MAT/PAT/STEP I, II or III/TMUA/None of these/I don't know

2. (optional) What issues, if any, do you see around support, cost, equity of access, etc., with these additional papers?

If you have answered 'none of these' or 'I don't know', in question 2, please now go to Section 3. Otherwise, please continue.

3. Are you personally taking/have you personally taken any such additional papers? Yes/No. If 'No', please go to Section 3.
4. Which additional paper(s) will you be taking/have you taken? Please tick all that apply: AEA (Advanced Extension Award)/MAT/PAT/STEP I, II or III/TMUA

5. What additional support outside ordinary A Level lessons will you have had in preparation for additional papers? (Please list source of support, e.g. college teachers, online tuition from AMSP, nrich sessions, university visiting support,….. and if possible the approximate quantity of that support)
6. What other support do you/did you need, or would you have appreciated?
7. Apart from any requirement to do such papers, what do you think you gain from preparing for them? Please identify as many aspects as possible.
8. Are there any downsides to your involvement with these papers, and if so, what?

Section 3: Support for challenge within and beyond A Level work

1. Are there other universities or course you might have applied for if they had not made additional requirements? Yes/No
   If 'yes', which and why did you not apply?
2. The main resources I use to support my study of mathematics within A Level lessons are:
3. The main resources I use to support my study of mathematics outside lessons are:
4. What access do you have to your mathematics teachers, outside lessons?
5. How challenging have you found maths A Levels or other courses in comparison with your other subjects? Please note any particular sources of challenge.
6. If you or your teacher are looking for extension or enrichment questions in maths, where are those sourced from?

Finally, if you would like to be sent a link to the final report in Summer 2020, please give the appropriate email address here (to be used for those purposes only):

B. Googlesurvey: Teacher views on the purpose, role and impact of additional 'extension' papers for entry to mathematics-intense courses at English universities:

(for y13 teachers any time before Easter 2020)

Thank you for completing this anonymous survey, which is part of a small study funded by the Joint Mathematical Council of the UK. The resulting report will be available on the JMC website from Summer 2020: if you would like to be sent a link to that, please give your email address at the end. We think the survey will take about 15 minutes. It has been approved by the UCL Research Ethics Committee (REC number 1286). We very much appreciate your time and thoughts: those help us understand better how university entry requirements, including any additional 'extension' papers for entry to mathematics and similar courses at English universities, are impacting both teachers, and students' experience and learning.

This survey has 3 sections: the first asks about you and your teaching experiences; the second (longest) asks about your perceptions of any additional entrance papers your students might have, or are, studying for; the third asks about the different sorts of support your students have access to in relation to meeting their university offers.

Section 1:

1. Which is your school or college? (This information will not be quoted but enables us to contextualise your answers).

2. I am happy for you to use what I say in this survey, keeping both me and my school/college anonymous, for purposes of research, academic papers and marketing. Y/N
3. I understand you might keep the data securely for up to ten years for such purposes. Y/N
4. My own highest degree, subject and university are
5. The number of years I have been teaching post-16 (pre-university) mathematics is
6. My school/college offers the following pre-university mathematics courses to year 13 in 2019-20 (please tick all that apply): A Level mathematics/AS Further Mathematics/A Level Further Mathematics/ IB Mathematics Standard level/IB Mathematics Higher Level/IB Further mathematics Higher Level
7. The total number of students in my school/college taking at least one of these pre-university courses in year 13 in 2019/20 is about
8. In our current year 13, the number of students we have who are hoping to go on to read a mathematically-intense course (e.g. mathematics, physics, engineering, …) at a Russell Group university is about

Section 2:

1. Some competitive university courses in mathematics-related subjects require, or recommend, that students take additional university entrance papers.
   Which students considering mathematics-intensive courses would your school/college encourage to apply for courses which demand, or recommend, entry for such additional papers? Which would they discourage? Why? Please answer as fully as you are able.
2. Which of the following additional mathematics or physics papers are some of your year 13 students sitting this year? Please tick all that apply. AEA (Advanced Extension Award)/MAT/PAT/STEP I/STEP II or III/TMUA/None this year, but fairly recently there has been some such entry/None recently/I don't know
3. What issues, if any, do you see around support, cost, equity of access, etc., with these additional papers for students at your centre or elsewhere?

If you have answered 'none recently' or 'I don't know' to question 2, please now go to Section 3. Otherwise, please continue.

4. (optional) What additional support outside ordinary (A Level/IB) mathematics lessons do your students typically have in preparation for additional papers? (Please list source of support, e.g. college teachers, online tuition from AMSP, nrich sessions, university visiting support,….. ***and if possible, quantify that support)***
5. (optional) Are you personally teaching for/have you personally taught for any such additional papers, or had close contact with teachers who have? Yes/No. If 'No', please go to Section 3.
   If 'yes', which papers? AEA/MAT/PAT/STEP I/STEP II or III/TMUA

   In questions 6-10, please distinguish between the different papers where you feel able to do so.
6. What do you think are the benefits to students who succeed with such papers, in the short- and in the medium-term?
7. What are the benefits (or costs) to those who enter for them but do not succeed?
8. What are the related challenges for schools/colleges, if any?
9. What is the *range* of students who would benefit from preparing for such papers, and what support would they require?
10. What are the disadvantages and costs for students who could benefit from preparing for such papers but cannot access in-school/college support to do so?

Section 3 (the shortest!)

1. What access do your year 12/13 students have to your mathematics teachers, outside lessons?
2. What sources of support do you encourage your year 12/13 mathematics students to access when they're really challenged with their work?
3. If you or your colleagues are looking for mathematics extension or enrichment questions for years 12 and 13, where do you source those?
4. Is it beneficial for a wider range of year 12/13 mathematics students, than those entered for additional papers, to embed consideration of the questions in STEP, MAT AEA and TMUA in their regular classwork, and is that something you or members of your department do? Please explain your response.

Thank you so much for your input. If there are any other issues related to progression to university courses that are mathematics-intensive that you feel might inform the study, do please explain them here:
Finally, if you would like to be sent a link to the final report in Summer 2020, please give the appropriate email address here (to be used for those purposes only):

C. Interview for mathematics academics

(for academics any time in February or March 2020)

Thank you for completing this interview, which is part of a small study funded by the Joint Mathematical Council of the UK. The resulting report will be available on the JMC website from Summer 2020. We think the interview will take about 20 minutes. We very much appreciate your time and thoughts: those help us understand better how university entry requirements, including any additional 'extension' papers for entry to mathematics and similar courses at English universities, are impacting both teachers, and students' experience and learning. Thank you for completing the consent form.

This interview has 3 sections: the first asks about your experiences with student transition; the second (longest) asks about your perceptions of the benefits of any additional entrance papers or other enrichment opportunities your students might have participated in, and your thinking about such papers more widely.

Semi-structured interview based around:

Section 1: Your background experience

1. Tell me about your HE role and experience, and any specific reflections you have re student transition to HE maths-intensive courses.
2. To what extent do you think ALs in maths/FM prepare them for that? (Prompt: the new ALs)
3. And do you consider at all any experiences students have with UKMT IMC/SMC and succeeding rounds? Your thoughts re the benefits to students of participating in those?
4. So can I just clarify what your institution takes wrt FM, and any additional papers such as AEA, STEP, MAT, TMUA? Why do you take that line? And the evidence you're drawing on to make that decision? What impact do you think that has on your entry or attractiveness?

Section 2: Your thoughts re the benefits or costs of such papers

1. What are the benefits to students who succeed with these papers? Evidence?
2. Any issues you perceive of access or equity (including re girls)?

3. What are the benefits (or costs) to those who enter for them but do not succeed?
4. What is the range of students who would benefit from preparing for such papers, and what support would they require?
5. What are the disadvantages and costs for students who could benefit from preparing for such papers but are not supported to do so?
6. Would it be beneficial for a wider range of A-level students to embed consideration of the questions in STEP, MAT AEA and TMUA in their regular classwork?
7. What are the transferable benefits of having engaged with such questions at different levels of intensity, for example the benefits for university progression? So how much difference does it make if they actually take the papers?

   Thank you so much for your input. If there are any other issues related to progression to university courses that are mathematics-intensive that you feel might inform the study, do please explain them.

D. Interviews with support providers *focused on how and why their support functions as it does, and their lens on the following questions:*

1. What are the benefits to students who succeed with these papers?
2. What are the benefits (or costs) to those who enter for them but do not 'succeed'?
3. What is the range of students who would benefit from preparing for such papers, and what support would they require?
4. What are the disadvantages and costs for students who could benefit from preparing for such papers but are not supported to do so?
5. Is it beneficial for a wider range of A-level students to embed consideration of the questions in STEP, MAT AEA and TMUA in their regular classwork?
6. What are the transferable benefits of having engaged with such questions at different levels of intensity, for example the benefits for university progression?

Appendix 3: Steps taken to address research trustworthiness and integrity

Authenticity	• The expertise of the main researcher encompassed personal and school/college teacher support for student development towards engagement with these papers; the design of the study and analysis/interpretation were also evaluated at intervals by a colleague in the mathematics department at UCL, a former provider of enrichment sessions focused on these papers. • The study benefited from the involvement at all stages, and access to all (by then anonymised) data of a postgraduate student studying a 'research methods' course at UCL, himself a Cambridge mathematics graduate. He acted as an 'auxiliary researcher' in order to

	further his grasp of research methods. This supported a deliberate cultivation of reflexivity in relation to approach and data.
Credibility	• Approach and findings are presented in detail, so open to scrutiny, in this document, and will be shared with BSRLM • Engagement with participant school/college centres sustained over most of a day, though more limited for last school/college participants, and for most support providers and participants because of pandemic and/or distance/availability constraints. • All interactions preceded by familiarisation with related websites, so as to support contextually-informed conversations. • Cross-researcher triangulation of subset of emergent interpretations. • All (anonymised) data available for appropriate subsequent use. • All participants (other than to school/college students involved in focus groups) had the opportunity to provide email addresses which were used to offer validation of the (anonymised) accounts in this report. • Proactive seeking out of participants where interpretation was problematic
Transferability	• Purposive sampling as analysed, with 'thick description' provided in full report. • Limits to transferability as identified, though sample was designed to be potentially 'telling' as described. • Arguments about transferability on a wider scale rest on the 'telling'-ness of the sample achieved: all schools/colleges were state funded, recognised by Ofsted as at least 'Good' and usually, 'Outstanding', and with comparatively good access to well-qualified mathematics teachers who might reasonably have the potential to support students working towards such papers, in a meaningful way.
Dependability	• All data coded, then at least 10% of all data re-coded independently by auxiliary researcher, and discrepancies resolved.
Confirmability	• Cross-researcher triangulation of emergent interpretations at multiple scales. • Methodology, including interpretation, will be validated at a high level by lead researcher's institutional research group, and with interested others in a variety of conference fora. • UCL 'Safe Data Haven' curation of anonymised data, for subsequent use and re-use, as authorised by participants.

- All fieldwork assured participants were assured the research set out to value their uninhibited responses.
- All interviews and focus groups were recorded and (sometimes part-) transcribed. Selections from recordings transcribed all conversations directly relevant to research foci. However, there is scope for bias in selecting from available data including transcriptions, and in notes from visits. The researcher has attempted to maintain a stance of 'typicality' from the data, with a subset validated by the auxiliary researcher.
- Recordings were kept by all researchers until after transcription, for sampled validation against transcripts
- Survey responses were accessible to the auxiliary researcher for validation against downloaded spreadsheet.
- High level interpretations of data will be open to the mathematics education and mathematics education research communities for validation, via a variety of channels.