A new mathematics GCSE curriculum for post-16 resit students

Final report

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Contents

About MEI

MEI is an independent national charity committed to improving maths education. MEI's support for maths education includes developing curriculum specifications and schemes of assessment, providing professional development opportunities for teachers, and publishing teaching and learning resources. Most of our work is directed towards the maths education of 11–18 year-olds, addressing both academic and vocational pathways, and including maths in other subjects.

MEI's developments of post-16 mathematics curricula and qualifications include the development of the Critical Maths curriculum,[1] which informed the development of the level 3 Core Maths qualifications.

MEI's experience of supporting providers of resit GCSE Mathematics includes working with the Stoke-on-Trent Mathematics Excellence Partnership (MEP) and taking part in the Education Endowment Foundation's research into the contextualisation of resit GCSE Mathematics.[2]

For more information about MEI see https://mei.org.uk/

[1] MEI (2013). *Critical Maths: a mathematics-based thinking curriculum for Level 3*

[2] Runge, J., Munro-Lott, N., & Buzzeo, J. (2019). *Embedding contextualisation in English and mathematics GCSE teaching: Pilot report*. Education Endowment Foundation.

Foreword

Current GCSE Mathematics resit policies and practices are not working well. Many thousands of post-16 students each year resit a GCSE Mathematics qualification designed for 16 year-olds and intended, in part, to support progression to higher-level mathematical study that is not relevant to them. GCSE Mathematics does not address the needs of the large majority of students who currently resit it. Less than a quarter of resit students achieve the level 2 pass (grade 4 or above) in GCSE Mathematics that would open doors for them in future education and employment. Most resit students leave education poorly equipped to use mathematics effectively as adults and with a negative view of mathematics that they may well communicate to future generations.

It is right that young people who have not achieved a level 2 pass in GCSE Mathematics at the end of year 11 should continue to study mathematics post-16, because competence and confidence with using mathematics at this level will be valuable to them for whatever they choose to do in adult life.

Many resit students would benefit from a different post-16 level 2 mathematics curriculum, designed to meet their needs. This project has attempted to design such a curriculum.

To be of value, such a curriculum must meet some key criteria:

- It must not be a 'soft option': it should be just as demanding to achieve a level 2 pass through the new curriculum as through GCSE Mathematics.
- It must carry the same currency with employers and Higher Education Institutions as GCSE Mathematics, which means it must lead to a GCSE qualification.
- It must focus on mathematics that is directly relevant to the students' future needs in education, everyday life and the workplace.
- It must motivate the students, so that they engage positively with learning mathematics.

We believe we have succeeded in establishing that it is possible to develop a post-16 mathematics curriculum that meets these criteria and hope that this work will result in the development of a new mathematics GCSE qualification for post-16 students. This could help many more young people succeed in mathematics education, so that they are able to use mathematics effectively as adults. This would bring significant benefits, both to the students taking the new qualification and to society as a whole.

Charlie Stripp MBE, MEI Chief Executive, January 2020

Executive summary

Increasing numbers of post-16 students in England resit GCSE Mathematics, but there is widespread concern that the GCSE Mathematics qualification does not meet the needs of the majority of these students. This raises the question: *What mathematics curriculum would be appropriate for these students?*

In consultation with key stakeholders and drawing on national and international research, MEI undertook a project to develop an alternative mathematics GCSE curriculum that is better suited to the needs of the majority of students who have not achieved a level 2 pass in GCSE Mathematics by age 16.

The project has shown that a new mathematics GCSE qualification could be developed for post-16 students that would meet the project's aim. The proposed new qualification would focus on the maths needed for everyday life and work, while retaining the currency and rigour of foundation tier GCSE Mathematics. It would have the same level of demand as foundation tier GCSE Mathematics and so would be limited to GCSE grades 1 to 5.

Background

Educational policy in England[3] requires full-time students aged 16–18 who have not achieved grade 4 or higher in GCSE Mathematics to continue studying mathematics. Students who achieved grade 3 at age 16 are required to study for GCSE Mathematics.[4] This policy was introduced following a report by Professor Alison Wolf,[5] which found that '*English and Maths GCSE (at grades A*–C) are fundamental to young people's employment and education prospects.*' Unfortunately, the GCSE Mathematics resit success rate is very low. Almost 180,000 students resat GCSE Mathematics in summer 2019 but only 22.3% achieved a level 2 pass (grade 4 or above).[6] Those who do not achieve a level 2 pass by age 18 are seriously disadvantaged in the employment market.[7]

16–18 year-olds with grade 3 or below in GCSE Mathematics often lack confidence in mathematics and the prospect of 'more of the same' is very de-motivating.[8] As a result, many do not improve their performance and can be left with a lasting sense of failure and a reinforced negative attitude towards mathematics.[9] This may prevent them from future engagement with learning and using mathematics. These negative attitudes can be transmitted to their families and others and may last a lifetime. Much effort and money is being spent on a system that gives many young people a negative experience of mathematics education that is detrimental to them as individuals and to our society as a whole.

[3] Education and Skills Funding Agency (2019). *Guidance: 16 to 19 funding: maths and English condition of funding*

[4] From 2019/20, students with grade 2 and below in GCSE Mathematics at age 16 can achieve level 2 Mathematics post-16 either by taking GCSE Mathematics or by taking Functional Skills Mathematics level 2.

[5] Wolf, A. (2011). *Review of Vocational Education – The Wolf Report*

[6] Joint Council for Qualifications (2019). *GCSE (Full Course) Results Summer 2019*

[7] Wolf, A. (2011). *Review of Vocational Education – The Wolf Report*

[8] Higton, J., Archer, R., *et al.* (2017). *Effective practice in the delivery and teaching of English and Mathematics to 16–18 year olds.* DfE.

[9] Johnston-Wilder, S., Lee, C., *et al.* (2015). *Developing mathematical resilience in school-students who have experienced repeated failure*

The project

The project ran from December 2018 to December 2019. Curriculum content and exemplar examination papers were written, informed by reviews of national and international evidence, and a small-scale feasibility study was conducted with key stakeholders to establish whether the proposed curriculum and assessment would be fit for purpose and practicable.

How and what should post-16 GCSE Mathematics students learn?

Four key themes emerged from our review of evidence:

Motivation and confidence

Students required to resit GCSE Mathematics understand the importance of gaining a GCSE qualification in mathematics but are hampered by their lack of confidence in mathematics. Applying mathematics to contexts from everyday life is motivating for students, and the value they attach to being able to do this enables them to develop resilience.

Progression

Student progress is measured using the points system in the post-16 mathematics progress measure.[10] Nationally, the average progress points score from post-16 students resitting GCSE Mathematics is close to zero, with most students who achieved grade E or below at age 16 making negative progress by age 18.

Relevant content and skills

The ability to use basic mathematics in complex situations is needed in employment, as well as personal life. This includes the use of digital tools such as online calculators and spreadsheets.

The qualifications landscape

The GCSE brand is highly valued by employers and higher education institutions (HEIs), so for an alternative post-16 mathematics qualification to have credibility, it should be a GCSE.

A review of qualifications at a similar level to GCSE Mathematics, from England and other countries, was conducted. This included consideration of qualification structure and content.

These reviews, together with our experience of drafting exemplar teaching resources, were used to inform the development of the curriculum. To determine the mathematical content, we started by compiling a list of quantitative skills all adults should possess. This included contexts from the 'General life and personal interest' section from the foundation tier GCSE context grid[11] and the 'Essentials of Numeracy for All' poster[12] from National Numeracy. The Essentials of Numeracy were defined by National Numeracy working with employers, unions, charities and maths experts such as Cambridge Maths.[13] Once we had a list of skills, we grouped them into four themes to provide four teaching units in the outline curriculum, as outlined in the following table. The associated mathematical content is detailed in Appendix 2 of the main report and in the draft curriculum document.

[10] DfE (2019). *16 to 18 accountability measures: technical guide*
[11] MEI (2017). *Contextualisation Toolkit*
[12] National Numeracy (2013). *The Essentials of Numeracy for All*
[13] www.nationalnumeracy.org.uk/essentials-numeracy

Financial understanding	Working with measures and shape
• Understanding discounts in the sales • Understanding household bills • Estimating the cost of weekly food shopping • Splitting a restaurant bill • Shopping around for the best mobile phone deal • Comparing prices for differently sized packages • Budgeting for a holiday or major purchase • Personal budgeting • Managing a budget at work • Understanding interest rates when saving and borrowing	• Being able to read a measuring scale • Knowing your height and weight • Converting between imperial and metric units • Buying enough paint to decorate a room • Using shapes in designing a garden or craft project • Making and interpreting measurements to decide whether a piece of furniture or household appliance will fit in a given space • Understanding a map or scale drawing • Understanding measurements relating to personal fitness and health • Giving the right quantity of medicine to children
Planning activities	**Understanding quantitative information**
• Estimating time needed for tasks • Planning a schedule • Understanding staff shifts on a rota • Planning a meal or party for a large number of people • Giving and following directions • Understanding journey times • Understanding a map or scale drawing • Understanding timetables	• Recording numerical information accurately so others can understand • Making sense of statistics in the news • Interpreting the results of an opinion poll and understanding why different polls may produce different results • Understanding results of elections • Understanding food labels • Understanding statistics relating to personal fitness and health • Understanding risk in the news in relation to health

Outline quantitative skills for a new post-16 GCSE Mathematics qualification

It would be helpful for teaching and examining the proposed curriculum if all contexts which could occur in examinations were listed; the lists in this table are not sufficiently detailed to be exhaustive but they indicate the kinds of skills which students would develop.

How should the proposed GCSE be assessed?

Many of the qualifications reviewed for this project are either modular or have a stepping-stone qualification at a lower level. That is not the case for GCSE Mathematics. Although, theoretically, Functional Skills Mathematics level 2 can be used as a stepping stone, this is not what Functional Skills qualifications are designed for. Moreover, there is no evidence that Functional Skills Mathematics level 2 is being successfully used as a stepping stone to GCSE Mathematics.

The proposed assessment structure for the post-16 mathematics GCSE is shown in the table below.

	Paper 1	Paper 2	Paper 3
Style	Multiple choice questions assess basic skills. Questions may be in context or context-free. Results reported to centres for diagnostic purposes.	A mixture of short and long questions, all set in realistic contexts.	A mixture of short and long questions, all set in realistic contexts.
Time	1 hour	1.5 hours	1.5 hours
Number of marks	40	80	80
% of total qualification	20%	40%	40%
Calculator allowed?	Yes	No	Yes
Availability	Twice a year[14]	November and June	In same series as Paper 2

Proposed assessment structure for a new post-16 GCSE Mathematics qualification

Paper 1 is designed to be taken early, as a stepping stone to the whole GCSE. It would allow students to receive prompt feedback and experience success; students would be allowed one resit opportunity for Paper 1.

Papers 2 and 3 are taken at the end of the course and assess the whole curriculum content. As part of this development project, we wrote exemplar papers; these are included as Appendix 3 in the main report.

Comparing the demand of the proposed post-16 mathematics GCSE with GCSE Mathematics

No More Marking Ltd[15] are national experts in the use of comparative judgement in assessment.[16] They compared the difficulty of the exemplar papers with the summer 2017 GCSE (9 to 1) Mathematics assessments and with the AQA Functional Skills Mathematics level 2 specimen papers.[17] The GCSE Mathematics papers are a natural set of qualifications for comparison with the exemplar papers for the new curriculum because the new curriculum is designed as an alternative to the current GCSE Mathematics for post-16 students.

[14] Papers could be automatically compiled based on defined parameters and available online in a one-week window to allow for centres not having enough computers for all candidates. Alternatively, a paper-based assessment, available at two sittings, could be used with an online bank of practice questions for diagnostic assessment.
[15] www.nomoremarking.com/
[16] No More Marking was used by Ofqual to compare GCSE (9 to 1) Mathematics papers; see *GCSE maths: Final research report and regulatory summary* (2015).
[17] Barmby, P., & Wheadon, C. (2019). *A comparative judgement study of MEI GCSE exam items*. No More Marking Ltd.

The comparative judgement found that the difficulty of exemplar Papers 2 and 3 for the new curriculum was in line with foundation tier GCSE Mathematics papers. Paper 1 is easier than foundation tier GCSE Mathematics papers; this is in line with its design as a diagnostic paper to be taken early in the course to check whether students have the basic skills in place to enable them to succeed. The comparative judgement found that Functional Skills Mathematics level 2 was harder than foundation tier GCSE Mathematics.

What about Functional Skills Mathematics qualifications?

An alternative level 2 mathematics qualification for post-16 students already exists – Functional Skills level 2. The national policy has recently changed to allow students with grade 2 and below in GCSE Mathematics at age 16 to achieve level 2 Mathematics by taking Functional Skills Mathematics. Moreover, a programme to reform Functional Skills qualifications has led to new versions available for first teaching from September 2019.[18] This raises the question of whether the new Functional Skills Mathematics removes the need for any alternative GCSE qualification.

A recent survey conducted by Pye Tait[19] shows that a greater proportion of employers consider Mathematics and English GCSE to be essential when recruiting for entry and admin roles, compared to those who consider Functional Skills Mathematics and English essential.

The dominance of the GCSE brand is very well established and is easily understood by employers and HEIs. It seems unlikely that the reformed Functional Skills Mathematics will provide the credibility needed for employers and HEIs to change their selection criteria. Consequently, Functional Skills qualifications can be seen as having a lower currency by students, and this is likely to reduce their motivation.

The dominance of the GCSE brand was confirmed by findings in an article in *FE Week* in November 2019:

> *Colleges claimed they did not want 'to limit our learners' progress' and highlighted 'the strong emphasis that employers and education establishments put on GCSEs' as part of their reasoning.*
>
> *Criticism was also directed towards the alternative Functional Skills qualification for its 'cliff-edge pass or fail'.*[20]

The comparative judgement conducted by No More Marking[21] confirms that, as a level 2 qualification, level 2 Functional Skills Mathematics is more demanding than foundation tier GCSE Mathematics, which allows students to demonstrate achievement at level 1 or level 2.

[18] Ofqual (2018). *Decisions on Functional Skills Qualification reform – English and mathematics*
[19] Pye Tait Consulting (2019). *Perceptions of Vocational and Technical Qualifications: Wave 2*
[20] Mersinoglu, Y.C. 'Why colleges are choosing GCSE resits over alternatives', *FE Week*, 15 November 2019
[21] Barmby, P., & Wheadon, C. (2019). *A comparative judgement study of MEI GCSE exam items*. No More Marking Ltd.

Conclusions, recommendations and next steps

This project shows that the proposed new GCSE has the potential to improve mathematics learning for resit students and to recognise this through a rigorous, credible post-16 mathematics GCSE qualification. The proposed stepping stone, Paper 1, would improve student confidence and the contextualised content would increase student motivation and resilience, leading to higher success rates.

Success in this qualification would give young people the competence and confidence in the fundamental mathematics they need to function as effective citizens. It would also prepare young people for further programmes of study, including Core Maths and vocational courses, that do not require prior learning of abstract mathematics. The proposed qualification is suitable for the government to adopt as a means for young people to achieve the level 2 maths requirement of 16–19 study programmes and T level programmes.

In order to make a success of an alternative GCSE, employers and HEIs would need to understand its purpose so that parents, teachers, schools and colleges could be confident in choosing it as an option for students.

Main recommendation

A new mathematics GCSE should be developed for post-16 students that focuses on the maths needed for everyday life and work. It should be clearly branded as a GCSE qualification and afforded equal status to a GCSE Mathematics qualification at the same grade, both for progress measures and for entry to employment or higher education.

The findings set out in the report should be considered by the DfE, Ofqual and the awarding bodies to inform the development of the new GCSE; this may require changes to regulation or to usual assessment practices at GCSE.

The new mathematics GCSE should have the following features:

- It should be limited to foundation tier (grades 1 to 5) and available to post-16 students only.
- It should have higher grade boundaries than foundation tier GCSE Mathematics to ensure that students gain the qualification by demonstrating the ability to succeed in the mathematics they are likely to encounter in life and work, rather than by merely achieving a minimal number of marks.
- It should incorporate a stepping-stone assessment element that can be taken before the final assessment, to test basic skills and provide a more supportive pathway for students who have experienced limited success with mathematics. This stepping stone should attract points in the post-16 maths progress measure. The whole GCSE should have the same progress measure points as GCSE Mathematics.
- It should exclude content from foundation tier GCSE Mathematics that is not directly relevant to everyday applications but should include a small amount of additional content that is not included in foundation tier GCSE Mathematics, such as risk, financial applications and the basic use of spreadsheets to perform quantitative tasks. This aligns with the skills students need in daily life and is consistent with feedback received from employers.

Next steps

- The outline curriculum content and proposed assessment structure in this report should be used as the basis for a post-16 GCSE, limited to foundation tier.
- Development should be overseen by the DfE and Ofqual, as usual for GCSE qualifications, but with changes to regulations where needed. These would include enabling:
    - the qualification to be for post-16 students only
    - the first paper to be sat early and count towards the full GCSE.
- The title of the GCSE should make it clear that it covers the mathematics needed for adults to possess the quantitative skills needed in daily life. Possible titles include GCSE Core Maths, GCSE Numeracy.
- The content of the GCSE should be based on the outline curriculum and contain sufficient detail to enable teachers to know what they need to teach (and what is not included).

The contexts for assessment questions should be drawn from those included in the final subject content.

1 Introduction

1.1 Project summary

Increasing numbers of students resit GCSE Mathematics after the age of 16 but there is widespread concern that the GCSE Mathematics qualification does not meet the needs of the majority of these students.

Professor Sir Adrian Smith, in his report[22] of his review of mathematics education for 16–18 year-olds in England, recommended:

> *Recommendation 5: In view of the low GCSE resit success rates and new GCSE requirements, the Department for Education should review its 16–18 resit policy with the aim that a greater proportion of students without a grade C or equivalent attain appropriate mathematical understanding by age 18. Specifically, there should be fresh consideration of appropriate curricula and qualifications for these students and the extent to which current policy incentivises these to be offered.*

The CBI/Pearson Education and Skills Survey report *Education and learning for the modern world*[23] highlights how important mathematics (and English) GCSE qualifications are for the future of young people as individuals and for the country:

> *… currently about a third of students do not achieve a pass (grade 4) in English or Mathematics, with provisional figures in October 2019 showing 35.6% of students didn't achieve a standard pass in both English and Maths GCSE. We need to see rapid action to raise standards among this cohort or risk productivity and social justice at a local, regional and national level.*

Current policies for post-16 level 2 mathematics education are not working, and we believe a new curriculum is needed that can motivate post-16 students to engage with learning mathematics and develop the mathematical skills they need for adult life and work. The aim of this project was to investigate whether it was possible to develop such a curriculum and so address Sir Adrian Smith's recommendation 5 and the concerns of employers.

In consultation with key stakeholders, from January 2019 to October 2019, MEI developed a new mathematics curriculum exclusively for post-16 students, designed specifically to incorporate content and contexts most useful for their needs, thus allowing them to gain valuable skills in applying mathematics and leave the course with a more positive and receptive attitude towards the subject. Our reviews of research and consultation with teachers indicate that this curriculum would motivate more students to succeed and could also help to reduce the negative perception of the relevance and value of mathematics held by many in our society.

The curriculum features required to meet the needs of these students are outlined in the project aims in section 1.3 and considered further in section 6. A key factor is that the curriculum must lead to a GCSE qualification if it is to have currency with employers and higher education institutions (HEIs), so it must be as demanding as GCSE Mathematics.

We obtained formative feedback from small samples of teachers and employers to inform development; this feedback is considered in section 5. We wanted to ensure that the curriculum and associated assessments were of an appropriate standard for a mathematics

[22] Smith, A. (2017). *Report of Professor Sir Adrian Smith's review of post-16 mathematics*
[23] CBI/Pearson (2019). *Education and learning for the modern world: CBI/Pearson Education and Skills Survey report 2019*

GCSE, so an independent review of the exemplar papers using comparative judgement was conducted by Dr Patrick Barmby and Dr Chris Wheadon of No More Marking Ltd,[24] national experts in the use of comparative judgement in assessment.[25] Their report is published as an external appendix to this report.[26]

Through this project we have shown that it is possible to develop a post-16 mathematics GCSE curriculum that can meet the needs of those who fail to achieve a level 2 pass in mathematics by age 16. We believe our proposed curriculum can address recommendation 5 of the Smith review and the concerns of employers by enabling far more young people to develop the mathematical understanding they need for life and work.

1.2 Background

The issue

Educational policy in England[27] requires full-time students aged 16–18 who have not achieved grade 4 or higher in GCSE Mathematics to continue studying mathematics. Students who achieved grade 3 at age 16 are required to study for GCSE Mathematics.[28] This policy was introduced following a report by Professor Alison Wolf,[29] which found that *'English and Maths GCSE (at grades A*–C) are fundamental to young people's employment and education prospects.'* Unfortunately, the resit success rate is very low. Almost 180,000 students resat GCSE Mathematics in summer 2019 but only 22.3% achieved a level 2 pass.[30] Those who do not achieve a level 2 pass by age 18 are seriously disadvantaged in the employment market.[31]

16–18 year-olds with grade 3 in GCSE Mathematics, or lower, often lack confidence in mathematics and the prospect of 'more of the same' is very de-motivating.[32] As a result, many do not improve their performance and can be left with a lasting sense of failure and a reinforced negative attitude towards mathematics.[33] This may prevent them from future engagement with learning and using mathematics. These negative attitudes can be transmitted to their families and others, and may last a lifetime. Much effort and money is being spent on a system that gives many young people a negative experience of mathematics education that is detrimental to them as individuals and to our society as a whole.

[24] www.nomoremarking.com/
[25] 'No More Marking was used by Ofqual to compare GCSE (9 to 1) Mathematics papers; see *GCSE maths: Final research report and regulatory summary* (2015).
[26] Barmby, P., & Wheadon, C. (2019). *A comparative judgement study of MEI GCSE exam items*. No More Marking Ltd.
[27] Education and Skills Funding Agency (2019). *Guidance: 16 to 19 funding: maths and English condition of funding*
[28] From 2019/20, students with grade 2 and below in GCSE Mathematics at age 16 can achieve level 2 Mathematics post-16 either by taking GCSE Mathematics or by taking Functional Skills Mathematics level 2.
[29] Wolf, A. (2011). *Review of Vocational Education – The Wolf Report*
[30] Joint Council for Qualifications (2019). *GCSE (Full Course) Results Summer 2019*
[31] Wolf, A. (2011). *Review of Vocational Education – The Wolf Report*
[32] Higton, J., Archer, R., *et al.* (2017). *Effective practice in the delivery and teaching of English and Mathematics to 16–18 year olds.* DfE.
[33] Johnston-Wilder, S., Lee, C., *et al.* (2015). *Developing mathematical resilience in school-students who have experienced repeated failure*

Why is this happening?

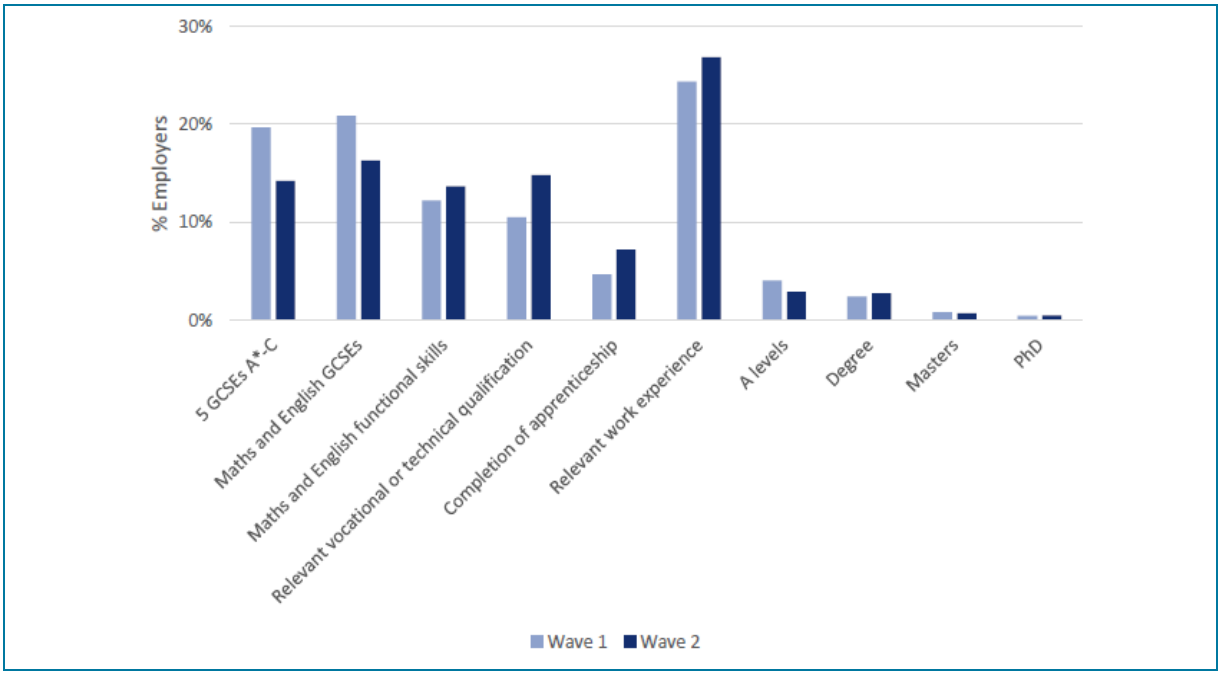
The GCSE Mathematics curriculum, which is designed for pre-16 year-olds, attempts to do two things:

1. prepare students for further academic study of mathematics, and
2. develop the knowledge and skills to apply mathematics to practical problems encountered in the workplace and other aspects of life.

Most resit students need to focus only on the second of these, and so would benefit from a different qualification that could help motivate them to do this. Of course, some students resitting GCSE Mathematics have ambitions to follow a course of study which includes higher-level mathematics, and they should resit GCSE Mathematics.

What other recent or current developments are there in the field?

An alternative level 2 mathematics qualification for post-16 students already exists – Functional Skills Mathematics level 2; however, Functional Skills qualifications have failed to gain traction with employers and HEIs. A 2017 Ofqual-commissioned report of employer perceptions of qualifications found that just over 50% of employers valued Functional Skills qualifications as highly as other English and mathematics qualifications.[34] A more recent survey conducted by Pye Tait[35] shows that a greater proportion of employers consider Mathematics and English GCSE to be essential than feel the same about Functional Skills, as shown in the chart from their report reproduced below. However, the proportion that recognises Functional Skills as essential is greater in Wave 2 (conducted in 2018/19) than in the earlier Wave 1 survey (conducted in 2017/18).



Responses to the question, 'When recruiting new employees, are any of the following essential for entry and admin roles?' in Wave 2 (2018/19, base 3130) compared with Wave 1 (2017/18, base 2750); reproduced from Pye Tait (2019)

[34] Ofqual (2017). *Employer Qualifications Perceptions Survey: data annex*
[35] Pye Tait Consulting (2019). *Perceptions of Vocational and Technical Qualifications: Wave 2*

Some HEIs require a level 2 pass in GCSE Mathematics (grade 4 or above, or equivalent) as a pre-requisite for entrance; it is less common for Functional Skills Mathematics to be accepted as an equivalent. A programme to reform Functional Skills qualifications has led to the development of new versions, available for first teaching from September 2019.[36] However, the dominance of the GCSE brand is very well established, and it seems unlikely that the reformed Functional Skills Mathematics will provide the credibility needed for employers and HEIs to change their selection criteria.

Another relevant development is the new T level technical qualifications, to be introduced between 2020 and 2023. They will become a third major option available to 16–19 students, alongside A levels and apprenticeships. To achieve a T level, students must achieve a minimum level of maths and English. The government has stated[37] that this will be set at level 2, to align with the existing policy on maths and English requirements for level 3 apprenticeships. Therefore, students may meet the T level mathematics requirement through achievement of either a level 2 pass in GCSE Mathematics or level 2 Functional Skills Mathematics.

How does this project relate to policy or practice developments?

In February 2019, just after the start of this project, the government confirmed a change to its policy on resit GCSE Mathematics.[38] The policy had previously required young people aged 16–18 who have not achieved grade 4 or higher in GCSE Mathematics to continue working towards it, with full-time students who achieved grade 3 required to study GCSE Mathematics. The change allows students with grade 2 or below in GCSE Mathematics by the end of KS4 the option to sit Functional Skills Mathematics level 2, rather than resitting GCSE Mathematics.

When we started the project, national policy was for students to work towards GCSE Mathematics because *'English and Maths GCSE (at grades A*–C) are fundamental to young people's employment and education prospects.'*[39] Because of the importance of GCSE Mathematics for students' future prospects, we embarked on a project to develop an alternative GCSE for students who are post-16. Other numeracy qualifications already exist and were reviewed as part of this project; they do not have the currency or national recognition to provide an alternative to GCSE.

During the course of the project, Functional Skills Mathematics level 2 has been redeveloped and is now accepted as an end point for level 2 Mathematics study for some students, rather than GCSE. This raises the question of whether Functional Skills Mathematics at level 2 already provides a suitable alternative for all students; this is considered in detail on the next two pages in this report and in sections 5.4 and 6.1. We conclude that Functional Skills Mathematics at level 2 does not provide a suitable alternative because it has less currency than GCSE qualifications with employers and HEIs, and it is less likely to enable students to demonstrate progression.

There have been corresponding changes to the English and maths progress measures,[40] as shown in the table on the next page.

[36] Ofqual (2018). *Decisions on Functional Skills Qualification reform – English and mathematics*
[37] DfE (2017). *Implementation of T level programmes: Government consultation*
[38] Education and Skills Funding Agency (2019). *16 to 19 funding: maths and English condition of funding*
[39] Wolf, A. (2011). *Review of Vocational Education – The Wolf Report*
[40] DfE (2019). *16 to 18 accountability measures: technical guide*

Progress measures are calculated for individual students and then averaged to compare institutions. They apply to students who have a GCSE Mathematics grade below 4 at the end of KS4 and are on a study programme of at least 150 hours. Students on apprenticeship programmes are not included in the measure. Students can make positive or negative progress. Negative progress is capped at −1 for each student. A student who does not take a GCSE or stepping-stone qualification post-16 would have a progress measure of −1.

Current points: 2017, 2018 and 2019 tables				**Future points: 2020 performance tables**			
Points awarded	**Grade achieved**			**Points awarded**	**Grade achieved**		
	9–1 GCSEs	**Legacy GCSEs**	**Functional Skills**		**9–1 GCSEs**	**Legacy GCSEs**	**Functional Skills**
8	9	A*		9	9		
				8.5		A*	
7.7	8			8	8		
7	7	A		7	7	A	
6.3	6			6	6		
6		B		5.5		B	
5.7	5			5	5		
5	4	C		4	4	C	
				3.5			L2
4	3	D	L2	3	3	D	
3	2	E		2	2	E	L1
2.5			L1				
2		F		1.5		F	
1.7							
1.5							
1	1	G		1	1	G	
0.8				0.75			EL 3
0.4			Entry level	0.5			EL 2
				0.25			EL 1

Current and future points for the maths progress measure[41]
The tables have been cropped to show information for GCSE and Functional Skills only.
The colours are as in the DfE technical guide: green for level 2, salmon for level 1 and yellow for entry level.

For the purposes of post-16 progress measures, students who have a grade 2 GCSE can now make progress by passing level 2 Functional Skills Mathematics. However, anecdotal evidence from school teachers and college lecturers suggests that it is more difficult for these students to pass Functional Skills Mathematics at this level than to gain a higher GCSE grade. This anecdotal evidence is confirmed by the comparative judgement work undertaken as part of this project;[42] level 2 Functional Skills is a level 2 qualification, whereas foundation tier GCSE includes all of level 1 and the lower end of performance at level 2. Moreover, Functional Skills is a pass/fail qualification and students who take Functional Skills but do not pass cannot demonstrate progression.

[41] DfE (2019). *16 to 18 accountability measures: technical guide*
[42] Barmby, P., & Wheadon, C. (2019). *A comparative judgement study of MEI GCSE exam items*. No More Marking Ltd.

For example, consider a student who has gained grade 1 at GCSE Mathematics at the end of KS4. The grade boundaries for GCSE Mathematics foundation tier[43] suggest that this student has a very incomplete understanding of mathematics; passing Functional Skills Mathematics at level 1 would allow the demonstration of progress, but failing the qualification gives a negative progress. Entering GCSE again and getting another grade 1 gives a zero progress score; this is considered better than passing Functional Skills Mathematics at entry level 3, which gives a negative progress score (0.75 − 1 = −0.25). This means that, from a school's or college's point of view, it makes more sense for students to resit GCSE Mathematics than to do Functional Skills Mathematics, and this is reflected in exam entry patterns.

It is not easy to find national data comparing entries for GCSE Mathematics with entries to Functional Skills Mathematics for post-16 students who are working towards level 2 Mathematics. The Association of Colleges has supplied the following data, obtained from Individual Learner Returns for 2018:

- *38,490 college students were entered for Functional Skills Mathematics level 1.*
- *6,640 college students were entered for Functional Skills Mathematics level 2.*

The numbers of entries are low compared to over 100,000 students resitting GCSE.

From the point of view of individual students, progress measures are irrelevant, but the selection criteria used by employers and HEIs will continue to have a major influence over which level 2 mathematics qualifications young people prefer to take. The new mathematics GCSE we propose would offer an alternative option, better suited to the needs of young people, employers and HEIs, for the government to consider when reviewing its policies.

Previous work which informed the project

The development work done in this project was informed by MEI's previous curriculum development work and by other qualifications and research, as outlined below:

- similar qualifications that do not meet the government's current resit GCSE Mathematics policy
- research into the mathematical needs of students and the workplace,[44] [45] and the benefits of learning mathematics in relevant contexts[46]
- experience of supporting providers of resit GCSE Mathematics, in particular through the Stoke-on-Trent Mathematics Excellence Partnership (MEP) and the Education Endowment Foundation's research into the contextualisation of resit GCSE Mathematics[47]
- previous developments of post-16 mathematics curricula and qualifications, in particular the development of the Critical Maths curriculum,[48] which informed the development of the level 3 Core Maths qualifications.

[43] In summer 2019 the average grade 1 boundary for GCSE Mathematics across the four specifications for GCSE Mathematics was 11%.
[44] Hodgen, J., & Marks, R. (2013). *The Employment Equation: Why our young people need more maths for today's jobs*. The Sutton Trust.
[45] Education & Training Foundation (2015). *Making maths and English work for all*
[46] Education & Training Foundation (2014). *Effective Practices in Post-16 Vocational Maths*
[47] Runge, J., Munro-Lott, N., & Buzzeo, J. (2019). *Embedding contextualisation in English and mathematics GCSE teaching: Pilot report*. Education Endowment Foundation.
[48] MEI (2013). *Critical Maths: a mathematics-based thinking curriculum for Level 3*

1.3 Aims

The aim was to establish the feasibility of a new mathematics GCSE curriculum for post-16 resit students, which:

1. Has content relevant to the practical application of mathematics
2. Focuses on the acquisition of key mathematical skills and their application in meaningful contexts
3. Wherever practical, supports learning in contexts young people can relate to
4. Has sufficient rigour to meet the requirements of a GCSE qualification; it is intended that the new post-16 mathematics GCSE would have a maximum grade of 5, and would be designed so that students achieving grades 4 or 5 (a level 2 pass) would have to demonstrate a good understanding of the fundamental mathematics needed to function as effective citizens
5. Prepares young people for further programmes of study, including Core Maths and vocational courses, that do not require prior learning of abstract mathematics
6. Is suitable for the government to adopt as a means for young people to achieve the level 2 maths requirement of 16–19 study programmes and T level programmes.

We did not aim to create an 'easier' mathematics GCSE: MEI has a strong reputation for academic rigour in its qualification development. Our intention was to create a curriculum for a mathematics GCSE that is more appropriate for the target students, allowing them to gain valuable skills in applying mathematics and to leave the course with a more positive and receptive attitude towards the subject. This would motivate more students to succeed, and could also help to reduce the negative perception of the relevance and value of mathematics held by many in our society.

1.4 Objectives

The objectives were:

- to produce an initial design for the new curriculum, together with supporting materials, including sample teaching and learning resources and exemplar examination papers
- to conduct a small-scale study to establish whether a sample of key stakeholders considered the proposed curriculum fit for purpose and practicable
- should the study find that the proposed curriculum is feasible,
    - to work with the Advisory Committee on Mathematics Education (ACME) and awarding bodies to make a case to the government to accept a new mathematics GCSE for resit students, based on the new curriculum, as an alternative to GCSE Mathematics, within its post-16 resit policy
    - to make the outputs from the project freely available to awarding bodies and Ofqual, to support them in developing the associated mathematics GCSE qualification.

1.5 Outline of methodology

The project was planned to take place in three phases.

Phase 1, December 2018 – April 2019

During this phase, appropriate content for the curriculum was identified and plans were made to obtain feedback.

The way that content was selected is outlined in sections 2.4 and 3.3; the draft curriculum can be found in Appendix 2. A more detailed version is published separately as an external appendix to the report. The exemplar examination papers are in Appendix 3.

Phase 2, May 2019 – September 2019

During this phase, feedback on the draft curriculum and exemplar examination papers was sought and analysed. Following feedback from teachers and lecturers, changes were made to the exemplar examination papers before Dr Patrick Barmby and Dr Chris Wheadon of No More Marking Ltd[49] used comparative judgement to compare the difficulty of the exemplar papers with current GCSE Mathematics papers to ensure that we obtained objective evidence of the level of difficulty of the papers.

Feedback from teachers and employers can be found in sections 5.2 and 5.3 respectively, with reflections on the feedback in section 5.4. The main findings of the No More Marking comparative judgement study are outlined in section 4.4, with the full report published as an external appendix.[50]

Phase 3, September 2019 – December 2019

In the final phase, findings were compiled and further feedback was sought to inform the production and publication of the final project report.

Recommendations and conclusions are in section 6.

[49] No More Marking was used by Ofqual to compare GCSE (9 to 1) Mathematics papers; see *GCSE maths: Final research report and regulatory summary* (2015).
[50] Barmby, P., & Wheadon, C. (2019). *A comparative judgement study of MEI GCSE exam items*. No More Marking Ltd.

2 Emerging themes from the review of research

2.1 Overview

Key themes from research into how the mathematical experiences of students likely to be resitting GCSE Mathematics could be improved are explored below. They are grouped under four themes: 'Motivation and confidence', 'Progression', 'Important content and skills', and 'The qualifications landscape'. Each of these themes must be addressed in order to understand how a new qualification could be developed that would better serve a large proportion of those students who are required to resit GCSE Mathematics.

2.2 Motivation and confidence

Greatbach and Tate's review of evidence[51] reported that contextualisation was important for improving student confidence and engagement:

> *Applying mathematical skills to work contexts improves both confidence and engagement. Wherever possible, the teaching and learning of English and maths should be contextualised through using materials and resources which are relevant to learners' contexts; linking the learning to specific vocations or employment sectors.*

MEI worked with the Association of Employment and Learning Providers (AELP)[52] on a pilot project to explore embedding contextualisation in mathematics and English teaching post-16. The report on the pilot[53] found that contexts relating to everyday life were more motivating for students than contexts directly related to their vocational areas:

> *Teachers reported concerns about the challenge of applying contextualised knowledge to a non-contextualised GCSE exam. They also reported students' tendency to respond better to real-life, rather than vocational, contextualisation*

A more contextualised mathematics GCSE qualification would address teachers' concerns about the value of using contextualisation within their teaching. Furthermore, the use of contexts related to everyday life would prevent difficulties with the teaching and assessment of mixed groups of students who are working towards different vocations: everyday contexts are relevant to all students.

The importance of student motivation is well expressed in a translation of a report published by the Danish Ministry of Education:[54]

> *If the teaching of mathematics is not capable of producing a minimum amount of enthusiasm for the subject amongst the recipients, even the best founded, designed and implemented educational plans will fall short.*

[51] Greatbatch, D., & Tate, S. (2018). *Teaching, leadership and governance in further education*. DfE.
[52] www.aelp.org.uk/about/
[53] Runge, J., Munro-Lott, N., & Buzzeo, J. (2019). *Embedding contextualisation in English and mathematics GCSE teaching: Pilot report*. Education Endowment Foundation.
[54] Niss, M. A., & Højgaard, T. (eds.) (2011). *Competencies and Mathematical Learning: Ideas and inspiration for the development of mathematics teaching and learning in Denmark*. Roskilder Universitet.

Although the work of Carey *et al.* on mathematical anxiety[55] was focused on primary and secondary students, they reported that

> *Researchers have shown a consistent relationship between maths anxiety and maths performance in adolescents and adults.*

This suggests that improving the confidence of post-16 students who have had limited success with mathematics is important. This is confirmed by the work of Johnston-Wilder *et al.*[56] who worked with FE teachers to enable students to develop resilience in mathematics and so overcome their anxiety:

> *The learning outcomes were that the delegates would be able to understand and apply key ideas of 'mathematical resilience', and use these in order to support learners to overcome emotional barriers to learning mathematics, using activities which make mathematics ALIVE (accessible, linked, inclusive, valuable, engaging) and enable them to support the development of mathematical resilience in learners.*

Johnston-Wilder *et al.* gave financial mathematics as a particular example of mathematics which is valued by students.

2.3 Progression

Robey and Jones, in their research with students who were either resitting or had resat either maths or English at GCSE,[57] found that students were aware of the importance of a level 2 pass at GCSE for opening doors to job opportunities and HEI courses:

> *"I'm thinking of going to university and one of the main entry requirements is getting your GCSE maths grade C. So I thought in order for me to pursue my career and university I have to get the basic requirements." [Maths learner]*

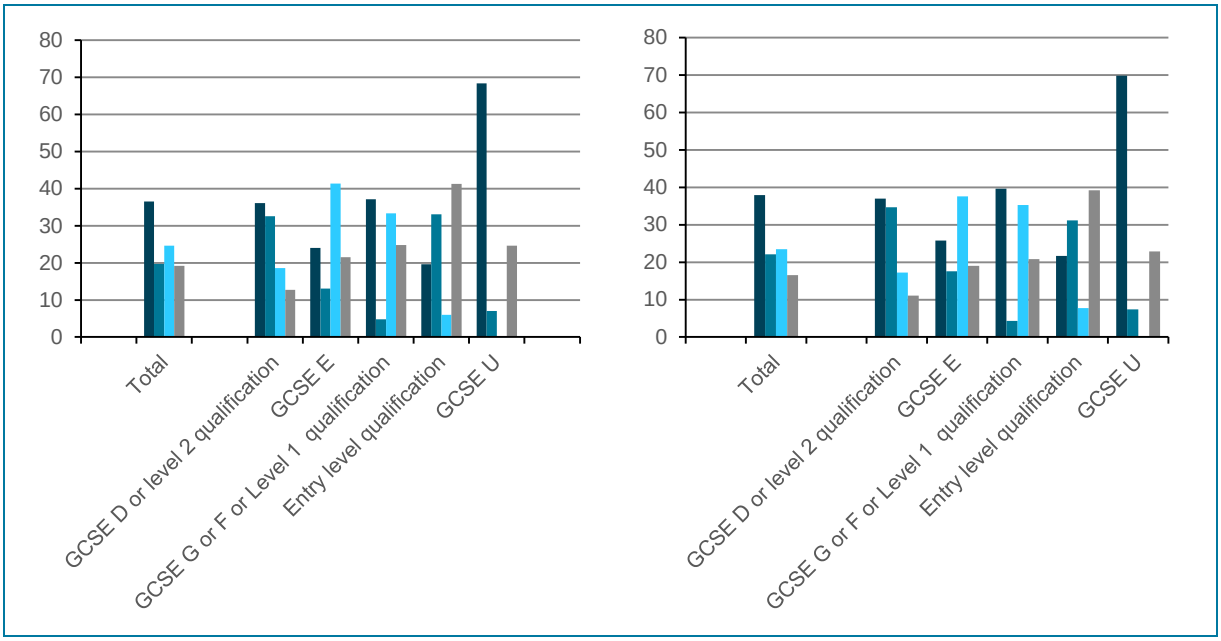
The average progress for post-16 students working towards resitting GCSE mathematics tends to be negative. Velthuis *et al.* analysed the progression by attainment at age 16.[58] Figure 19 from their report is reproduced in the left-hand chart on the next page. Progress is measured by the points for progress measures, as shown in the table in section 1.2 above.

The latest data available at the time of writing are for students completing KS4 in 2015/16.[59] These students took the A* to G GCSEs at the end of KS4. The main text reports that in 2018 overall progress for 18 year-olds who had failed to achieve a level 2 pass at the end of KS4 was positive for the first time:

> *2018 was the first year that average progress was positive for both English and maths. Average progress was 0.06 and 0.05 for English and maths respectively.*

Progression has been analysed by attainment at age 16 in the right-hand chart, in the same way as for the earlier data.

[55] Carey, E., Devine, A., *et al.* (2019). *Understanding Mathematics Anxiety: Investigating the experiences of UK primary and secondary school students*. University of Cambridge.
[56] Johnston-Wilder, S., Pardoe, S., *et al*. (2016). *Developing teaching for mathematical resilience in further education*
[57] Robey, C., & Jones, E. (2015). *Engaging learners in GCSE maths and English*. NIACE
[58] Velthuis, S., Lupton, R. *et al*. (2018). *The characteristics and post-16 transitions of GCSE 'lower attainers'*. University of Manchester.
[59] DfE (2019). *A level and other 16 to 18 results: 2017 to 2018 (revised)*



Proportions of young people who improved their maths attainment as measured by the point score (darkest), kept the same attainment (mid), and lowered their attainment (light), by prior attainment at Key Stage 4: left-hand chart 2016/17 data (2014/15 KS4 cohort), [60] right-hand chart 2017/18 data (2015/16 KS4 cohort); on both charts the grey bars represent the percentages that did not enter any approved exams

Although the positive average progress in 2017/18 is welcome, it remains the case that students with grade E, or equivalent, at the end of KS4 are more likely to make negative progress than positive progress. It would be interesting to analyse progression in this way for students who have taken the 9 to 1 GCSEs at age 16, once the data are available.

2.4 Important content and skills

The RSA review of international perspectives on mathematics education[61] suggested that students who struggle with mathematics should focus on content which will be most useful for them:

> *Students who struggle with mathematics in secondary schools need content that will support basic numeracy and be relevant for the workplace and everyday life.*

Hough *et al.*[62] highlighted the importance of choosing suitable contexts when working with post-16 GCSE resit students:

> *Post-16 students bring a significant amount of 'life-experience' to the mathematics classroom which can be drawn on through the use of carefully chosen contexts which connect it to mathematics; strategies and procedures are more likely to make sense and there is less need to resort to memorising rules and procedures. In pilot work, students working out the 'best buy' in a supermarket already had a number of*

[60] Velthuis, S., Lupton, R. *et al*. (2018). *The characteristics and post-16 transitions of GCSE 'lower attainers'*. University of Manchester.
[61] Norris, E. (2012). *Solving the maths problem: international perspectives on mathematics education*. RSA.
[62] Hough, S., Solomon, Y., *et al.* (2017). *Investigating the impact of a Realistic Mathematics Education approach on achievement and attitudes in Post-16 GCSE resit classes*. Manchester Metropolitan University.

informal strategies which could be modelled in a ratio table, ultimately leading to more formal ideas.

Jonas's report[63] on numeracy skills among adults highlighted the relevance of numeracy for everyday life:

> *Some proficiency in numeracy is needed to perform many of the most common tasks of everyday life, not just work-related ones. In the area of health, the skills that aid medical decision-making and the understanding of indications in mathematical form (such as the risks and effects of treatment) are 'literally, a matter of life and death' (Reyna and Brainerd, 2007). In today's healthcare systems, the burden of decision-making is being increasingly transferred to patients, who therefore have a growing need to understand numerical information about their own health and manage their care path effectively.*

Hodgen and Marks[64] spoke of the prevalence of *'simple maths in complex settings'* in the workplace and identified the following core areas which are mostly to be found in GCSE Mathematics:

- *Number, particularly mental maths, approximation, estimation and proportional reasoning*
- *Using and interpreting calculators and spreadsheets*
- *Statistics and probability, including data collection, interpretation and representation*
- *Algebra, particularly graphical representation and diagrams*
- *Geometry and measures, including 2D and 3D representation*

The use and interpretation of spreadsheets is identified as a core area in the list above, but is not contained in GCSE Mathematics content. A new mathematics GCSE qualification could include all of the above list.

Research from the Education and Training Foundation (ETF) went further,[65] identifying that some of the current GCSE Mathematics content is not relevant for employment:

> *There is an apparent disconnect noted between maths learned in school and maths required in daily working and personal life, and a challenge in ensuring that maths and numeracy teaching is relevant for employers. Employees now tend to require an ability to apply basic maths in complex situations, meaning that swathes of the current GCSE curriculum are not relevant to employment practices. The lack of employer input to the development of curriculum content acts as a barrier to the usefulness of GCSE qualifications in vocational settings.*
>
> *Interviewees further noted that employers look for GCSE qualifications but may not fully understand the relevance of other, alternative qualifications for work; it was suggested that 'what employers really want is a numerate workforce who are confident and competent with numbers – that's what we should be doing a better job of delivering'. The same maths specialist actively promoted the development of a new maths qualification tailored to meeting employer needs.*

[63] Jonas, N. (2018). *Numeracy practices and numeracy skills among adults*. OECD.
[64] Hodgen, J., & Marks, R. (2013). *The Employment Equation: Why our young people need more maths for today's jobs*. The Sutton Trust.
[65] Education & Training Foundation (2014). *Effective Practices in Post-16 Vocational Maths*

The review of the PIAAC (Programme for the International Assessment of Adult Competencies) numeracy assessment framework undertaken by Tout *et al.*[66] highlighted the importance of graphical and digital literacy:

> *The review found that 21st century digital technologies provide tools and processes that mediate thinking as well as action and are not just devices that can be used to complete manual, hands-on tasks more efficiently.*

The report included the following table of examples of the use of technology when using numeracy in different areas.

Category	Related to	Connections with digital information and technology
Numeracy for practical purposes	Aspects of the physical world that involve designing, making, and measuring	e.g., many aspects of measuring are now digital – theodolites, inclinometers, medical equipment/monitors, etc. e.g., design aspects are now available digitally, via software such as Computer-aided design (CAD) or online design software for kitchen/house planning
Numeracy for interpreting society	Interpreting and reflecting on numerical and graphical information in public documents and texts	e.g., much digital information is presented in digital and graphical formats, often dynamic in nature, including the use of spreadsheets for analysis. Even common software such as Word has sophisticated graphic and data options available. e.g. use of data, statistics and quantitative information through social and mass media for advertising, news and political information dissemination, etc.
Numeracy for personal organisation	Numeracy requirements for personal organisational matters involving money, time and travel	e.g., digital diaries, online banking, online shopping and planning, GPS and Google maps
Numeracy for knowledge	Mathematical skills needed for further study in mathematics, or other subjects with mathematical underpinnings or assumptions	The degree of technology inclusion is dependent on the programs of study—some are technology intensive, others less so. But often it is expected to be able to use and work with sophisticated digital and technological tools, including calculators, software, etc.

Four categories of numeracy use and their connections with technology; reproduced from Tout *et al.* (2017)

[66] Tout, D., Coben, D., *et al.* (2017). *Review of the PIAAC Numeracy Assessment Framework: Final Report*. Camberwell, Australia: Australian Council for Educational Research.

2.5 The qualifications landscape

Noyes *et al.*[67] expressed the view that it is appropriate to have different pathways but that it is important that they are equally esteemed:

> *We do not believe that a one-size-fits-all model is appropriate. We wish to see a highly flexible set of interlinking pathways that provide motivation, challenge and worthwhile attainment across the whole spectrum of abilities and motivations, but avoid the danger of returning to the O-level/CSE 'sheep and goats' divide.*
>
> *At a structural level we note the difficulties the new 'Use of Mathematics' qualification has encountered. The piloted GCSE Use of Mathematics could not be implemented beyond the pilot as a GCSE as it did not comply with regulations that changed during the project.*

The ETF review[68] of what employers and learners need from the maths and English qualifications taken by young people and adults recognised the widespread trust of GCSE and the suitability of the Functional Skills qualifications:

> *It is clear from discussions with employers that GCSE is a qualification they trust and many use it to select employees.*
>
> *Functional Skills are gaining widespread recognition across small and large employers. Employers who know about them like the approach they embody i.e. applied skills, flexible assessment and problem solving.*

National Numeracy's report[69] commented on a finding from the OECD/INFE International Survey of Adult Financial Literacy Competencies (2016):

> *Of those 16–24 year olds with a grade C or above in GCSE maths, less than a quarter were at an equivalent level when their numeracy was tested – suggesting that GCSE is no guarantee of good numeracy.*

[67] Noyes, A., Drake, P., *et al.* (2011). *Evaluating Mathematics Pathways: Final Report*. DfE.
[68] Education & Training Foundation (2015). *Making maths and English work for all*
[69] National Numeracy (2017). *A new approach to making the UK numerate*

3 How and what should post-16 GCSE Mathematics students learn?

3.1 Overview

The large majority of GCSE Mathematics resit students have no aspiration to progress on to study mathematics or mathematics-related disciplines at a higher level, but much of the GCSE Mathematics curriculum is geared towards preparation for further mathematical study. Furthermore, post-16 students associate GCSE Mathematics with being at school rather than with making progress in learning useful skills.

For these reasons, many GCSE Mathematics resit students see resitting GCSE Mathematics as irrelevant to them. This leads to poor motivation. Many students can see no point in resitting GCSE Mathematics beyond the need to achieve a level 2 pass. This is often reinforced by the way they are taught. Rather than being taught in a way that enables them to develop a sustainable understanding of mathematics, teachers concentrate on exam technique in the hope that this will give students the best chance of achieving a level 2 pass. The reason for learning the mathematics is lost. This results in under-achievement and reinforces a negative view of mathematics that can last a lifetime and contributes to sustaining the negative perception of mathematics held by many in our society: that it is difficult to learn and of little practical value.

Informed by our research, in the sections below we discuss how more active, engaging teaching approaches and a more relevant, contextualised curriculum could help to address these problems. This can provide a far more positive learning experience for these students, which can help them to achieve their potential.

3.2 Teaching approaches and materials

In order to develop our curriculum, we started by considering existing, successful teaching approaches that we had trialled in our work to support providers of resit GCSE Mathematics, in particular through the Stoke-on-Trent Mathematics Excellence Partnership (MEP) and the Education Endowment Foundation's research into the contextualisation of resit GCSE Mathematics,[70] along with the well-designed resources produced by the Standards Unit.[71] We asked colleagues who were experienced in teaching students resitting GCSE Mathematics to develop teaching materials which exemplified the kinds of things that the students should be learning. Three sets of teaching materials they produced are briefly described below, and are published alongside this report.

Minimum wage

This resource takes account of the age of the target students, who are likely to be in employment soon, and addresses the topic of finance; this featured as an important topic in our reviews of both relevant research and similar qualifications.

Our initial thoughts about the kind of course that would be suitable as an alternative to GCSE resit were that it might be something like Core Maths but at level 2. However, discussions with the project Advisory Group highlighted the gaps in understanding which

[70] Runge, J., Munro-Lott, N., & Buzzeo, J. (2019). *Embedding contextualisation in English and mathematics GCSE teaching: Pilot report*. Education Endowment Foundation.
[71] Swan, M. (2005). *Improving learning in mathematics: challenges and strategies*. Department for Education and Skills Standards Unit.

GCSE resit students typically have. This is different from Core Maths students, who are the same age but have succeeded at GCSE and so have a better foundation of understanding to build upon. Hence, this resource addresses telling the time and understanding units of time, as well as minimum wage, in order to enable students to consolidate basic skills in an area where experience shows that many have gaps in their knowledge.

Ratio

This resource exemplifies that the same mathematics can be used in different contexts and uses bar modelling as a way of working with ratios to solve problems. It recognises that some students will already have working strategies for working with ratios but also introduces effective strategies for those who do not.

The resource covers an important topic with a variety of applications but also addresses preparation for exam-style questions.

Working with data

This resource uses skills which students will have already met at KS4; it also uses real data, which they are less likely to have encountered at KS4. The resource provides the opportunity to revise important statistics and probability skills and to use them in interpreting real data.

The resource also covers drawing statistical graphs using a spreadsheet. Students will have drawn the graphs by hand at KS4; use of a spreadsheet allows them to learn a new skill as well as enabling them to rapidly produce graphs to discuss and interpret.

3.3 Developing an outline curriculum

It is important that students resitting GCSE Mathematics gain a credible qualification that has currency with employers and HEIs. It is also important that they are enabled to use mathematics in a range of contexts which they are likely to encounter in everyday life.

We used our review of relevant research and existing qualifications, and our experience of drafting some exemplar teaching resources, to inform the development of the curriculum.

We began by listing the kinds of quantitative tasks all adults should be able to do, then we considered the appropriate mathematical content to support these. Content was organised using contexts, in order to support a different approach to teaching. It was then referenced to GCSE Mathematics content, to help teachers see how it relates to the GCSE Mathematics curriculum.

To compile a list of quantitative skills all adults should possess, we included contexts from the 'General life and personal interest' section from the foundation tier GCSE context grid[72] and the 'Essentials of Numeracy for All' poster[73] from National Numeracy. The Essentials of Numeracy were defined by National Numeracy working with employers, unions, charities and maths experts such as Cambridge Maths.[74] We also considered work on personal finance education, which is clearly relevant to students and has been shown to motivate mathematical learning.[75] Once we had a list of skills, we grouped them into four themes to

[72] MEI (2017). *Contextualisation Toolkit*
[73] National Numeracy (2013). *The Essentials of Numeracy for All*
[74] www.nationalnumeracy.org.uk/essentials-numeracy
[75] Spielhofer, T., Kerr, D. & Gardiner, C. (2010). *Personal Finance Education: Effective practice guide for schools*. Slough: NFER.

provide four teaching units in the curriculum, as outlined below. The associated mathematical content is detailed in Appendix 2 and in the draft curriculum document.

It would be helpful for teaching and examining the proposed curriculum if all contexts which could occur in examinations were listed; the lists in this table are not sufficiently detailed to be exhaustive but they indicate the kinds of skills which students would develop.

Financial understanding	**Working with measures and shape**
• Understanding discounts in the sales • Understanding household bills • Estimating the cost of weekly food shopping • Splitting a restaurant bill • Shopping around for the best mobile phone deal • Comparing prices for differently sized packages • Budgeting for a holiday or major purchase • Personal budgeting • Managing a budget at work • Understanding interest rates when saving and borrowing	• Being able to read a measuring scale • Knowing your height and weight • Converting between imperial and metric units • Buying enough paint to decorate a room • Using shapes in designing a garden or craft project • Making and interpreting measurements to decide whether a piece of furniture or household appliance will fit in a given space • Understanding a map or scale drawing • Understanding measurements relating to personal fitness and health • Giving the right quantity of medicine to children
Planning activities	**Understanding quantitative information**
• Estimating time needed for tasks • Planning a schedule • Understanding staff shifts on a rota • Planning a meal or party for a large number of people • Giving and following directions • Understanding journey times • Understanding a map or scale drawing • Understanding timetables	• Recording numerical information accurately so others can understand • Making sense of statistics in the news • Interpreting the results of an opinion poll and understanding why different polls may produce different results • Understanding results of elections • Understanding food labels • Understanding statistics relating to personal fitness and health • Understanding risk in the news in relation to health

Outline quantitative skills for a new post-16 mathematics GCSE Mathematics qualification

The teaching units in the outline curriculum are clearly related to real-world, practical applications that are obviously relevant to the target students. Making the applications of the mathematics explicit will ensure students can see the practical value of learning mathematics at this level.

A curriculum designed in this way can be taught using meaningful contexts that can motivate and engage students who are not interested in further mathematical study but will be interested in ensuring they can use mathematics effectively in their everyday lives.

Further details of the draft curriculum are given in Appendix 2.

4 Assessment structure

4.1 Overview

The qualification must be assessed in a way that enables the students to demonstrate fluency with the mathematics they need and, as far as possible, that they are able to use it in meaningful contexts. In considering how this might be achieved in practice, we reviewed similar mathematics qualifications, developed a rationale and then drafted an assessment structure that we feel can meet the needs of the target students.

We then prepared exemplar assessment materials and had them externally evaluated to judge whether they were at the appropriate standard, using a comparative judgement study; this is outlined in section 4.4 with the full report published separately as an external appendix.

In section 4.5, we reflect on our proposed assessment structure and the outcomes of the comparative judgement.

4.2 Review of similar qualifications

Many countries do not have high-stakes examinations at age 16.[76] In order to consider suitable content for possible alternatives to GCSE resit, current and previous qualifications which addressed numeracy and/or were aimed at post-16 learners and which were at a similar level to GCSE were reviewed. These included SQA National 5 examinations from Scotland, GCSE Numeracy from Wales and Mathematical Applications from the Irish Leaving Certificate Applied.

We compared the content of the following qualifications with that of foundation tier GCSE Mathematics. The comparison was necessarily approximate because different qualifications state content with different wording, which often implies a different emphasis.

Qualifications reviewed

- AQA Level 1/Level 2 Certificate in Use of Mathematics (Foundation tier) (2012–2018)
- WJEC GCSE Mathematics Numeracy (Intermediate tier) (2017 onwards)
- Linked pair Applications of Mathematics (Foundation tier) (2011–2016, resits 2017)
- Level 2 Functional Skills Mathematics (from 2020)
- SQA National 5 Applications of Mathematics (2018 onwards)
- SEG Modular Mathematics 2520T and 2520X (for post-16) (1999, with similar specifications in previous years)
- Edexcel Level 2 Award Number and Measures (2011 onwards)
- Edexcel Level 2 Award Statistics (2013 onwards)
- New Zealand adult numeracy (current)
- Mathematical Applications from the Irish Leaving Certificate Applied (1990s onwards)

[76] Gray, S.L. (2018). 'No other European country tests children at 16: let's scrap pointless GCSEs', *The Guardian*, 21 August 2018.

Observations about qualification structure

Older qualifications used in England from the above list tend to be modular or unitised. The table below shows which qualifications are modular and also which of them allow use of a calculator throughout.

	Calculator throughout	Calculator for some
Modular	Linked pair Applications of Mathematics (Foundation tier) (2011–14)	SEG Modular Mathematics 2520T and 2520X
	AQA Level 1/Level 2 Certificate in Use of Mathematics (Foundation tier)	
	New Zealand adult numeracy	
	Mathematical Applications from Irish Leaving Certificate Applied (part of a larger modular programme)	
Linear	Linked pair Applications of Mathematics (Foundation tier) (2014–16)	Edexcel Level 2 Award Number and Measures
	Edexcel Level 2 Award Statistics	Level 2 Functional Skills Mathematics (from 2020)
		SQA National 5 Applications of Mathematics
		WJEC GCSE Mathematics Numeracy (Intermediate Tier)

The *Evaluating Mathematics Pathways: Final Report*[77] had this to say about the Level 1/2 Certificate in Use of Mathematics:

> *The pilot GCSE Use of Mathematics is to become a Level 1/2 Certificate post-pilot. This pilot qualification was also well received and the new Certificate has the potential to provide a more worthwhile learning experience for students leaving school at 16 without achieving a grade C in GCSE Mathematics.*

It may be that the positive views of the Use of Mathematics qualification were, at least partly, to do with its modular nature, which allowed students to achieve success in part of the qualification before going further. It should also be noted that the following linear qualifications from the above list have a lower-level qualification with a similar title which could be taken as a stepping stone:

- Level 2 Functional Skills Mathematics (from 2020)
- SQA National 5 Applications of Mathematics (2018 onwards)
- Edexcel Level 2 Award Number and Measures (2011 onwards)
- Edexcel Level 2 Award Statistics (2013 onwards)

[77] Noyes, A., Drake, P., *et al.* (2011). *Evaluating Mathematics Pathways: Final Report*. DfE.

4.3 Rationale for proposed assessment structure

Grades available

GCSE grades 1 to 5 are available (to be consistent with foundation tier of the current GCSE Mathematics).

Balance of content

The content weightings for current GCSE Mathematics are shown below.[78]

Domain area	Weighting of marks per Assessment Series	
	Foundation tier	Higher tier
Number	25%	15%
Algebra	20%	30%
Ratio, proportion and rates of change	25%	20%
Geometry	15%	20%
Probability and statistics	15%	15%

Content areas have been recorded for the items in the exemplar assessments for comparison purposes. The weightings for the exemplar papers for our proposed curriculum are as follows.

Domain area	Exemplar papers
Number	25.5%
Algebra	5.5%
Ratio, proportion and rates of change	22%
Geometry	24.5%
Probability and statistics	22.5%

Further consideration would need to be given to what weighting of content areas, if any, is appropriate for the proposed curriculum. In view of the applied nature of the curriculum, it might be desirable to allow more flexibility in these weightings than is allowed in current GCSE Mathematics. This suggests that possible weightings for the new curriculum could be as follows.

Domain area	Approximate weighting
Number	25%
Algebra	5%
Ratio, proportion and rates of change	25%
Geometry	22%
Probability and statistics	23%

[78] Ofqual (2017). *GCSE Subject Level Conditions and Requirements for Mathematics*

Assessment objectives

The assessment objectives and weightings for GCSE Mathematics are shown in the following table. To ensure that the exemplar papers are similar in demand to foundation tier GCSE Mathematics, the assessment objectives for items on the exemplar papers have been recorded with the aim of achieving overall weightings that are broadly similar. However, no attempt has been made to record the strands of assessment objectives or to achieve compliance with the detailed rules in the GCSE subject level guidance for mathematics. The proposed qualification uses mathematics in realistic contexts so use of mathematical models has been incorporated into AO3, as it is for A level Mathematics. This additional element in AO3 is shown in *italics* in the table.

Assessment objectives	Weighting	
	Higher	Foundation
AO1 Use and apply standard techniques Students should be able to: • accurately recall facts, terminology and definitions • use and interpret notation correctly • accurately carry out routine procedures or set tasks requiring multi-step solutions.	40%	50%
AO2 Reason, interpret and communicate mathematically Students should be able to: • make deductions, inferences and draw conclusions from mathematical information • construct chains of reasoning to achieve a given result • interpret and communicate information accurately • present arguments and proofs • assess the validity of an argument and critically evaluate a given way of presenting information. Where problems require candidates to 'use and apply standard techniques' or to independently 'solve problems' a proportion of those marks should be attributed to the corresponding Assessment Objective.	30%	25%
AO3 Solve problems within mathematics and in other contexts Students should be able to: • translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes • make and use connections between different parts of mathematics • interpret results in the context of the given problem • evaluate methods used and results obtained • *use mathematical models* • evaluate solutions to identify how they may have been affected by assumptions made. Where problems require candidates to 'use and apply standard techniques' or to 'reason, interpret and communicate mathematically' a proportion of those marks should be attributed to the corresponding Assessment Objective.	30%	25%

Comparing problem solving in the new Functional Skills Mathematics level 2 with GCSE Foundation tier Mathematics

At the time of writing this report, new Functional Skills Mathematics qualifications have been accredited. The aims for Functional Skills Mathematics at levels 1 and 2 include the following:

> *Functional Skills Mathematics qualifications at these levels should:*
>
> - *Indicate that students can demonstrate their ability in mathematical skills and their ability to apply these, through appropriate reasoning and decision making, to solve realistic problems of increasing complexity*[79]

The aims of Functional Skills Mathematics are similar to the curriculum we have developed; it is also now possible for students who gain GCSE grade 2, or below, at KS4 to work towards Functional Skills Mathematics at level 2 rather than GCSE Mathematics. This raises the question of whether Functional Skills Mathematics at level 2 already does what the proposed curriculum is aiming to do. The approaches to regulating problem solving in Functional Skills Mathematics and GCSE Mathematics are compared in the table below.

	Functional Skills Mathematics level 2	**Foundation tier GCSE**
Requirements	• 25% of total marks are allocated to questions or tasks which assess underpinning skills, and • 75% of total marks are allocated to questions or tasks which assess problem solving	• 50% AO1 Use and apply standard techniques • 25% AO2 Reason, interpret and communicate mathematically • 25% AO3 Solve problems within mathematics and in other contexts
Guidance	We expect problem solving questions and tasks to involve the type of cognitive operations and processes typically encountered in everyday life. The context within which a question or task is set should be relevant and not superfluous to the question or task. Additional guidance about the appropriate level is given.	Where problems require candidates to 'use and apply standard techniques' or to 'reason, interpret and communicate mathematically' a proportion of those marks should be attributed to the corresponding Assessment Objective.

The wording of the requirements for Functional Skills Mathematics suggests that items that contain some problem solving would have all their marks counted towards the 75% problem-solving allocation. At GCSE, each individual mark is assigned to one of the three assessment objectives, and it is unusual for an item to have all its marks assigned to AO3 (problem solving).

AQA level 2 Functional Skills Mathematics has a simple structure, with section A of each paper testing underpinning skills and section B testing problem solving. Mark allocations for the specimen papers are shown in the following table.

[79] Ofqual (2019). *Functional Skills Mathematics Conditions and Requirements*

Marks	Paper 1	Paper 2	Totals
Section A	8	12	20
Section B	12	48	60
Total	20	60	80

This confirms that the 75% problem solving includes all the marks from items (question parts) which include some problem solving.

GCSE Mathematics includes marks allocated to AO2 (mathematical reasoning) as well as AO3 (problem solving) and AO1 (standard techniques). Questions which involve problem solving often include some reasoning; there isn't a clear-cut distinction between problem solving and reasoning at GCSE level. OCR sample papers at Foundation tier, for the specifications for teaching from 2015, have the following mark allocations to GCSE assessment objectives:

Marks	Paper 1	Paper 2	Paper 3	Totals
AO1	50	50	50	150
AO2	25	25	25	75
AO3	25	25	25	75
Total	100	100	100	300
Items which include some AO3	44	43	43	130
Items which are AO1 only	17	18	19	54

43% of the marks are for items which include problem solving and only 18% for items which only assess standard techniques. This may vary between specifications and also between different years for the same specification, but it is consistent between the three papers in this suite. So, although the 25%/75% split between underpinning skills and problem solving in Functional Skills Mathematics appears to indicate that there is a greater emphasis on problem solving in Functional Skills Mathematics than in foundation tier GCSE, this is not necessarily the case.

Rationale for proposed structure of assessments

Students who have not achieved a level 2 GCSE Mathematics pass pre-16 generally lack confidence in their mathematical ability.

Many of the qualifications reviewed for this project are either modular or have a stepping stone which leads to the qualification. That is not the case for GCSE (9 to 1). Although, theoretically, Functional Skills Mathematics level 2 can be used as a stepping stone, this is not what Functional Skills Mathematics was designed for, and there is no evidence that Functional Skills Mathematics level 2 is being used as a stepping stone to GCSE. Moreover, the comparative judgement conducted by No More Marking[80] confirms that, as a level 2 qualification, level 2 Functional Skills Mathematics is more demanding than foundation tier GCSE Mathematics, which includes all of the level 1 and some of the level 2 curriculum.

[80] Barmby, P., & Wheadon, C. (2019). *A comparative judgement study of MEI GCSE exam items*. No More Marking Ltd.

The proposed assessment structure for the post-16 mathematics GCSE is shown in the table below.

	Paper 1	Paper 2	Paper 3
Style	Multiple choice questions assess basic skills. Questions may be in context or context-free. Results reported to centres for diagnostic purposes.	A mixture of short and long questions, all set in realistic contexts.	A mixture of short and long questions, all set in realistic contexts.
Time	1 hour	1.5 hours	1.5 hours
Number of marks	40	80	80
% of total qualification	20%	40%	40%
Calculator allowed?	Yes	No	Yes
Availability	Twice a year[81]	November and June	In same series as Paper 2

The multiple choice format for Paper 1 means it could be marked quickly and also provide centres with diagnostic information. It would also allow students to receive prompt feedback and experience success, and could be taken as a stepping-stone element part way through the course; students would be allowed one resit opportunity for Paper 1. The test has been designed to be taken online; this would allow it to incorporate the use of technology such as spreadsheets and online calculators in a similar way to that proposed in the PISA 2021 framework for mathematics.[82] In the exemplar papers, a start is made on incorporation of technology by allowing use of a calculator.

The importance of students developing basic skills is highlighted by Watson *et al.*:[83]

> *While improvement of mathematical thinking and self-esteem are appropriate goals for educationists, students also need to be achieving in ways recognised by the outside world. So final examination results are important, but students will be better prepared for these if they understand some mathematics, and feel confident to tackle unfamiliar problems, while being sure they have some fundamental tools, such as arithmetic and calculator use.*

[81] Papers could be automatically compiled based on defined parameters and available online in a one-week window to allow for centres not having enough computers for all candidates. Alternatively, a paper-based assessment, available at two sittings could be used with an online bank of practice questions for diagnostic assessment.
[82] *PISA 2021 Mathematics Framework: Examples*. OECD.
[83] Watson, A., De Geest, E., & Prestage, S. (2003). *Deep Progress in Mathematics: The Improving Attainment in Mathematics Project*. University of Oxford.

Papers 2 and 3 will be timed, written examination papers, sat at the end of the course. There is no restriction on the content which may appear on either paper. There should be no more than two papers at the end of the course, to minimise the possibility that students who do not feel as if they have succeeded in one paper fail to attend the next paper. This will also make it easier for post-16 students who need to take time off work or to arrange childcare in order to sit an examination.

No paper should be longer than 1½ hours, because students who qualify for additional time will take longer than this, with some needing to take twice as long as the published examination time. It is likely that a greater proportion of resit students will qualify for additional time compared to the general population, and it is undesirable for these students to have an examination lasting longer than 3 hours.

Students who have attended for at least two of the three papers should have had the opportunity to gain enough marks to get a grade. The overall grade should be based on the total mark over the three papers. Candidates who, for any reason, have failed to attend an assessment should at least be graded on the basis of what they have done. Candidates who miss an assessment for illness should be considered more favourably than this, in line with general rules applicable to all GCSE qualifications.

It has generally been accepted for examining mathematics that 'a mark a minute' is about right in terms of mark allocation. The proposed written assessments are consistent with this; for each 90-minute paper, 80 marks are available. The multiple-choice paper, which is intended to provide a confidence-boosting stepping stone, allows well over one minute for each mark to allow students plenty of thinking time.

4.4 Level of difficulty and comparative judgement of proposed assessment

Level of demand and grade boundaries

For the foundation tier of GCSE Mathematics, the requirements for level of demand are as follows:[84]

> *An awarding organisation must, therefore, take all reasonable steps to ensure that the marks available for each assessment within the foundation tier are targeted as follows.*
>
> - *50% of those marks must be targeted at a Level of Demand consistent with grade 1 to the lower part/half of grade 3.*
> - *50% of marks must be targeted at a Level of Demand consistent with the upper part/half of grade 3 to grade 5.*

This suggests that the following proportions of the paper are likely to be accessible to students who get particular grades:

Grade	**1**	**2**	**3**	**4**	**5**
Proportion of paper accessible (%)	20	40	60	80	100

[84] Ofqual (2017). *GCSE Subject Level Conditions and Requirements for Mathematics*

Candidates in an examination will not get all the questions right, even when they can do them, because everyone makes mistakes. Assuming that a typical candidate who just achieves a given grade will get 80% of the marks which are accessible to people working at that grade[85] leads to the hypothetical grade boundaries in the following table. Average grade boundaries, as a percentage of all marks available and rounded to the nearest whole number, from across the foundation tier Mathematics GCSEs from summer 2017, 2018 and 2019 are shown for comparison. Grade boundaries vary from year to year and between specifications. The June 2019 foundation tier grade boundaries for Pearson Edexcel were the highest; these are also shown in the table to give an idea of the variation.

Grade	1	2	3	4	5
Hypothetical grade boundary (%)	16	32	48	64	80
Average June 2017 grade boundary (%)	10	22	35	47	63
Average June 2018 grade boundary (%)	10	21	37	51	66
Average June 2019 grade boundary (%)	11	24	38	52	66
Edexcel June 2019 grade boundary (%)	15	30	46	62	77

For students aiming to achieve grade 4, the target students for this curriculum, aiming to achieve 50% does not encourage them to feel a sense of success with mathematics. Consequently, the exemplar examination papers are deliberately set at a lower demand than typical foundation tier GCSE (9 to 1) papers. They will still aim to meet the level of demand requirements but will intentionally aim low with the demand in each of the following categories in the hope of coming close to the hypothetical grade boundaries in the table above. The June 2019 grade boundaries for Edexcel show that this is possible.

- *50% of those marks must be targeted at a Level of Demand consistent with grade 1 to the lower part/half of grade 3.*
- *50% of marks must be targeted at a Level of Demand consistent with the upper part/half of grade 3 to grade 5.*

The first sitting of the 9 to 1 Mathematics GCSEs was in summer 2017; there was, at the same time, a sitting of the old A* to G specifications, mainly for students resitting. The average grade boundaries for the June 2017 A* to G Mathematics GCSEs are shown in the table below.

Grade	G	F	E	D	C
Average June 2017 grade boundary (%)	21	32	42	54	65

The 9 to 1 GCSEs are intended to be more rigorous than the A* to G GCSEs. For grade C, in 2017 showing competence in two-thirds of the curriculum is, arguably, a better foundation for using mathematics in later life than a grade 4 pass showing competence in a half – even though the 9 to 1 curriculum at foundation tier is larger than the A* to C curriculum.

[85] MEI has used this ‘rule of thumb’ in qualification development for some years.

Comparative judgement

No More Marking compared the difficulty of the exemplar papers with the summer 2017 GCSE Mathematics 9 to 1 assessments and with the AQA Functional Skills Mathematics level 2 specimen papers. The GCSE foundation tier papers are a natural set of qualifications for comparison with the exemplar papers for the proposed curriculum because the proposed curriculum is designed as an alternative to the current GCSE Mathematics for post-16 students.

The emphasis on working in context in the proposed curriculum is similar to Functional Skills Mathematics; together with the national change to allow students with grade 2 at KS4 to work towards level 2 Functional Skills Mathematics post-16 instead of GCSE; this raises the question of whether the proposed curriculum is needed. However, anecdotal evidence from FE lecturers suggests that students find level 2 Functional Skills Mathematics harder than GCSE foundation tier, and this view is supported by the comparative judgement report.

There are too many Functional Skills Mathematics qualifications to include them all in the comparison. The AQA qualification was chosen because the design, with section A assessing underpinning skills and section B assessing working in everyday contexts, was quite similar to the design for the proposed new mathematics GCSE curriculum assessment.

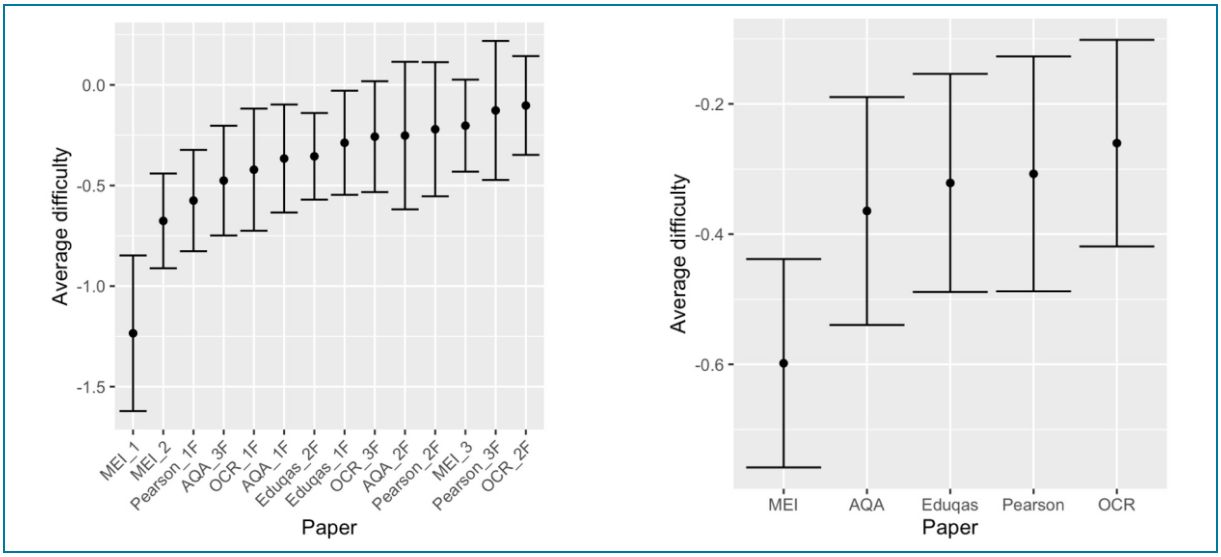
No More Marking made use of data from earlier comparative work on GCSE Mathematics for Ofqual,[86] which addressed both foundation and higher tier GCSE, so the difficulties are reported on the same scale, with zero representing the mean difficulty of all items included in the earlier work on sample assessments.[87]

The comparative judgement found that the difficulty of exemplar Papers 2 and 3 for the proposed curriculum was in line with foundation tier GCSE (9 to 1) papers. Paper 1 is easier than foundation tier GCSE (9 to 1) papers; this is in line with its design as a diagnostic paper to be taken early in the course to check whether students have the basic skills in place to enable them to succeed. The charts on the next page show 95% confidence intervals for the mean difficulty of each paper and for the qualifications as a whole. The whole No More Marking report is published alongside this report.[88] The overall finding was as follows:

> *In answer to the research question 'Are the new MEI GCSE exam papers comparable in difficulty to existing GCSE papers?', from the results of the analysis, we have concluded that overall the MEI papers are not significantly different in difficulty to previous GCSE Foundation papers. However, we did find that ideally, the difficulty of the MEI papers could be increased a little so that they are more in line with the comparison Foundation papers. In particular, we suggest that the difficulty of MEI paper 1 could be increased, perhaps by including multiple choice questions containing multiple steps rather than those with more straightforward factual recall.*

It is worth noting that some of the questions which were judged to be very easy, such as a question about telling the time on Paper 1, are testing content which many resit students do not find easy but which is needed for students to be able to use mathematics confidently.

[86] Ofqual (2017). *An evaluation of the difficulty of the assessments and the characteristics of the problem-solving (AO3) items*
[87] Ofqual (2015). *A Comparison of Expected Difficulty, Actual Difficulty and Assessment of Problem Solving across GCSE Maths Sample Assessment Materials*
[88] Barmby, P., & Wheadon, C. (2019). *A comparative judgement study of MEI GCSE exam items*. No More Marking Ltd.



Average difficulties of the MEI and comparison GCSE Foundation papers: left-hand chart individual papers, right-hand chart whole qualifications; reproduced from Barmby and Wheadon (2019)[89]

Implications of the difficulty of the exemplar papers for grade boundaries

Exemplar Papers 2 and 3 are similar in difficulty to the GCSE foundation tier papers for June 2017 and so might be expected to have similar grade boundaries.

Grade	1	2	3	4	5
Average June 2017 grade boundary (%)	10	22	35	47	63

Exemplar Paper 1 is easier and students can resit it, so it is reasonable to expect them to score well on this paper. If we make a simplistic assumption that every student gets 80% on this paper, which is worth 20% of the qualification overall, then this would raise the grade boundaries shown above as follows:

Grade	1	2	3	4	5
Mark on Paper 1 (out of 20)	16	16	16	16	16
Boundary for Papers 2 and 3 (out of 80)	8	18	28	38	50
Possible grade boundary on the exemplar papers (%)	24	34	44	54	66

Only the grade 1 boundary seems out of line with where we might reasonably expect grade boundaries to be at foundation tier, and it is likely that students working at grade 1 would not score as high as 80% on Paper 1. In practice, grade boundaries will be set after examinations have taken place, taking account of actual student performance. It is difficult to predict what this will be, but it is hoped that a curriculum which is more relevant to students' needs would result in better performance in examinations.

[89] [89] Barmby, P., & Wheadon, C. (2019). *A comparative judgement study of MEI GCSE exam items*. No More Marking Ltd.

There are a number of ways in which the overall difficulty of the qualification on the proposed curriculum could be increased, if this were felt to be desirable. Some possibilities are considered in section 4.5.

4.5 Reflections on the proposed assessment structure

Should Paper 1 be made as difficult as Papers 2 and 3?

This would raise the mean difficulty of the suite of papers to make them more comparable with summer 2017 foundation tier GCSE. However, Paper 1 is deliberately set at an easier level to make it suitable as a diagnostic paper to be taken early in the course and a stepping stone to the rest of the qualification. Making it more difficult would make it less useful. If it were felt necessary to raise the demand of the assessment, it would be better just to remove it from the qualification, relying on two papers at the end of the course instead.

Should Paper 1 be non-calculator?

GCSE Mathematics typically has one-third of the assessment without a calculator; the proposed curriculum has 40% non-calculator. Part of the reason for including Paper 1 is to check that students have the basic skills needed to be able to use mathematics confidently in their lives and work. The necessary skills students must demonstrate include being able to estimate sensibly and do basic calculations without a calculator. These skills are currently assessed in Paper 2, which is the non-calculator paper.

The ability to interpret the output of technology tools such as spreadsheets and the kinds of automatic calculators that are available online are important skills that are not currently assessed in GCSE Mathematics; the proposed online Paper 1 offers a way to assess these important skills. For this reason we consider that Paper 1 should be online, with Paper 2 fulfilling the requirement for non-calculator assessment.

The rules for GCSE Mathematics state that:[90]

> *an awarding organisation must ensure that between 33 and 50 per cent of the total marks available in those assessments are allocated to questions or tasks which must be completed by Learners without the use of a calculator.*

Should we retain the three-paper structure?

The feedback from teachers (in section 5.2) indicates that the information from the diagnostic assessment in Paper 1 would be helpful. The paper also provides a stepping stone to the whole qualification for students and a way of assessing the basic underpinning skills which are important not only for success at GCSE but also for being able to use mathematics confidently in future life.

Would it be possible to increase the difficulty of at least one of Papers 2 and 3?

The approximate mean difficulties of the three exemplar papers can be read off from Figure 8 in the No More Marking comparative judgement study,[91] and are summarised in the first row of the following table. The average has been calculated using the weightings for each paper.

[90] Ofqual (2017). *GCSE Subject Level Conditions and Requirements for Mathematics*
[91] Barmby, P., & Wheadon, C. (2019). *A comparative judgement study of MEI GCSE exam items*. No More Marking Ltd.

	Paper 1	Paper 2	Paper 3	Average
Mean difficulty, for papers used in the study	−1.25	−0.7	−0.2	−0.61
Mean difficulty with a harder Paper 3	−1.25	−0.7	0.5	−0.33

The content and philosophy of the proposed curriculum would allow Paper 3 to be more like AQA Functional Skills Mathematics Paper 2; this was found to be harder than foundation tier GCSE Mathematics in the comparative judgement. The second row of the table shows what would happen if the mean difficulty of Paper 3 was increased to 0.5; this would make the paper as difficult as a GCSE higher tier paper and increase the overall difficulty of the suite of papers to be in line with foundation tier GCSE. A mean difficulty of 0.5 is achievable, comparing with the difficulty of AQA Functional Skills Mathematics level 2. However, this is intended to be a level 1 and 2 qualification, and the higher grade boundaries resulting from having the papers easier than summer 2017 foundation tier GCSE would allow students who succeeded in obtaining a level 2 pass to show a good understanding of the curriculum and so enable them to have the quantitative skills needed for effective citizenship.

How should grade boundaries be set?

When setting grade boundaries for the summer GCSE examinations, the awarding bodies make use of statistical information to ensure that different specifications are awarded to the same standard. The statistical information is based on the KS2 performance of students who are taking the GCSE aged 16.[92] Once the grade boundaries are set using the information from these students, they apply to all students, including those resitting GCSEs.

For the November sitting of GCSE, the vast majority of students are resitting the qualification so a different approach is used. The statistical information used for awarding is based on the average GCSE grades at the end of KS4 for the 17 year-olds.[93]

If the proposed curriculum is used as the basis for an alternative mathematics GCSE for students resitting, then some students would sit the new qualification. Other students would continue to resit GCSE Mathematics, if that were more appropriate to their needs. It is likely that the students resitting GCSE Mathematics would have a higher average GCSE grade in the summer than students resitting the proposed mathematics GCSE. This should not limit the potential performance of the students sitting the mathematics GCSE based on the proposed curriculum. A different approach to setting grade boundaries should be used, in order not to disadvantage students based on KS4 performance. Information from comparative judgement of examination papers after they had been sat could be used to provide statistical information to inform the setting of appropriate grade boundaries that allowed reliable grade setting.

[92] Jadhav, C. (2016). 'Comparing like with like in 2017', *The Ofqual blog*, 1 December 2016.
[93] Jadhav, C. (2017). 'November 2017 exams and summer series 2018', *The Ofqual blog*, 17 November 2017.

5 Feedback

5.1 Overview

Once we had designed the curriculum and assessment structure and developed exemplar assessments, we sought feedback from two key groups:

- experienced teachers of GCSE Mathematics, who would be able to give informed views on the suitability of the content and assessment relative to GCSE Mathematics, and on the practicalities of implementing the proposed curriculum in school and college settings
- employers, who require their employees to be able to apply mathematics in a work context, and who could provide advice on what was required in the proposed curriculum to meet their needs.

The samples we used were small, to gain feedback which would indicate whether the development was proceeding along the right lines; larger samples were not practical in the context of a one-year development project. The feedback is presented and analysed below.

5.2 Feedback from teachers

We obtained feedback from 26 teachers who filled in an online questionnaire. All teachers who were asked for feedback were provided with the draft curriculum and the exemplar examination papers. Six teachers attended a focus group meeting in July 2019. Teachers had been provided with the curriculum and exemplar papers in advance of the meeting. These were discussed in detail at the meeting, including an item-by-item discussion of the exemplar papers. A larger number of teachers, recruited from events relating to GCSE resit, volunteered to give feedback but did not find time to do so. Although the sample was not large, it fulfilled the purpose of gaining formative feedback for the draft curriculum and exemplar papers from a group of teachers with relevant experience.

The feedback was used to improve the exemplar papers before No More Marking conducted the comparative judgement; the papers which were judged and the mark schemes are included in Appendix 3 of this report.

The feedback is presented without comment in this section, and discussed in section 5.4.

Feedback from online questionnaire (26 teachers)

Responses to questions with a choice of responses

Do you work in a centre which teaches GCSE Mathematics resit?

Yes	No
23	3

What type of centre do you work in?

General FE college	20
School with sixth form	2
Specialist land-based FE college	1

Please comment on the overall difficulty of the sample papers.

Too hard	0
About right	21
Too easy	3

The first paper is designed to be online on demand with feedback to centres.

	Helpful	Somewhat helpful	Not helpful	Don't know
Would the feedback about student misconceptions from this paper be helpful to teachers?	21	3	0	1
Would being able to do this paper early in the course and getting the results before the other two papers be helpful for students?	17	7	1	0

Would an online, on demand paper be manageable for your centre?

Manageable	11
Manageable, with difficulty	7
Not manageable	4
I am not in a centre at present	2
Don't know	1

Comments from free-response questions

Content included but not appropriate	Content not included but should be
• Fractions • Not sure about imperial units • Think I saw something on equations of straight-line graphs • Some of the uses of imperial measures • Venn diagrams • How much algebra is needed? Do they need to deal with algebraic simplification in order to interpret a real-life straight graph? I think that the fundamental concept is the one of rates of change, which can be explored in so many contexts that are relevant to students • Imperial measurements, we should be pushing for metric units to convert the nation to a metric nation • Risk percentages • Metric to imperial units • I would query the appropriateness of Standard form and Pythagoras in general life, the majority of learners would never use these skills, and if required for their career they would be acquired in their main course or in the workplace	• Didn't spot anything to do with social media/mobile phones • Using fractions and ratio together. This seems like an important skill and would be encountered in everyday life • More practical application to personal finance would enhance what is already there • Mortgages, Tax, Loans and understanding the risks involved • Trigonometry – used in vocational courses such as carpentry and construction • Surface area and volume of composite solids • More shape work (triangles) • You could emphasise financial literacy – tax rates, difference between NI and Tax, pensions, ISA's, mortgage versus rent over time • Test their knowledge of the area and the circumference of a circle. Maybe do that in a question on the volume and the surface area of a cylinder • I would have included one or two straightforward questions asking for fractions, decimals and percentages to be converted to and from one another

Other comments

- The list of excluded topics is longer than the list of extra topics. While this makes it easier to teach it may be harder to justify.
- I would like there to be more connection to understanding how the maths works in the real world. I know it is there in the themes but, for example, working out and comparing two loan payments, but then having to justify taking the loan that would cost more in the long term because you can afford the payments on that loan better. I think this would counterbalance what could be considered a reduced or simplified content.
- Links well to Core Maths so could be encouraging for students who want to study further.
- It seems a long winded version of the reformed Functional Skills Mathematics level 2 assessment, which I believe contextualises relevant topics for the post 16 cohort in a superior way.
- I like the fact that imperial measures are included in the content. This is so important for our learners given that as a society we continue to use both!
- I like that it leans towards a more modular structure, but would like some longer, multi skill questions.

- It is clear the ‘maths for maths sake’ has gone from this qualification (hooray) and I think the functional aspects and the contexts make it interesting and relevant and sellable as something different.
  Regarding algebra – taking out all the basic operational skills eg collecting like terms is definitely appropriate; however some students may see this reduction as making this qualification of less value. Many students with a grade 3 can perform the skills – by rote, admittedly – so there may be feelings in the early days (assuming the proposal is accepted!) of wasted maths.
- The draft syllabus looks OK. The draft assessments are far too easy.
- Like the contextualised comments to more modern situations e.g. lie detectors and use of Excel spreadsheets.
- The content seems well thought out and I agree with the choice in content that has been removed. I agree with all 3 areas that have been added; particularly the financial applications and the units of measurement.

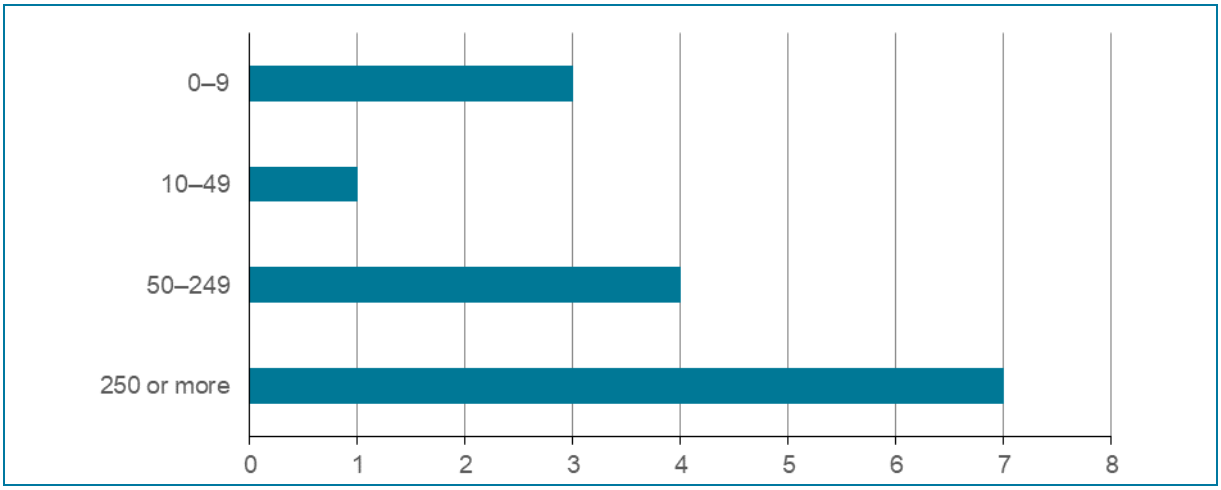
Feedback from a focus-group meeting, 10 July 2019 (6 teachers)

Content included but not appropriate	Content not included but should be
• Spreadsheets – there was universal disapproval of this being in – one reason was that teachers may not be able to teach it and it’s a new topic so extra to teach when time is a pressing issue • Standard form – it was felt that it shouldn’t be there at all in the syllabus • Is Pythagoras necessary? • Density • Box plots • Reciprocal graphs • Elevations (plans are OK)	• There was a feeling basic angles – measuring/turning – should be somewhere • Basic algebra. Some conceptual understanding of algebra is good – not necessarily in letters, but perhaps describing a situation. A step before spreadsheets perhaps? • Negative numbers/subtraction/division

5.3 Feedback from employers

Telephone interviews were conducted with employers; they were not asked to look at the draft curriculum or exemplar examination papers as we did not feel it was reasonable to ask employers to devote time to this. We were able to obtain helpful feedback from 15 employers, as shown below.

What type of organisation are you involved with?	Number
Education	4
Other	2
Construction	2
Public administration and defence; compulsory social security	2
Arts, entertainment and recreation	1
Employer organisation or association	1
Human health and social work activities	1
Manufacturing	1
Wholesale and retail trade; repair of motor vehicles and motorcycles	1



Responses to the question, 'How many employees in your organisation?' ***(15 responses)***

Owing to the short timescale allowed for the project and the difficulty of finding employers willing to give feedback on a mathematics curriculum, we had not intended to survey a large sample. The results from this small sample must be treated with caution but there are some interesting indicators.

The feedback is presented in this section, and discussed further in section 5.4.

Questions relating to new employees

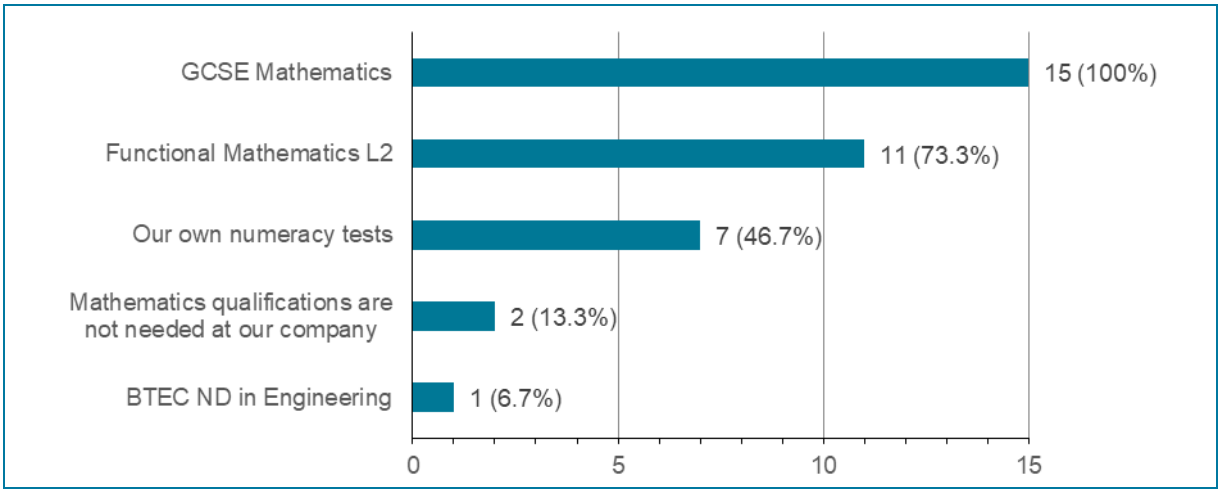
The table below shows that most of the employers we interviewed are generally happy with the mathematics skills of their staff and are willing to provide further training where needed. The only question where fewer than half the employers agreed was the question relating to confidence and positive attitude towards mathematics; nearly half the employers agreed that new employees were confident, but an equal number disagreed.

Statement/question	Strongly disagree	Disagree	Not sure	Agree	Strongly agree	Other
New employees have the basic mathematics skills needed for work	0	2	2	9	1	Depends on role
New employees can solve problems in the workplace using mathematics	0	2	3	9	1	
New employees are confident and have a positive attitude towards mathematics	1	6	1	7	0	
Do you teach new employees the mathematics that they need for the job?	0	1	1	9	4	
New employees can use mathematics in different contexts	0	1	1	12	1	

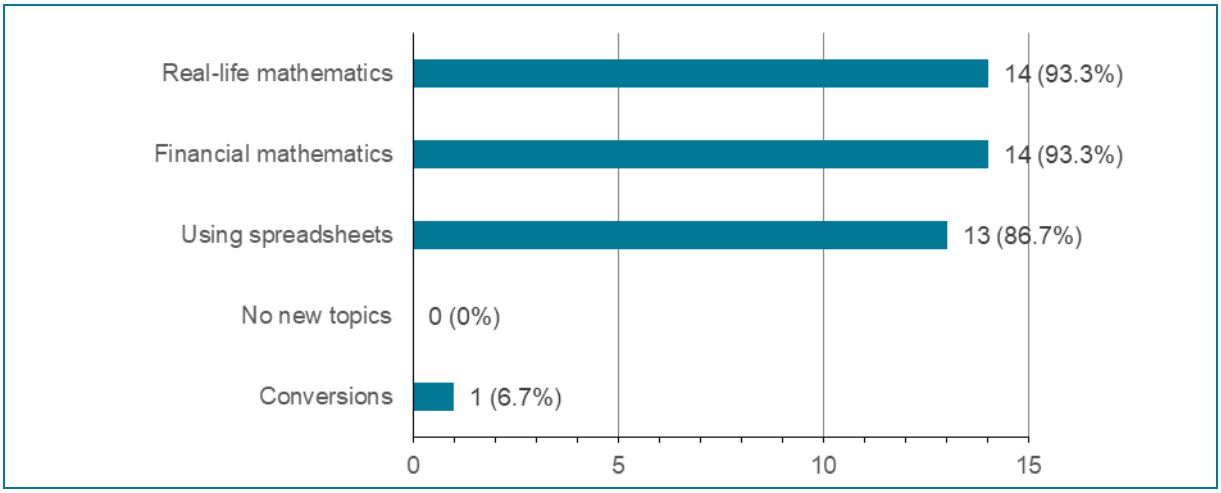
Employers were asked to indicate how useful they thought particular mathematical skills were in their workplace. The table below shows the numbers responding in each category. There was general agreement that all the content we asked about was either sometimes useful or very useful.

Skill	Not very useful	Sometimes useful	Very useful	Not sure
Mental arithmetic	0	1	14	0
Pen-and-paper methods of calculation	0	6	9	0
Estimation	1	3	11	0
Using a calculator	0	2	13	0
Analysing data	0	4	10	1
Interpreting graphs and charts	0	7	8	0
Using a formula in a spreadsheet	1	8	6	0
Creating a spreadsheet	0	8	7	0
Reading and understanding results from monitoring equipment	0	1	14	0

Questions relating to recruitment



Employers' responses to the question, 'When recruiting new employees which of these do you recognise?'(it can be more than one) ***(15 responses)***



Employers' responses to the question, 'If there was a new and different GCSE mathematics for students resitting what topics would you like to see in the qualification?' ***(15 responses)***

Additional topics suggested by employers

- Finance related to personal finance, financial literacy. Interpreting a salary slip and finance related to business, profit and loss.
- Industry specific skills, different skills needed for plumbers and decorators, electricians and engineers being the main ones.
- Anything that it is relevant to the work in the NHS, so relevant to different skills and also building confidence.
- Visual estimation of area and quantities in an operational setting. Ability to react quickly and do real world problem solving when tools go down, and act with fluency.
- Practical elements – how it links to clinical (real life examples of training to be a nurse).
- General understanding between mm and cm and imperial.

- Constructing graphs and interpreting graphs, percentages, simple equations, spreadsheets.
- Problem solving, and handling different types of data.
- Ability to do practical maths, so using maths in real contexts but actually doing practical examples. Spatial awareness, if you have a hole this big and a piece of metal this big what do you have to do to make it fit.
- Analysing data, interpret data and make conclusions from it. Really key to our organisation to be able to take in a whole load of data and make decisions from it. Being comfortable with numbers, every recommendation is backed up by numbers to make the right decisions.

5.4 Reflections on feedback

General remarks

Practical considerations are important. GCSE resit students have two terms post-16 before their examinations; they are often taught by teachers with limited confidence in mathematics so there will be a need for professional development. The wide range of teachers in FE colleges may well find that it is easier to convince students to learn content which is more directly applicable.

Students may not want to take the course, and there are very large numbers of students in some centres. It is not possible to introduce much new content in the limited time, given that students lack confidence with the content which they have already been taught.

Should spreadsheets be included?

Understanding how compound interest works is an important part of financial understanding. The natural way to teach compound interest at this level is by using a spreadsheet, and it is natural for students to ask how the spreadsheet is calculating the numbers. Employers are keen for spreadsheet skills to be included. Bearing in mind the practicality of teaching the course in limited time, it seems appropriate to include understanding simple spreadsheet formulae and being able to construct simple formulae – understanding that A2 is a variable and how this changes as the formula is copied down or across the sheet. However, students need not understand that a formula must start with = or that * needs to be used rather than a multiplication sign. If constructing a spreadsheet formula for themselves, they would have access to online help to enable them to get the syntax correct.

Should standard form be included?

Using a spreadsheet for compound interest will lead to some numbers appearing in standard form. Standard form also arises in biological and medical contexts. It is important for students to understand numbers in standard form, whether they appear in print or on a calculator or spreadsheet display. However, they do not need to be able to write numbers in standard form or to do calculations with them, with or without a calculator.

Should Pythagoras be included?

The applications of Pythagoras that students might typically encounter in everyday life are in the area of DIY, for example using a 3,4,5 triangle to measure a right angle or calculating the diagonals when making a rectangular gate. These tasks are done by a minority of the population and there are alternatives to the use of Pythagoras, so there is a strong argument

for removing it from the curriculum. It has been included at present but further consideration of the qualification content should take place at a future date.

Should density be included?

It is important for students to understand compound units such as speed and rates of pay, but the specific example of density is a better fit to GCSE science than to everyday life and so is more appropriate for students in KS4 than for students retaking GCSE. Density will be removed as a specific example. However, compound units such as miles per gallon or litres per km should be included. The informal use of compound units such as mg per ml to compare different formulations of a medication is important in order to understand whether these have similar strengths; this has been included in the proposed curriculum.

Should box plots be included?

Box plots are in higher tier GCSE so will be new content to students who have previously sat foundation tier. Although box plots are widely used in education-related reports, in particular those to do with examinations, they are less common in the press so it is not as important for students to understand them as it is for students to understand more common statistical diagrams. They have been removed from the curriculum. Understanding distributions is important but this can be done through bar charts and frequency charts, which are easier to interpret and also include more detail.

Should reciprocal graphs be included?

Students will already have encountered reciprocal graphs at KS4 and they do provide a way of showing inverse proportion graphically so perhaps they should stay in.

Should elevations be included as well as plans?

Interpreting plans, scale drawings and maps are related skills which are commonly used in everyday life; interpreting and drawing elevations is very much less common so elevations have been removed from the curriculum.

Should basic understanding of angles be included?

This is fundamental to being able to use angles in applications such as pie charts and scale drawings – it has been made more explicit in the proposed curriculum.

Should there be more emphasis on basic algebra?

Students need to be able to use simple formulae, and some basic algebra is needed to support this – mainly understanding the conventions for algebraic notation. It's harder to see why collecting like terms, multiplying out or factorising might be needed.

To what extent should calculations with fractions and negative numbers be included?

It is important that students can use fractions as operators and simple fractions as numbers. Adding and subtracting directed numbers in the context of temperature is also something they should be able to do, but multiplying or dividing two negative numbers does not arise in everyday contexts – nor do most fraction calculations.

Should imperial units be included?

Some imperial units continue to be used in everyday life but the question arises of how far this should go. Students may encounter ounces when using an old recipe book and pounds and stones when weighing themselves, but should these be in the curriculum or something that students look up when they need it?

Should Venn diagrams be included?

Students will have used Venn diagrams at KS4 and they are useful for working in contexts where probability is involved so it seems reasonable to leave them in.

What about using fractions and ratio together?

Some of the ratio and proportion curriculum statements from GCSE foundation tier content were initially removed because they seemed obscure; these have now been added to the proposed curriculum, with exemplification of what they mean, in context.

Should there be more emphasis on personal finance?

This is important new content in the proposed curriculum, and the specific examples given in feedback have been incorporated explicitly in the curriculum.

Should trigonometry or other industry specific content be included?

Although trigonometry is used in some vocational contexts, it is less applicable for use in everyday life so will not be included. Industry-specific content would be relevant to some students but less relevant for most students.

Should risk percentages be included?

This is where students are most likely to encounter probability in everyday life so it has been left in.

Should conditional probability be included?

Conditional probability is the one area of content from the higher tier GCSE Mathematics content which has been included in the proposed curriculum. No-one has commented on this in the feedback, other than in the meeting with teachers, where the feedback was as follows:

> *Loved the question. Usually they see ovals not rectangles in frequency tree questions.*

Conditional probability has been included because an understanding of conditional probability is important in understanding health-related information and also for interpreting forensic evidence, which citizens are likely to encounter when doing jury service. Frequency trees are an important tool for developing understanding of conditional probability; these are in foundation tier GCSE but the content is not built on to lead to this important application. We took the decision that the more mature students studying the proposed curriculum should have access to an understanding of conditional probability.

Should there be more shape work?

Although there has been one request to include more shape work, it's not specific so it is difficult to judge why this might be considered a good fit with the rest of the curriculum.

Should surface area and volume of composite solids be included?

This isn't something with wide applications in everyday life so has been left out.

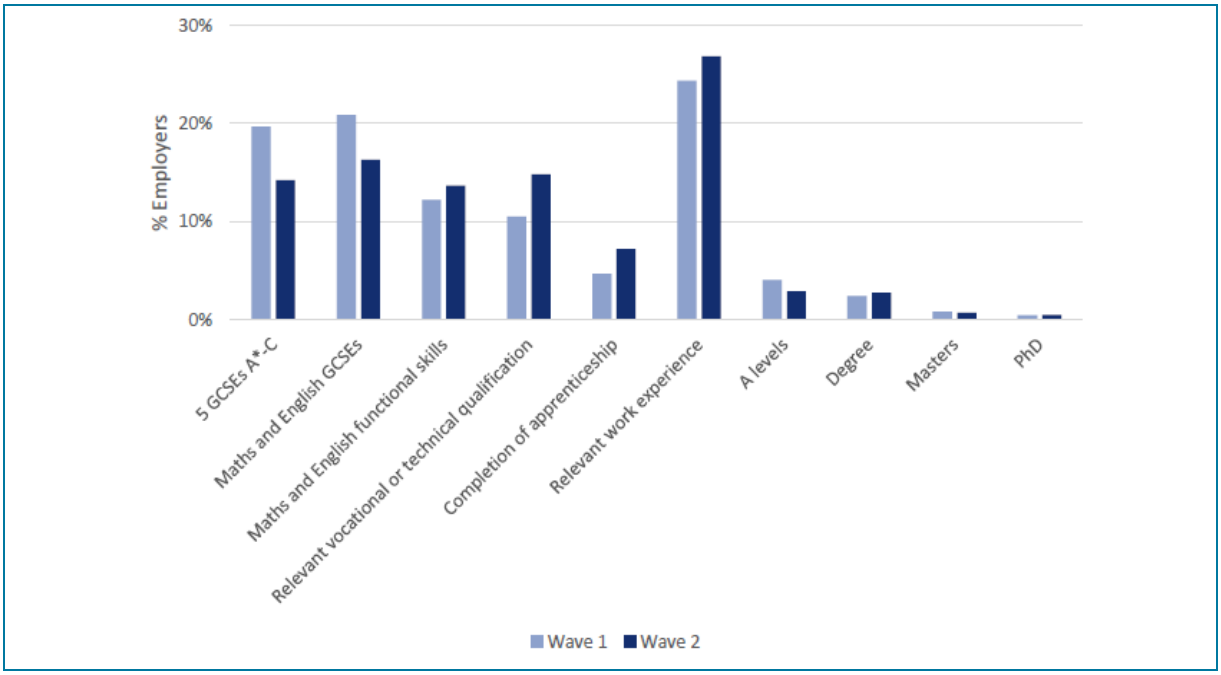
Why is the list of excluded topics longer than the list of extra topics?

The emphasis has been on constructing a curriculum which is appropriate for the target students and allows suitable progression for those who are successful. Some topics are more important than others; this is not always reflected in the number of words needed to describe them. Different topics take different amounts of time to teach, and it will take time to teach some of the new content. There is no intention to make the course equal in volume to current foundation tier GCSE. However, there is certainly enough content for a GCSE full-course qualification.

Does the reformed Functional Skills Mathematics level 2 remove the need for a new curriculum?

Functional Skills Mathematics has been reformed; there is a lot of commonality of content and intention between level 2 Functional Skills Mathematics and the curriculum we are designing. It may be that Functional Skills Mathematics qualifications provide suitable assurance of students' proficiency in mathematics. However, our (limited) survey of employers suggests that GCSE is more widely recognised. The much larger survey conducted by Pye Tait[94] shows that a greater proportion of employers consider Mathematics and English GCSE to be essential when recruiting for entry and admin roles, compared to those who consider Functional Skills Maths and English essential, as shown in the chart from their report, reproduced below.

However, the proportion which recognise Functional Skills as essential is greater in Wave 2 than in the earlier Wave 1 survey.



Responses to the question, 'When recruiting new employees, are any of the following essential for entry and admin roles?' in Wave 2 (2018/19, base 3130) compared with Wave 1 (2017/18, base 2750); reproduced from Pye Tait (2019)

[94] Pye Tait Consulting (2019). *Perceptions of Vocational and Technical Qualifications: Wave 2*

Should the grading structure be the same as GCSE?

As a foundation tier GCSE, the grades will be 5 to 1. It is important for employer (and other stakeholder) recognition that all GCSEs have the same grading structure.

Should vocational skills for particular vocations be added?

The EEF *Embedding contextualisation in English and mathematics GCSE teaching* pilot report[95] found that students tend to respond better to real-life, rather than vocational, contextualisation. Real-life contexts also have wider applications than contexts related to particular vocations so it seems more appropriate to focus on them in a general qualification.

Should Paper 1 be online, multiple choice?

Having the paper multiple choice and online would allow for prompt feedback to centres about student performance so that teaching can be tailored to what students need to learn. It would also allow for questions with an element of interactive IT to be developed. However, the manageability of this for centres needs to be considered. It might be feasible to have each of the two sittings in a one-week window to help centres overcome any practical difficulties they might have with conducting large-scale online assessment.

Multiple-choice questions allow for machine marking and identification of student misconceptions. It would be possible to have Paper 1 as a written paper with quick feedback to centres. It would also be possible to have an online bank of questions like those on Paper 1, which centres could use for diagnostic assessment early in the course and as exam preparation for Paper 1. Paper 1 could then be paper based and available on two specific dates each year.

It would be interesting to investigate whether students find it less intimidating to see one question at a time on screen or whether having the whole paper to work through is more accessible.

Are the exemplar papers too easy?

The comparative judgement concludes that the papers are *'not significantly different in difficulty to previous GCSE Foundation papers'*, though it suggests they are a little easier than the foundation tier GCSE (9 to 1) papers from June 2017; this is discussed in section 4.4 of this report.

[95] Runge, J., Munro-Lott, N., & Buzzeo, J. (2019). *Embedding contextualisation in English and mathematics GCSE teaching: Pilot report*. Education Endowment Foundation.

6 Conclusions, recommendations and implementation

6.1 Conclusions

We believe we have met the six aims for the curriculum which were set out in section 1.3. The following table shows how the aims have been met.

Aim	Comment
1. Has content relevant to the practical application of mathematics	The review of similar qualifications we have undertaken, together with the development process of starting with a consideration of the contexts in which adults should be able to use mathematics and the associated mathematical content, provides assurance that aims 1 to 3 are met. The contexts used are related to everyday life and so are relevant to most young people.
2. Focuses on the acquisition of key mathematical skills and their application in meaningful contexts	
3. Wherever practical, supports learning in contexts young people can relate to	
4. Has sufficient rigour to meet the requirements of a GCSE qualification. It is intended that the new post-16 mathematics GCSE would have a maximum grade of 5, and would be designed so that students achieving grades 4 or 5 (a level 2 pass) would have to demonstrate a good understanding of the fundamental mathematics needed to function as effective citizens	The comparative judgement work undertaken by No More Marking,[96] concludes that the exemplar papers are '*not significantly different in difficulty to previous GCSE Foundation papers'.* This suggests aim 4 has been met. The relative merits of making the papers harder and making grade boundaries higher are discussed in section 4.4.
5. Prepares young people for further programmes of study, including Core Maths and vocational courses, that do not require prior learning of abstract mathematics	Aims 5 and 6 have also been met. The content of the proposed curriculum is suitable for progression to Core Maths and a variety of vocational courses and will be suitable as a support for T level programmes. It will also be suitable for mature learners who wish to gain a mathematical foundation as a prelude to further learning at level 3 and beyond.
6. Is suitable for the government to adopt as a means for young people to achieve the level 2 maths requirement of 16–19 study programmes and T level programmes	

There is widespread concern that the GCSE Mathematics qualification is not fit for purpose for the majority of those who do not achieve a level 2 pass in GCSE Mathematics by the age of 16. Almost 180,000 students resat GCSE Mathematics in summer 2019 but only 22.3% achieved a level 2 pass. Those who do not achieve a level 2 pass by age 18 are seriously disadvantaged in the employment market.

This exploratory work by MEI has shown that a potential new mathematics GCSE could be developed for post-16 students, which focuses on the mathematics needed for everyday life and work. The dominance of the GCSE brand is very well established, and it seems unlikely that the reformed Functional Skills Mathematics will provide the credibility needed for employers and HEIs to change their selection criteria, hence the need for a new GCSE

[96] Barmby, P., & Wheadon, C. (2019). *A comparative judgement study of MEI GCSE exam items*. No More Marking Ltd.

qualification. The dominance of the GCSE brand was confirmed by findings in an article in *FE Week* in November 2019:

> *Colleges claimed they did not want 'to limit our learners' progress' and highlighted 'the strong emphasis that employers and education establishments put on GCSEs' as part of their reasoning.*
>
> *Criticism was also directed towards the alternative Functional Skills qualification for its 'cliff-edge pass or fail'.*[97]

The small-scale feasibility testing we have done shows that the curriculum which we have developed is acceptable to teachers and employers and so could form the basis of an alternative mathematics GCSE for post-16 students.

6.2 Recommendations

It is important that students who have not succeeded in gaining a level 2 pass at GCSE by age 16 are able to gain a credible qualification that has currency with employers and HEIs. It is also important that they are enabled to use mathematics in a range of contexts which they are likely to encounter in everyday life.

Many students who have failed to gain a level 2 mathematics qualification by age 16 continue to fail to do so post-16; allowing Functional Skills Mathematics level 2 as an alternative to GCSE Mathematics will not solve this problem. Our current national policy for post-16 mathematics education at level 2 is failing to meet students' needs and turning many against mathematics for life, which also has a negative impact on our culture. Based on our findings when undertaking this project, and mindful of the needs of students, we have the following main recommendation.

Main recommendation

A new mathematics GCSE should be developed for post-16 students that focuses on the maths needed for everyday life and work. It should be clearly branded as a GCSE qualification and be afforded equal status to a GCSE Mathematics qualification at the same grade, both for progress measures and for entry to employment or higher education.

The findings set out in this report should be considered by the DfE, Ofqual and the awarding bodies to inform their development of the new GCSE; this may require changes to regulation or to usual practices at GCSE.

The new mathematics GCSE should have the following features:

- It should be limited to foundation tier (grades 1 to 5) and available to post-16 students only.
- It should have higher grade boundaries than the current foundation tier GCSE Mathematics to ensure that students are able to demonstrate the ability to succeed in the mathematics they are likely to encounter in life and work, rather than by merely achieving a minimal number of marks.
- It should incorporate a stepping-stone assessment element that can be taken before the final assessment, to test basic skills and provide a more supportive pathway for students who have experienced limited success with mathematics. This stepping stone should

[97] Mersinoglu, Y.C. 'Why colleges are choosing GCSE resits over alternatives', *FE Week*, 15 November 2019

attract points in the maths progress measure. The whole GCSE should have the same progress measure points as GCSE Mathematics.

- It should exclude content from foundation tier GCSE Mathematics that is not directly relevant to everyday applications but should include a small amount of additional content which is not included in foundation tier GCSE Mathematics, such as risk, financial applications and the basic use of spreadsheets to perform mathematical tasks. This aligns with the skills students need in daily life and is in line with feedback received from employers.

In order to make a success of an alternative GCSE, employers and HEIs would need to understand its purpose so that parents, teachers, schools and colleges could be confident in choosing it as an option for students. Section 6.3 summarises the key issues that the DfE, Ofqual and wider stakeholders would want to work through/towards to maximise the success of the new GCSE, and so improve mathematics learning, recognised by success in appropriate qualifications, for young people.

6.3 Next steps and implementation

This section outlines the steps that different groups would need to take in order to successfully implement the recommendations in section 6.2.

The DfE and Ofqual

- The outline curriculum content and proposed assessment structure in this report should be used as the basis for a post-16 GCSE, limited to foundation tier.
- Development should be overseen by the DfE and Ofqual, as usual for GCSE qualifications, but with changes to regulations where needed. These would include enabling:
    - the qualification to be for post-16 students only
    - the first paper to be sat early and count towards the full GCSE.
- The title of the GCSE should make it clear that it covers the mathematics needed for adults to possess the quantitative skills needed in daily life. Possible titles include GCSE Core Maths, GCSE Numeracy.
- The content of the GCSE should be based on the outline curriculum and contain sufficient detail to enable teachers to know what they need to teach (and what is not included).
- The contexts for assessment questions should be drawn from those included in the final subject content.
- The feasibility of online assessment should be explored.
- Suitable teaching materials and training for teachers should be made available prior to the start of teaching of the new qualification. This should include training on using spreadsheets and working in realistic contexts.
- Progress measure points should be assigned to Paper 1 so that the progress which students have made by succeeding in this stepping stone is recognised. The whole GCSE should have the same progress measure points as GCSE Mathematics.

- Consideration should be given to supporting the setting of grade boundaries through statistical evidence which allows for the possibility that students might improve over time.

Teachers

- Decisions about which GCSE to enter students for should be taken after considering students' prior performance and future ambitions. Students should be considered on a case-by-case basis, but the following broad guidelines may be helpful:
    - Students who have narrowly missed getting grade 4 in GCSE Mathematics are likely to be best served by entering for the same GCSE again. Students who have a more limited understanding of the current GCSE content are likely to be better served by entering for the proposed qualification which has a smaller curriculum.
    - Students who are aiming at grades above 4 are likely to be better served by GCSE Mathematics. However, those who have a lot of progress to make to achieve their ambitions might aim at the new qualification, at least as far as Paper 1.
- Diagnostic information from Paper 1 of the proposed qualification should be used to inform teaching.

Employers

- Employers should recognise the new qualification as an equivalent to GCSE Mathematics where that is a requirement for particular roles.
- Employers should support employees to improve their quantitative skills, including through taking appropriate qualifications.

Higher education institutions

- HEIs should recognise the new qualification as an equivalent to GCSE Mathematics where that is a requirement for entry to courses where algebra is not a pre-requisite. This would be particularly helpful for mature students who have not taken a traditional route to university.
- HEIs should support students to improve their quantitative skills.

Appendix 1 Sample teaching resources

Three exemplar resources are published alongside this report. They are briefly described in section 3.2 above.

- Minimum wage
- Ratio
- Working with data

Appendix 2 Draft curriculum

The draft curriculum needs further refinement before it would be ready to form the basis of a new qualification. It is offered here as a work in progress. It is also published alongside this report with additional appendices showing how it relates to GCSE Mathematics.

The draft curriculum content has been informed by the content of the qualifications reviewed (see section 4.2) and by our own judgement of the content that is important for adults to be able to successfully navigate the mathematical demands of everyday life and work. Generally speaking, content from current foundation tier GCSE Mathematics that has been included in the draft curriculum is closely related to content included in many of the qualifications we reviewed. The only exceptions are the following Geometry and Measures GCSE content statements,[98] which are included in many of the reviewed qualifications but not in the draft curriculum:

2. use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line
3. apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)

These were omitted because, following discussions at the first Advisory Group meeting, it was felt that this content was not very useful in terms of applications to life or work at this level.

Purpose

This document is part of an exploration of the feasibility of a new mathematics GCSE curriculum which is appropriate to the needs of post-16 students who have already sat GCSE Mathematics and have not achieved at least a grade 4 (a level 2 pass).

Students who have narrowly missed achieving grade 4 and who wish to pursue more mathematical or academic pathways may be well served by resitting GCSE Mathematics. However, many students either resit GCSE Mathematics repeatedly without improving on their KS4 grade, or manage to achieve a pass grade by cramming for the examination, without mastering fundamental mathematical skills and concepts.

It is intended that the new post-16 mathematics GCSE would have a maximum grade of 5, and would be designed so that students achieving grades 4 or 5 (a level 2 pass) would have demonstrated a good understanding of the fundamental mathematics needed to function as effective citizens.

It is not intended to create an ‘easier’ mathematics GCSE: MEI has a strong reputation for academic rigour in its qualification development. Our intention is to create a mathematics GCSE that is more appropriate for the target students, allowing them to gain valuable skills in applying mathematics and supporting them to develop a positive attitude towards the

[98] Ofqual (2017). *GCSE Subject Level Conditions and Requirements for Mathematics*

subject. This would motivate more students to succeed and would help to improve the public perception of mathematics.

Practical considerations are important. GCSE resit students have two terms post 16 before examinations; they are often taught by teachers with limited confidence in mathematics so there will be a need for professional development. The wide range of teachers in FE colleges may well find that it is easier to convince students to learn content which is more directly applicable.

Students may not want to take the course, and there are very large numbers of students in some centres. It is not possible to introduce much new content in the limited time, given that students lack confidence with the content which they have already been taught.

Aims

Students who succeed on the course should:

- develop a positive, confident attitude to mathematics
- be able to use mathematical skills in contexts encountered in work, study and life where specialised mathematical skills are not required
- be prepared for further study that does not require prior learning of abstract mathematics, including Core Maths and vocational courses.

To achieve this, students will need to have fluency with key mathematical skills. Students will have mastered some of these skills before starting on this course but they will have gaps in their knowledge and understanding. GCSE Core Maths has been suggested as a possible name for a qualification based on this curriculum.

Exemplar contexts

The contexts listed in the following table exemplify what students should be able to do; they are not intended to form an exhaustive list. They are based on the Essentials of Numeracy from National Numeracy,[99] the New Zealand Learning Progressions for Adult Numeracy[100] and the 'General life and personal interest' section of the foundation tier GCSE context grid.[101]

The purpose of this section is to exemplify what students who have successfully completed a course based on this curriculum should be able to do, and so provide a basis for determining what content is appropriate to support competences like these. The content of the curriculum has been organised to show which content supports which types of context. It is intended that this contextualised organisation of content will support teaching and facilitate making links between mathematics teaching and other teaching, including vocational teaching.

[99] www.nationalnumeracy.org.uk/essentials-numeracy
[100] ako.ac.nz/knowledge-centre/learning-progressions-for-adult-numeracy/
[101] MEI (2017). *Contextualisation Toolkit*

Contexts from everyday life have been chosen rather than vocational contexts; these will be useful for a wider range of students and fairer as most students will be familiar with such contexts because they do not rely on specialist knowledge. Moreover, the EEF *Embedding contextualisation in English and mathematics GCSE teaching* pilot report[102] found that students tend to respond better to real-life, rather than vocational, contextualisation.

Financial understanding	**Working with measures and shape**
• Understanding discounts in the sales • Understanding household bills • Estimating the cost of weekly food shopping • Splitting a restaurant bill • Shopping around for the best mobile phone deal • Comparing prices for differently sized packages • Budgeting for a holiday or major purchase • Personal budgeting • Managing a budget at work • Understanding interest rates when saving and borrowing	• Being able to read a measuring scale • Knowing your height and weight • Converting between imperial and metric units • Buying enough paint to decorate a room • Using shapes in designing a garden or craft project • Making and interpreting measurements to decide whether a piece of furniture or household appliance will fit in a given space • Understanding a map or scale drawing • Understanding measurements relating to personal fitness and health • Giving the right quantity of medicine to children
Planning activities	**Understanding quantitative information**
• Estimating time needed for tasks • Planning a schedule • Understanding staff shifts on a rota • Planning a meal or party for a large number of people • Giving and following directions • Understanding journey times • Understanding a map or scale drawing • Understanding timetables	• Recording numerical information accurately so others can understand • Making sense of statistics in the news • Interpreting the results of an opinion poll and understanding why different polls may produce different results • Understanding results of elections • Understanding food labels • Understanding statistics relating to personal fitness and health • Understanding risk in the news in relation to health

[102] Runge, J., Munro-Lott, N., & Buzzeo, J. (2019). *Embedding contextualisation in English and mathematics GCSE teaching: Pilot report*. Education Endowment Foundation.

Progression

Students aged over 16 may be taking other courses at the same time as, or soon after, seeking to improve on their GCSE Mathematics grades.

Core Maths

For Core Maths courses, the content of foundation tier GCSE Mathematics is assumed knowledge:

> *It is assumed that students will already have confidence and competence in the content presented in standard and underlined type within the GCSE mathematics subject content. Students will make use of elements of this content when addressing problems within Core Maths but we do not expect these to be explicitly set out in qualification content.*[103]

In practice, most of the content of most Core Maths qualifications builds on GCSE foundation tier in the following subject areas:

- Number
- Ratio, proportion and rates of change
- Probability
- Statistics

Students who have confidence and competence in these areas of foundation tier GCSE Mathematics have the background needed to be able to progress to doing Core Maths.

T levels

At the time of writing, T levels are being developed. The Royal Society's Advisory Committee on Mathematics Education (ACME) has proposed a framework of General Mathematical Competences (GMCs) to inform the development of mathematical content and assessment in T levels:[104]

- Measuring with precision
- Estimating, calculating and error spotting
- Working with proportion
- Using rules and formulae
- Processing data
- Understanding data and risk
- Interpreting and representing with mathematical diagrams
- Communicating using mathematics
- Costing a project
- Optimising work processes

The content outline that follows supports the development of these GMCs.

[103] DfE (2018). *Core maths qualifications: technical guidance*
[104] The Royal Society (2019). *Mathematics for the T level Qualifications: a rationale for General Mathematical Competences (GMCs)*

Subject content: Financial understanding

Exemplar contexts

- Understanding discounts in the sales
- Understanding household bills
- Estimating the cost of weekly food shopping
- Splitting a restaurant bill
- Shopping around for the best mobile phone deal
- Comparing prices for differently sized packages
- Budgeting for a holiday or major purchase
- Personal budgeting
- Managing a budget at work
- Understanding interest rates when saving and borrowing

Example learning outcomes and supporting content from GSCE Mathematics[105]

Number: Structure and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Order amounts of money in £ and/or pence • Interpret and order negative numbers in the context of a budget; recognise whether a higher value is better or worse	1. order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
• Calculate the total amount to be paid (without a calculator in suitable context) • Estimate the total to be paid • Divide a bill equally (with or without adding a tip) • Recognise whether a profit or loss has been made • Use a spreadsheet to set up and manage a budget	2. apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
• Simplify working when calculating without a calculator • Interpret and construct a simple spreadsheet formula, knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these)	3. recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions; use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
• List possible options when making a financial decision • Decide which option is best, considering total cost and practical considerations such as spreading cost and avoiding waste	5. apply systematic listing strategies

[105] Ofqual (2017). *GCSE Subject Level Conditions and Requirements for Mathematics*

Example learning outcomes	Supporting content from GCSE Mathematics
• Use multipliers when working with interest rates	6. use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
• Interpret amounts when given in standard form, including understanding notation on a calculator or spreadsheet (converting a number into standard form is not required nor is calculating with numbers given in standard form)	9. interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer

Number: Fractions, decimals and percentages

Example learning outcomes	Supporting content from GCSE Mathematics
• Use a multiplier when working with percentages	12. interpret fractions and percentages as operators

Number: Measures and accuracy

Example learning outcomes	Supporting content from GCSE Mathematics
• Use and interpret decimals in the context of money • Round to the nearest pound or penny when appropriate	13. use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
• Estimate the total spend for a bill • Estimate in UK currency the cost of an item priced in a foreign currency	14. estimate answers; check calculations using approximation and estimation, including answers obtained using technology
• Round money to the nearest pound or penny, as appropriate • Know the interval that a rounded number of pounds could lie between	15. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding

Algebra: Notation, vocabulary and manipulation

Example learning outcomes	Supporting content from GCSE Mathematics
• Understand and use the equation of a straight-line graph	1. use and interpret algebraic notation, including: • ab in place of $a \times b$ • $3y$ in place of $y + y + y$ and $3 \times y$ • a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$ • $\frac{a}{b}$ in place of $a \div b$ • coefficients written as fractions rather than as decimals • brackets
• Use formulae for working out costs	2. substitute numerical values into formulae and expressions, including scientific formulae

Algebra: Graphs

Example learning outcomes	Supporting content from GCSE Mathematics
• Plot points for a time-series graph of cost or expenditure	8. work with coordinates in all four quadrants
• Use a straight-line graph for situations where there is a fixed charge plus a charge per unit • Draw and use a conversion graph • Find an equation for a straight-line graph showing cost	9. plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel lines; find the equation of the line through two given points, or through one point with a given gradient
• Interpret gradient and intercept of a straight-line graph in the context of cost or currency conversion	10. identify and interpret gradients and intercepts of linear functions graphically and algebraically
• Interpret graphs of cost against time and recognise the gradient as a rate of increase	14. plot and interpret graphs (including reciprocal graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

Algebra: Solving equations and inequalities

Example learning outcomes	Supporting content from GCSE Mathematics
• Use a graph to find how much you could afford in a situation where there is a fixed charge plus a charge per unit	17. find approximate solutions to linear equations in one unknown using a graph
• Write a formula for a situation where there is a fixed charge plus a charge per unit	21. translate simple situations or procedures into algebraic expressions or formulae

Ratio, proportion and rates of change

Example learning outcomes	Supporting content from GCSE Mathematics
• Convert between standard units of time	1. change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices,) in numerical contexts
• Express one cost as a fraction of another cost	3. express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
• Express two or three amounts of money as a ratio	4. use ratio notation, including reduction to simplest form
• Divide a sum of money into two or three parts in a given ratio • Solve ‘best buy’ problems	5. divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)

Example learning outcomes	Supporting content from GCSE Mathematics
• Find a multiplier when working with percentage increase or decrease	6. express a multiplicative relationship between two quantities as a ratio or a fraction
• Be able to use scale factors when working with rates of conversion or rates of pay, e.g. working for half a day gets half as much as working for a full day	7. understand and use proportion as equality of ratios
• Find percentage increase or decrease in financial contexts including profit and loss • Find a new value after a percentage increase or decrease • Find the original value before a percentage change, e.g. calculate the amount without VAT when given the amount including VAT	9. define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics
• Draw and use conversion graphs for currency exchange • Calculate amounts of money in the context of currency exchange	10. solve problems involving direct and inverse proportion, including graphical and algebraic representations
• Use rates of pay to find pay for different lengths of time, e.g. week, year • Use an appropriate inflation rate to estimate prices at a later time	11. use compound units such as speed, rates of pay, unit pricing
• Interpret gradient and intercept of a straight-line graph in the context of cost or currency conversion • Know that a graph for direct proportion is a straight line through the origin	14. interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct proportion
• Use a multiplier to calculate compound interest for loans and saving • Use average inflation rates	16. set up, solve and interpret the answers in growth and decay problems, including compound interest

Statistics

Example learning outcomes	Supporting content from GCSE Mathematics
• Use statistical diagrams in financial contexts	2. interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use

Additional content

Financial applications
Use mathematics in the context of personal, domestic and simple business finance including loan repayments and choosing the best loan to fit with personal circumstances, budgeting, VAT, understanding the idea of a measure of inflation; be able to calculate using exchange rates; be able to calculate income tax – includes understanding terms such as personal allowance and tax rate; be able to interpret a pay slip and use it in the context of budgeting
Be able to use and interpret the output of online calculators in the context of finance, e.g. for loans, savings, taxation, choosing the best deal for electricity or gas
Use spreadsheets to model financial, statistical and other numerical situations, including using mathematical content from the rest of the specification and the following additional content and skills: • use and interpret simple spreadsheet formulae including sums, differences, percentage change, using cell references or absolute cell references, four functions and powers e.g. = C2 + D2, = C2*1.05^D2 knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these) • select, draw and format suitable graphs and charts using a spreadsheet • interpret output from a spreadsheet in the context of solving a problem or making a decision

Subject content: Working with measures and shape

Exemplar contexts

- Being able to read a measuring scale
- Knowing your height and weight
- Converting between imperial and metric units
- Buying enough paint to decorate a room
- Using shapes in designing a garden or craft project
- Making and interpreting measurements to decide whether a piece of furniture or household appliance will fit in a given space
- Understanding a map or scale drawing
- Understanding measurements relating to personal fitness and health
- Giving the right quantity of medicine to children

Example learning outcomes and supporting content from GSCE Mathematics

Number: Structure and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Order amounts (including when given in different units) • Read a thermometer and order temperatures • Use negative numbers in the context of temperature	1. order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
• Calculate a total distance, knowing that all distances in the calculation must be in the same units • Calculate an area when the sides are given in fractions of a mile (or other unit) • Calculate area of a rectangle when sides are given in decimals • Calculate a missing measurement for a shape with known area or when working out area	2. apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
• Simplify working when calculating without a calculator • Construct a simple spreadsheet formula, knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these)	3. recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions; use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

Example learning outcomes	Supporting content from GCSE Mathematics
• Use powers in units and formulae when working with area and volume	6. use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
• Use simple fractions when working with measurements	8. calculate exactly with simple fractions

Number: Fractions, decimals and percentages

Example learning outcomes	Supporting content from GCSE Mathematics
• Convert fractions of a mile, fractions of an inch etc. to decimals and vice versa	10. work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)
• Be able to calculate a fraction of a distance	12. interpret fractions and percentages as operators

Number: Measures and accuracy

Example learning outcomes	Supporting content from GCSE Mathematics
• Use, interpret and convert between standard units	13. use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
• Estimate lengths, areas and volumes	14. estimate answers; check calculations using approximation and estimation, including answers obtained using technology
• Round a measure to a given degree of accuracy • Know when it might be necessary to round up, e.g. when estimating how much paint is required • Use inequality notation to specify a simple error interval	15. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding
• When given rounded measurements, calculate the limits of possible area, volume or length • Recognise that rounded measurements may give a misleading impression of whether something will fit into a gap	16. apply and interpret limits of accuracy

Algebra: Notation, vocabulary and manipulation

Example learning outcomes	Supporting content from GCSE Mathematics
• Use standard algebraic notation in formulae for areas and volumes	1. use and interpret algebraic notation, including: • ab in place of $a \times b$ • $3y$ in place of $y + y + y$ and $3 \times y$ • a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$ • $\frac{a}{b}$ in place of $a \div b$ • coefficients written as fractions rather than as decimals • brackets
• Substitute numerical values into formulae for area, volume and speed • Use a formula to calculate the amount of medication needed for a child	2. substitute numerical values into formulae and expressions, including scientific formulae
• Understand and use standard mathematical formulae in the context of measures	5. understand and use standard mathematical formulae

Algebra: Graphs

Example learning outcomes	Supporting content from GCSE Mathematics
• Be able to plot points on a graph when solving a problem	8. work with coordinates in all four quadrants
• Draw and use a graph of length against width for a rectangle of given area when solving a problem	12. recognise, sketch and interpret graphs of linear functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$

Algebra: Solving equations and inequalities

Example learning outcomes	Supporting content from GCSE Mathematics
• Use a graph to convert from one unit to another or vice versa	17. find approximate solutions to linear equations in one unknown using a graph
• Write a formula for converting from one unit to another	21. translate simple situations or procedures into algebraic expressions or formulae

Ratio, proportion and rates of change

Example learning outcomes	Supporting content from GCSE Mathematics
• Change between different standard units	1. change freely between related standard units (e.g. time, length, area, volume/capacity, mass) in numerical contexts
• Express a length, area or volume as a fraction of another one • Calculate and interpret waist-to-hip ratio	3. express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1

Example learning outcomes	Supporting content from GCSE Mathematics
• Understand that e.g. a distance twice as long as another has the same relationship in any units	7. understand and use proportion as equality of ratios
• Be able to change between one distance being a fraction of another to the ratio between the distances and vice versa	8. relate ratios to fractions and to linear functions
• Work out a percentage increase or decrease in a measurement	9. define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics
• Use compound units informally (e.g. mg per ml) to compare strengths of similar medicines such as those designed for different ages of children and so understand that they have different concentrations	11. use compound units

Geometry and measures: Properties and constructions

Example learning outcomes	Supporting content from GCSE Mathematics
• Sketch and interpret diagrams for designs for garden or craft projects • Interpret vertical, parallel and perpendicular in 3D in a room or building • Use reflection and rotation symmetry to describe a design	1. use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description
• Know and use the properties of special types of quadrilaterals in the context of a craft or DIY project	4. derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language
• Sketch and interpret plans for simple buildings	13. construct and interpret plans of 3D shapes

Geometry and measures: Mensuration and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Convert between different units of length and mass • Use the correct units for area and volume • Know that capacity and volume can be measured in the same units • Know that a litre of water weighs 1 kg	14. use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)
• Measure lines and angles on a design • Know that a full turn is 360° and that a right angle is 90°	15. measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings
• Estimate the total wall area of a room given length, width and height, and use this to estimate the amount of paint required • Work out the capacity of a fish tank and calculate its total mass when filled with water	16. know and apply formulae to calculate: area of triangles, volume of cuboids and other right prisms (including cylinders)
• Work out the distance round a circular pond or flower bed • Work out the area of a circular table and estimate the amount of varnish needed to paint the top surface	17. know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes;
• Use Pythagoras' theorem to check whether a corner is a right angle	20. know and use the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$

Additional content

Financial applications	
Work out or estimate the cost of a craft or DIY project	Use mathematics in the context of personal, domestic and simple business finance including loan repayments, budgeting, VAT, understanding the idea of a measure of inflation; be able to calculate using exchange rates
Units of measurement	
Be able to read a measurement from a measuring scale and tell the time using both digital and analogue clocks; understand 24-hour clock and am and pm	
Work with commonly used units of measurement, including imperial units: mm, cm, m, km, litres, ml, cl, g, kg, pounds, stones, inches, miles, pints, gallons. Students should be able to convert between units in the metric system and know the following conversions relating to imperial units: • 14 lb = 1 stone • 8 pints = 1 gallon • 5 miles ≈ 8 km • 1 inch ≈ 2.5 cm • 1 lb ≈ 0.5 kg • 1 pint ≈ 0.5 litre • 1 gallon ≈ 4 litres	
Be able to work with more accurate (given) conversion factors and understand when it is appropriate to estimate	

Subject content: Planning activities

Exemplar contexts

- Estimating time needed for tasks
- Planning a schedule
- Understanding staff shifts on a rota
- Planning a meal or party for a large number of people
- Giving and following directions
- Understanding journey times
- Understanding a map or scale drawing
- Understanding timetables

Example learning outcomes and supporting content from GSCE Mathematics

Number: Structure and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Order times (both time of day and intervals); use the symbols =, ≠,<, >, ≤, ≥ • Order amounts given in different units	1. order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
• Calculate the total time taken for a task • Calculate the length of time a journey takes • Calculate the total cost of running an event • Calculate the cost per mile of a journey • Work out speed, distance or time given the other two quantities	2. apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
• Simplify working when calculating without a calculator • Construct a simple spreadsheet formula, knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these)	3. recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions; use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
• List possible options when making a plan • List the possible combinations of meals of 2 or 3 courses from a simple menu	5. apply systematic listing strategies
• Calculate with fractions of hours	8. calculate exactly with simple fractions

Number: Fractions, decimals and percentages

Example learning outcomes	Supporting content from GCSE Mathematics
• Convert fractions of an hour to decimal parts of an hour and vice versa, including when working out speed	10. work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)

Number: Measures and accuracy

Example learning outcomes	Supporting content from GCSE Mathematics
• Interpret measurements on a map or scale drawing • Use standard units for measurements when using an online calculator • Work with speed, distance and time • Work with compound units such as miles per gallon, litres per mile when working out the cost of a journey	13. use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
• Estimate the length of time needed for a journey • Estimate the cost of a party or other event	14. estimate answers; check calculations using approximation and estimation, including answers obtained using technology
• Round the amount of food needed for a large party to help with shopping • Express the time needed for a journey as an error interval • Compare an answer from a rule of thumb to an exact answer	15. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding

Algebra: Notation, vocabulary and manipulation

Example learning outcomes	Supporting content from GCSE Mathematics
• Use a formula to estimate the length of time needed for a journey	2. substitute numerical values into formulae and expressions, including scientific formulae
• Know that speed = distance/time	5. understand and use standard mathematical formulae

Algebra: Graphs

Example learning outcomes	Supporting content from GCSE Mathematics
• Use references (e.g. A4) to find a street on a map	8. work with coordinates in all four quadrants
• Sketch a graph of distance against time when travelling at constant speed • Sketch a graph of time against speed for travelling a fixed distance	12. recognise, sketch and interpret graphs of linear functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$

Example learning outcomes	Supporting content from GCSE Mathematics
• Use and interpret travel graphs and use them to solve problems	14. plot and interpret graphs (including reciprocal graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

Algebra: Solving equations and inequalities

Example learning outcomes	Supporting content from GCSE Mathematics
• Make a spreadsheet to calculate the cost of a large party	21. translate simple situations or procedures into algebraic expressions or formulae

Ratio, proportion and rates of change

Example learning outcomes	Supporting content from GCSE Mathematics
• Find a runner's speed in km per hour or miles per hour	1. change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices) in numerical contexts
• Plan a journey using a map, using the scale to work out distance • Interpret scale drawings • Make a simple scale drawing	2. use scale factors, scale diagrams and maps
• Express a scale on a scale drawing or model as a fraction	3. express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
• Use and interpret ratio for a map scale	4. use ratio notation, including reduction to simplest form
• Use a scale factor to find a distance either in reality or on the map/diagram	5. divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
• Express and use a scale factor for a map or scale drawing as either a fraction or a ratio	6. express a multiplicative relationship between two quantities as a ratio or a fraction
• Understand that ratios of distances in real life are the same in a scale drawing or map	7. understand and use proportion as equality of ratios
• Be able to change between one distance being a fraction of another to the ratio between the distances and vice versa	8. relate ratios to fractions and to linear functions

Example learning outcomes	Supporting content from GCSE Mathematics
• Estimate the time taken for a task when different numbers of staff are involved • Find the cost of an event for different numbers of people • Scale up the total time for a task from information from a sample/pilot • Rewrite a recipe to make it for a different number of people	10. solve problems involving direct and inverse proportion, including graphical and algebraic representations
• Work out average speed for a journey using an internet planner which gives distance and time	11. use compound units such as speed, rates of pay, unit pricing
• Interpret the gradient of a distance–time graph as speed	14. interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct proportion

Geometry and measures: Properties and constructions

Example learning outcomes	Supporting content from GCSE Mathematics
• Describe a scale drawing using correct terminology	1. use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description
• Recognise special quadrilaterals in scale drawings, plans and diagrams and use their properties when interpreting the scale drawing, plan or diagram	4. derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language
• Draw and interpret plans drawn to scale	13. construct and interpret plans of 3D shapes

Geometry and measures: Mensuration and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Work out the total time taken for a project	14. use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)
• Measure a bearing from a map or scale drawing • Interpret a scale drawing • Measure a route on a map	15. measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings
• Convert measurements from a scale drawing or map to work out areas	16. know and apply formulae to calculate: area of triangles, volume of cuboids and other right prisms (including cylinders)

Example learning outcomes	Supporting content from GCSE Mathematics
• Use formula for area or circumference of circle when making calculations from a scale drawing or map	17. know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes;

Additional content

Financial applications	
• Set up and interpret a spreadsheet to plan an event	Use spreadsheets to model financial, statistical and other numerical situations, including using mathematical content from the rest of the specification and the following additional content and skills: • use and interpret simple spreadsheet formulae including sums, differences, percentage change, using cell references or absolute cell references, four functions and powers e.g. = C2 + D2, = C2*1.05^D2 knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these) • select, draw and format suitable graphs and charts using a spreadsheet • interpret output from a spreadsheet in the context of solving a problem or making a decision
Units of measurement	
• Find the time that a task or journey is expected to end	Be able to read a measurement from a measuring scale and tell the time using both digital and analogue clocks; understand 24-hour clock and am and pm
• Use imperial or metric units of length in scale drawings and maps	Work with commonly used units of measurement, including imperial units: mm, cm, m, km, litres, ml, cl, g, kg, pounds, stones, inches, miles, pints, gallons. Students should be able to convert between units in the metric system and know the following conversions relating to imperial units: • 14 lb = 1 stone • 8 pints = 1 gallon • 5 miles ≈ 8 km • 1 inch ≈ 2.5 cm • 1 lb ≈ 0.5 kg • 1 pint ≈ 0.5 litre • 1 gallon ≈ 4 litres
• Convert between imperial and metric units of length when planning a journey	Be able to work with more accurate (given) conversion factors and understand when it is appropriate to estimate
• Use the output of an online map to work out average speed • Compare the distance given by an online map with the direct distance	Be able to interpret the output of an online map in the context of planning a journey

Subject content: Understanding quantitative information

Exemplar contexts

- Recording numerical information accurately so others can understand
- Making sense of statistics in the news
- Interpreting the results of an opinion poll and understanding why different polls may produce different results
- Understanding results of elections
- Understanding food labels
- Understanding statistics relating to personal fitness and health
- Understanding risk in the news in relation to health

Example learning outcomes and supporting content from GSCE Mathematics

Number: Structure and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Order risks using either percentages or fractions • Order amounts of nutrients in different units	1. order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
• Calculate percentages from raw data • Use a probability expressed as a fraction, decimal or percentage to calculate an expected frequency	2. apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
• Simplify working when calculating without a calculator • Construct a simple spreadsheet formula, knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these)	3. recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions; use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
• Understand the use of an exponential model for bacterial growth	6. use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
• Calculate expected frequency given the risk	8. calculate exactly with simple fractions
• Interpret populations given in standard form	9. interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer

Number: Fractions, decimals and percentages

Example learning outcomes	Supporting content from GCSE Mathematics
• Convert a probability expressed as a fraction to a decimal or percentage and vice versa • Be able to convert between a fraction of the population and the corresponding percentage of the population	10. work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)
• Work out a percentage or fraction of a population	12. interpret fractions and percentages as operators

Number: Measures and accuracy

Example learning outcomes	Supporting content from GCSE Mathematics
• Interpret units on a statistical graph • Give the appropriate units when constructing a graph • Work with compound measures such as people per km^2 or per 1000 people	13. use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
• Compare different people's estimates of the number of people in a crowd from a photograph • Estimate a value from a statistical diagram when the scale will not allow accurate reading off	14. estimate answers; check calculations using approximation and estimation, including answers obtained using technology
• Round a mean calculated from rounded values	15. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding
• Express an error interval for a mean calculated from rounded values	16. apply and interpret limits of accuracy

Algebra: Notation, vocabulary and manipulation

Example learning outcomes	Supporting content from GCSE Mathematics
• Understand and use a trendline formula for a scatter diagram produced by software	1. use and interpret algebraic notation, including: • ab in place of $a \times b$ • $3y$ in place of $y + y + y$ and $3 \times y$ • a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$ • $\frac{a}{b}$ in place of $a \div b$ • coefficients written as fractions rather than as decimals • brackets
• Substitute values into a formula for a line of best fit	2. substitute numerical values into formulae and expressions, including scientific formulae

Algebra: Graphs

Example learning outcomes	Supporting content from GCSE Mathematics
• Plot points for a scatter diagram	8. work with coordinates in all four quadrants
• Find the equation of a line of best fit for a scatter diagram	9. plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel lines; find the equation of the line through two given points, or through one point with a given gradient
• Interpret the gradient and intercept for the line of best fit on a scatter diagram	10. identify and interpret gradients and intercepts of linear functions graphically and algebraically
• Recognise when a straight-line model is appropriate for a scatter diagram, and when it is not	12. recognise, sketch and interpret graphs of linear functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$

Algebra: Solving equations and inequalities

Example learning outcomes	Supporting content from GCSE Mathematics
• Use a line of best fit to estimate an unknown value	17. find approximate solutions to linear equations in one unknown using a graph

Ratio, proportion and rates of change

Example learning outcomes	Supporting content from GCSE Mathematics
• Use and interpret units on a statistical diagram	1. change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices,) in numerical contexts
• Find the fraction of the sample or population in a particular category • Find the fraction or percentage of daily recommended amount in a particular food	3. express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
• Use ratio to compare risks • Use ratio to compare the size of different groups in a sample or population	4. use ratio notation, including reduction to simplest form
• Use a ratio from a sample to estimate the number of people	5. divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
• Understand risk given as either a fraction or a ratio	6. express a multiplicative relationship between two quantities as a ratio or a fraction

Example learning outcomes	Supporting content from GCSE Mathematics
• Relate a fraction of a population or sample to a ratio between two parts of a population or sample	8. relate ratios to fractions and to linear functions
• Find the percentage of the sample or population in a particular category • Interpret probability as a percentage of the population	9. define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics
• Work out the number of calories or grams of a nutrient in a portion of food when given the amount in 100 g	10. solve problems involving direct and inverse proportion, including graphical and algebraic representations
• Use and interpret compound units in statistical diagrams	11. use compound units such as speed, rates of pay, unit pricing
• Interpret the gradient of a line of best fit in context including knowing that it gives the increase in the dependent variable for an increase of 1 unit in the independent variable	14. interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct proportion

Geometry and measures: Mensuration and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Use standard units in statistical diagrams • Interpret different units on food labels	14. use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)

Probability

Example learning outcomes	Supporting content from GCSE Mathematics
• Record and analyse the outcomes of a game involving probability • Analyse the outcomes of a simple clinical trial	1. record describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees
• Calculate the expected number of people with a particular condition in a randomly selected group of given size	2. apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments
• Use expected frequencies to find a probability	3. relate relative expected frequencies to theoretical probability, using appropriate language and the 0–1 probability scale

Example learning outcomes	Supporting content from GCSE Mathematics
• Given the lifetime probability of suffering from a particular illness, find the probability of not suffering from that illness • use probabilities summing to one when working with tree diagrams or two-way tables	4. apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
• Use spreadsheet simulations to estimate probabilities	5. understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
• Use frequencies to show the results of a clinical trial in a two-way table, Venn diagram or tree diagram • Use a systematic strategy to work out the number of ways to do something	6. enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams
• Find the probability that all three children in a family are boys • For a genetic illness with a given probability of transmission, find the probability that two children are free of the disease	7. construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities
• Use expected frequencies to work out the probability of a false positive in a medical test	9. **calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables**

Statistics

Example learning outcomes	Supporting content from GCSE Mathematics
• Given the results of an opinion poll or survey, estimate the number of people with each view in the population	1. infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
• Show the nutrients in a group of foods using an appropriate statistical diagram • Show results from a survey or a series of surveys using an appropriate statistical diagram or table • Interpret statistical diagrams on websites or in political leaflets	2. interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use
• Use calculator or spreadsheet statistical functions to calculate quartiles and know that they split the data into four groups of equal size • Compare earnings for male and female workers or house prices in different parts of the country • Have an informal understanding of outliers as unusual values which may have a large effect on the mean and range and understand that the median is not affected by outliers	4. interpret, analyse and compare the distributions of data sets from univariate empirical distributions through: • appropriate graphical representation involving discrete, continuous and grouped data, • appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers,)

Example learning outcomes	Supporting content from GCSE Mathematics
• Compare age distributions in different countries or towns	5. apply statistics to describe a population
• Plot a scatter diagram of match ticket prices v league table position for football teams and understand that neither causes the other even if there is correlation • Recognise that less scatter means a stronger relationship • Use a scatter diagram of a time series to make a prediction and understand that a prediction close to the times where data are known is more likely to be accurate	6. use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing

Additional content

Financial applications	
• Interpret a time series of exchange rates over time	Use mathematics in the context of personal, domestic and simple business finance, including loan repayments, budgeting, VAT, understanding the idea of a measure of inflation; be able to calculate using exchange rates
• Interpret statistical output from a spreadsheet • Choose a suitable statistical graph from a spreadsheet menu	Use spreadsheets to model financial, statistical and other numerical situations, including using mathematical content from the rest of the specification and the following additional content and skills: • use and interpret simple spreadsheet formulae including sums, differences, percentage change, using cell references or absolute cell references, four functions and powers e.g. = C2 + D2, =C2*1.05^D2 knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these) • select, draw and format suitable graphs and charts using a spreadsheet • interpret output from a spreadsheet in the context of solving a problem or making a decision
Risk	
• Be able to interpret a statement about risk in the form 1 in *n* and convert this form to and from a probability given as a fraction or decimal	
• Be able to use a frequency tree in the context of risk to find a probability	
• Be able to use information about conditional increase in risk to form a frequency tree using natural frequencies and calculate associated probabilities in simple cases	
• Be able to interpret relative and absolute risk	

Appendix 3 Exemplar examination papers

The versions of the papers which were used for the comparative judgement report are reproduced in this appendix together with their mark schemes.

Alternative Mathematics GCSE resit curriculum

Exemplar Paper 1

Time 1 hour

Calculator allowed

40 marks

Answer all questions

Note: This exemplar paper is intended to be sat online. It has been prepared by MEI as part of an exploration of an alternative to GCSE resit. It is not part of any recognised qualification.

1 How many grams in a kilogram?

A 10 **B** 100 **C** 1000 **D** 10 000

2 One morning the temperature was −5 °C.
At lunchtime, the temperature is 3 °C.

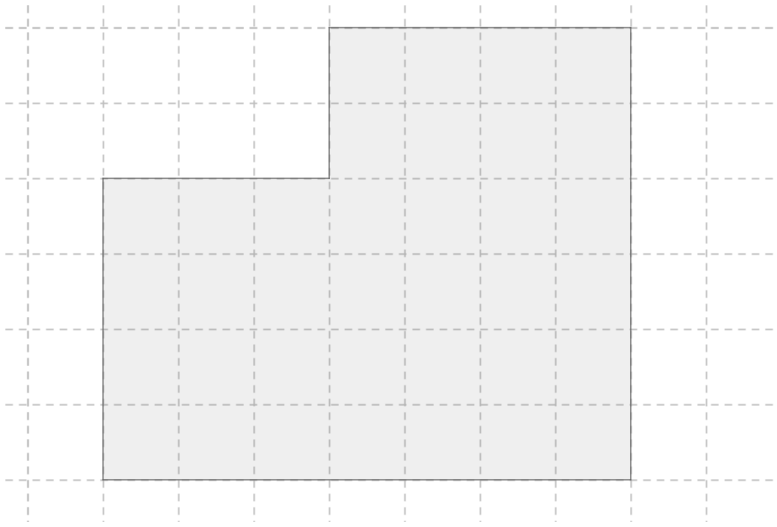
How many degrees has the temperature gone up by?

A 8 °C **B** 2 °C **C** −2 °C **D** −8 °C

3 $\frac{3}{4} - \frac{1}{2}$

A $\frac{2}{2}$ **B** $\frac{1}{4}$ **C** $\frac{2}{4}$ **D** $\frac{1}{0}$

4 The shape below is drawn on a 1 cm square grid.



What is the perimeter of the shape?

A 7 cm **B** 26 cm **C** 36 cm **D** 22 cm

5 Which of the digital clocks could be showing the same time as the clock below?

A 02:06	**B** 03:30
C 06:12	**D** 14:30

6 It takes 20 minutes to walk from Akim's home to the bus station.
He starts walking from home at 8.50 am.

What time does he arrive at the bus station?

A 8.30 am **B** 8.70 am **C** 9.10 am **D** 9.20 am

7 2.5 kg of potatoes cost £2.

How much does 1 kg of potatoes cost?

A 40p **B** 50p **C** 80p **D** £1

8 Which of the following is a million expressed in standard form?

A 10×10^6 **B** 1×10^5 **C** 1×10^4 **D** 1×10^6

9 £1 is worth 1.25 Euros.

What is 6 Euros in pounds?

A £7.50 **B** £5.75 **C** £4.80 **D** £4.08

10 Lee and Victor share £600 in the ratio 2 : 3.

How much does Lee get?

A £300 **B** £360 **C** £240 **D** £120

11 A 20p coin weighs 5.0 g
Wesley has been saving 20p coins.
His coins weigh 115.0 g.

How much money does Wesley have?

A £23 **B** £5.75 **C** £4.60 **D** £1.30

12 Which of the following measurements is the longest?

A 971 mm **B** 240 cm **C** 674.5 mm **D** 0.78 m

13 A café has the following safety rules for fridge and freezer temperatures.

Fridge	Between 0 °C and 5 °C
Freezer	−18 °C or lower

One day the fridge temperature is 0.1 °C and the freezer temperature is −15 °C.

Which of the following statements is true?

A Both temperatures are OK.
B The fridge is OK but the freezer is too cold.
C The fridge is OK but the freezer is too warm.
D Neither temperature is OK.

14 Round the number 0.012851 to 3 decimal places.

A 0.012 **B** 0.013 **C** 0.0128 **D** 0.0129

15 What is a 1 in 20 risk as a percentage?

A 0.05% **B** 5% **C** 19% **D** 20%

This information is for questions 16, 17, 18 and 19

Nadia's target is to walk at least 10 000 steps each day.
The numbers of steps for each day in one week are shown in the table below.

Mon	Tues	Weds	Thurs	Fri	Sat	Sun
8119	6156	9006	12 051	10 000	13 949	5910

16 On which day did Nadia walk the lowest number of steps?

A Monday **B** Wednesday **C** Friday **D** Sunday

17 On how many days did Nadia meet her target?

A 1 **B** 2 **C** 3 **D** 4

18 What is the median number of steps Nadia walks on a day that week?

A 9006 **B** 9313 **C** 10 000 **D** 12 051

19 How many **extra** steps did Nadia need to walk **altogether** that week to bring the mean number of steps per day up to 10 000?

A 687 **B** 4809 **C** 9313 **D** 70 000

20 A survey finds that 7.7% of young people are vegetarian.

How many vegetarians would you expect in a group of 650 young people?

A 8 **B** 50 **C** 84 **D** 85

21 Sadia's car has enough petrol to travel 50 miles.
She spends £25 on petrol and now has enough to travel 165 miles.

How much does petrol cost per mile for Sadia's car?

A £4.60 **B** £2.17 **C** 22p **D** 15p

This information is for questions 22 and 23

A survey asks drivers how many attempts it took to pass the driving test.
The results are shown in the table.

Attempts	Frequency
1	45
2	40
3	10
4	5

22 How many people replied to the survey?

A 4 **B** 25 **C** 45 **D** 100

23 What is the mean number of attempts?

A 1 **B** 1.75 **C** 2.5 **D** 25

24 A ready meal weighs 250 g.

What fraction of a kilogram is this?

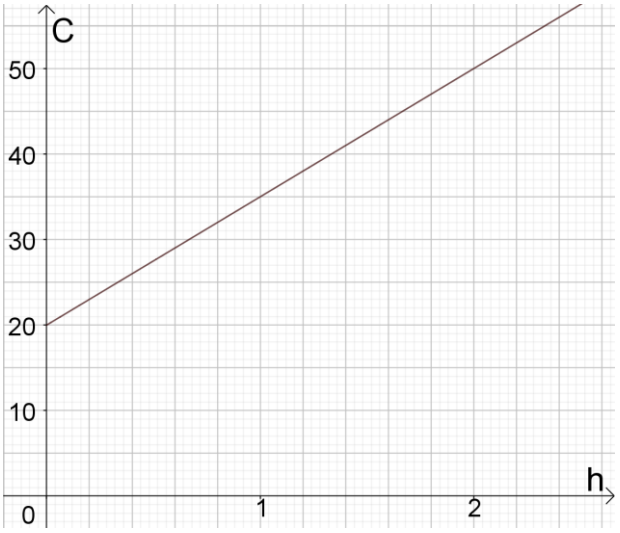
A $\frac{1}{4}$ **B** $\frac{1}{25}$ **C** $\frac{1}{40}$ **D** $\frac{1}{250}$

25 An internet journey planner estimates that a 60 mile car journey will take 1 hour 20 minutes.

What is the average speed for the journey?

A 40 mph **B** 45 mph **C** 50 mph **D** 60 mph

26 The graph below shows the cost, £C, for a person working h hours.



Which is the correct formula for the cost, £C?

A $C = 20h + 15$ **B** $C = 15h + 20$ **C** $C = 20 + h$ **D** $C = 20h$

27 £1000 is put into a bank account which pays 2.5% compound interest each year.

Which of the following is the way to calculate the total amount in the account after 2 years?

A $1000 \times 1.025 \times 2$ **B** 1000×1.025^2 **C** $1000 \times 2.5 \times 2$ **D** 1000×2.5^2

28 Janice buys a coffee on the way to work 5 days a week.
Each coffee costs £2.60**.**

Which of the following is the best estimate of how much she spends on coffee each month?

A £13 **B** £52 **C** £78 **D** £100

29 4 friends have a meal in a restaurant.
The total bill is £72.
They want to share the bill equally.
They want to pay roughly 10% extra to leave a tip.

How much should each person pay?

A £16 **B** £18 **C** £20 **D** £25

30 What do you multiply by to **reduce** a price by 30%?

A 0.3 **B** 0.7 **C** 1.3 **D** $\frac{1}{1.3}$

31 A flight takes off in London at 09:50, London time.
The flight lands in New York at 12:50, New York time.
The flight takes 8 hours.

What time is it in London when the flight lands?

A 04:50 **B** 12:50 **C** 14:50 **D** 17:50

32 The time that a good cyclist takes to travel any distance is about 75% of the time an average cyclist would take.
A good cyclist does a particular journey in 45 minutes.

Which is the best estimate of the time an average cyclist take to do the same journey?

A 34 minutes **B** 56 minutes **C** 1 hour **D** 70 minutes

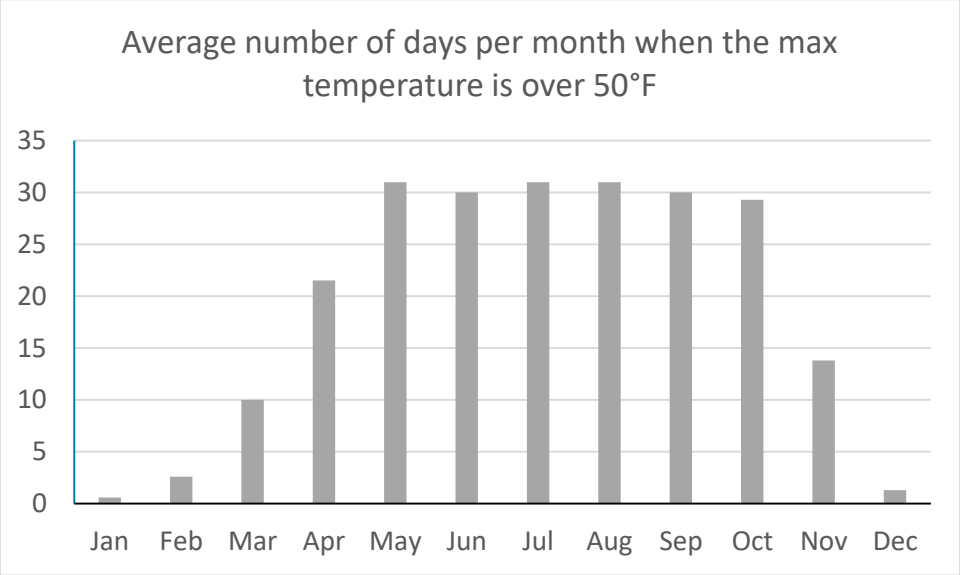
33 A restaurant has a special offer.
Customers choose a meal consisting of a starter, a main course and a pudding.
There are 3 starters, 5 main courses and 4 puddings on the menu.

How many possible meal choices are there?

A 4 **B** 5 **C** 12 **D** 60

The following information is for questions 34, 35 and 36

Temperatures in the USA are given in degrees Fahrenheit (°F).
The chart below shows the average number of days each month in Chicago when the maximum temperature is over 50 °F.



34 The formula for converting temperatures in °F to temperatures in °C is $C = \frac{5(F - 32)}{9}$.

Which of the following is the same as 50 °F?

A 10 °C **B** 24.2 °C **C** 81 °C **D** 246.4 °C

35 Which is the best estimate of the probability of a day in May in Chicago having a maximum temperature **below** 50°F?

A 0 **B** 50% **C** 31% **D** 100%

36 Which of the following is the best estimate of the number of days in a year when the maximum temperature in Chicago is above 50 °F?

A 12 **B** 180 **C** 230 **D** 365

37 A map has scale 1 : 10 000.

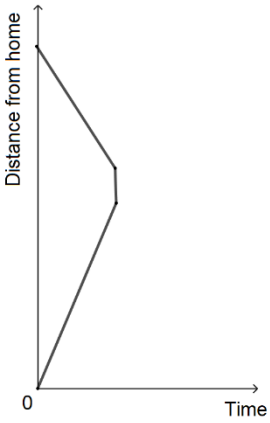
What distance is represented by 4 cm on the map?

A 4 m **B** 40 m **C** 400 m **D** 4 km

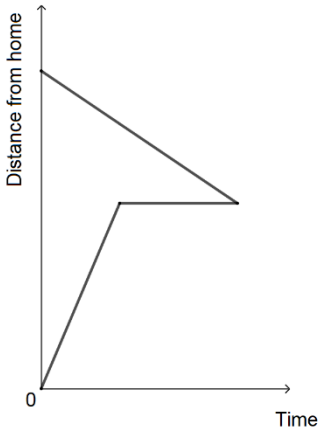
38 Bob walks to the shop at the end of his street.
He spends a few minutes in the shop.
Finally, he walks home slower than he walked to the shop.

Which of the following graphs is the best representation of Bob's journey?

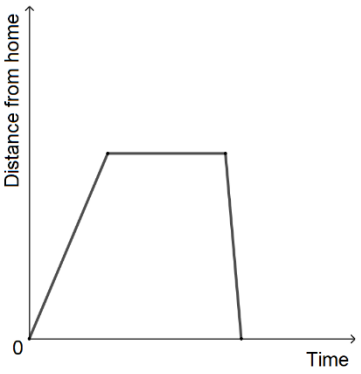
A



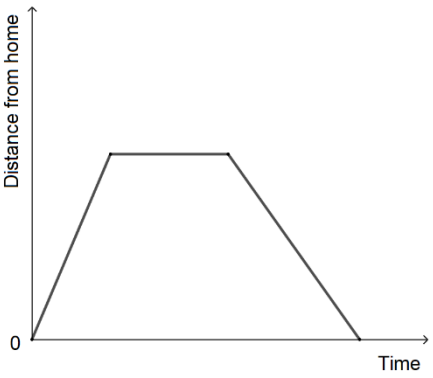
B



C



D



39 Hayley is going on holiday.
The maximum weight her suitcase can be is 20 kg.
Hayley's weighing scales give 19.4 kg as the weight of the case.
Her weighing scales are not accurate; they can be wrong by up to 5%.

Which of the following statements about the real weight of the suitcase is true?

A It could be up to 19.9 kg.
B It must be more than 20 kg.
C It must be exactly 20.37 kg.
D It could be more than 20 kg but is less than 21 kg.

40 Floor tiles measure 600 mm by 600 mm.
A pack of floor tiles has 3 tiles in it.

How many packs of floor tiles are needed for a rectangular floor 2.4 m by 3.6 m?

A 1 **B** 4 **C** 8 **D** 72

Exemplar Paper 1 mark scheme

Correct answer scores 1 mark.

Misconceptions associated with incorrect answers are shown below.

Number	Misconceptions associated with incorrect answers
1A	Thinks there are 10 g in a kg
1B	Thinks there are 100 g in a kg
1C	Correct answer
1D	Thinks there are 10 000 g in a kg
2A	Correct answer
2B	Realises the temperature has gone up but has found the difference between 3 and 5
2C	Has done 3 − 5
2D	Has found the difference but may have thought that one number negative and the other positive will result in a negative answer
3A	Subtracted numerators and denominators separately
3B	Correct answer
3C	Subtracted numerators and used common denominator
3D	Changed both to quarters then subtracted numerators and denominators
4A	Has given the width of the shape instead of the perimeter
4B	Correct answer
4C	Has worked out the area but thought it was the perimeter
4D	Has counted the squares on the inside round the edge instead of the total length of outside edges
5A	Read off the numbers nearest each hand on the clock rather than realising that the minute hand goes up in minutes rather than hours
5B	Hour read off incorrectly; may not realise that 14:00 is 2 o'clock in the afternoon
5C	Hour hand and minute hand the wrong way round
5D	Correct answer
6A	Has gone back in time 20 minutes instead of going forward
6B	Has treated the minutes like the decimal part of a number instead of working with 60 minutes in an hour
6C	Correct answer
6D	Expected to see 20 in the answer
7A	Has found cost of 0.5 kg
7B	Has subtracted 1.5 from each of cost and amount
7C	Correct answer
7D	Has halved given price to find cost of 1.25 kg
8A	This is 10 million and is not in standard form
8B	This is in standard form but is only 100 000
8C	This is only 10 000
8D	Correct answer

Number	Misconceptions associated with incorrect answers
9A	Has calculated 1.25 × 6 instead of 6 ÷ 1.25
9B	The number of Euros has gone up by 4.75. The number of pounds has been increased by the same amount
9C	Correct answer
9D	Has interpreted 4.8 as £4.08 instead of £4.80
10A	Has just found half the total amount
10B	Has found the amount that Victor gets instead of the amount Lee gets
10C	Correct answer
10D	Has found one part instead of two parts
11A	Has either found the number of 20p coins or just multiplied 20 by 115
11B	Has multiplied 5 by 115 or divided 115 by 20
11C	Correct answer
11D	Realises that 5 g increases by 110 g to get 115 go so has increased 20p by 110p
12A	Has chosen the largest number and ignored units
12B	Correct answer
12C	Has chosen the number with the most digits
12D	Has chosen the largest units
13A	Has not been able to order negative or decimal temperatures
13B	Has not been able to order negative temperatures correctly
13C	Correct answer
13D	Has not recognised that 0.1 is between 0 and 5
14A	Truncated instead of rounded
14B	Correct answer
14C	Truncated to have 3 sf
14D	Rounded to 3sf instead of 3dp
15A	Has expressed the risk as a correct decimal but not as a percentage
15B	Correct answer
15C	Has subtracted 1 from 20
15D	Has used the denominator instead of finding the percentage
16A	Has chosen the first day
16B	May have chosen this because of the zero digits
16C	This is the target number of steps or may have ignored the place value
16D	Correct answer
17A	Has interpreted the target as being exactly 10 000 steps
17B	Has interpreted the target as being more than 10 000 steps
17C	Correct answer
17D	May have counted the days when Nadia does not meet her target
18A	Correct answer
18B	Has calculated the mean
18C	Has stated the target
18D	Has selected the middle number but has not put the numbers in order

Number	Misconceptions associated with incorrect answers
19A	This is the average extra number of steps per day
19B	Correct answer
19C	This is the mean number of steps walked per day
19D	This is the total number of steps she needs to walk per week
20A	Has rounded 7.7 to nearest whole number
20B	Correct answer
20C	Has divided 650 by 7.7 and rounded down
20D	Has divided 650 by 7.7 and rounded up
21A	Has divided the number of miles by the number of pounds instead of the other way round
21B	Has misinterpreted 0.217…
21C	Correct answer
21D	Has assumed it is £25 for 165 miles instead of 115 miles
22A	Has chosen the number of groups
22B	Has chosen the mean frequency
22C	Has chosen the maximum frequency
22D	Correct answer
23A	Has chosen the mode
23B	Correct answer
23C	Has ignored frequencies and found mean of 1, 2, 3, 4
23D	Has found the mean frequency
24A	Correct answer
24B	Has used the numbers in the question to make a fraction and then changes to 25 in the denominator to make the answer look more reasonable
24C	May have cancelled incorrectly
24D	Has ignored units and used the numbers in the question to make a fraction
25A	Has reduced time by 20 mins to get an hour and reduced distance by 20 miles
25B	Correct answer
25C	Has worked out 60 ÷ 1.2
25D	Has rounded time down to an hour and approximated
26A	Gradient and intercept wrong way round
26B	Correct answer
26C	Correct intercept but has used 1 as gradient
26D	Has used intercept as gradient and got a line which goes through origin; the given line does not
27A	Correct multiplier used but has multiplied by 2 instead of squaring
27B	Correct answer
27C	Incorrect multiplier used and has multiplied by 2 instead of squaring
27D	Incorrect multiplier – has just used the percentage
28A	This is the cost for one week
28B	Correct answer
28C	This is the cost for 30 days. But Janice only works 5 days a week
28D	This is too much – may have chosen the only round number in the list

Number	Misconceptions associated with incorrect answers
29A	May have taken 10% off the bill. This will not be enough
29B	Has not added on 10%
29C	Correct answer
29D	Too much; the tip is £28 which is more like 40%. May have misread as 3 friends
30A	Multiplier for finding 30%, not for 30% reduction
30B	Correct answer
30C	Multiplier for 30% increase, not for 30% reduction
30D	multiplier for finding original before a 30% increase not for reducing by 30%
31A	Has subtracted 8 hours from landing time to find time in New York at take off
31B	Has ignored flight time and assumed London and New York have same time
31C	Has worked out that the difference between actual flight time and apparent flight time is 5 hours and added this to take off time instead of to landing time
31D	Correct answer
32A	Has found 75% of 45 minutes and rounded
32B	Has increased 45 minutes by 25% and rounded
32C	Correct answer
32D	Has added 25 (100 – 75) to 45
33A	Has averaged the numbers in the list
33B	Has picked the highest number
33C	Has added the numbers
33D	Correct answer
34A	Correct answer
34B	Has ignored the brackets when working out the numerator
34C	Has worked out the numerator correctly but subtracted 9 instead of dividing by 9
34D	Has typed the formula into a calculator but not included the brackets
35A	Correct answer
35B	Has assumed that half the days will have a max temperature below 50 °F; may be thinking there are two outcomes so equally likely
35C	Has read off from the bar chart
35D	Has found the probability of a day having a max temp above 50 °F
36A	Has counted the bars
36B	Has assumed about half the days in Chicago have a maximum temperature above 50 °F but it is more than half the days
36C	Correct answer
36D	This is the number of days in a year
37A	Does not know how to use the ratio scale
37B	May think there are 1000 cm in a metre
37C	Correct answer
37D	May think there are 100 m in 1 km

Number	Misconceptions associated with incorrect answers
38A	Has mixed up the two axes
38B	First two parts of the journey are correct but then Bob travels backwards in time rather than back home
38C	First two parts of the journey are correct but then Bob travels home quicker not slower
38D	Correct answer
39A	Has misinterpreted 5% as 0.5 kg
39B	Has realised the case could be too heavy but not that it might not be
39C	Has added 5% on to 19.4
39D	Correct answer
40A	Could have worked out area of floor in m^2 but then multiplied by 100 to get it into cm^2. Has worked out correct area of a tile in cm^2
40B	Could have worked out that it takes 4 tiles one way and 6 the other way but then added to get 10 tiles so 3 and a bit packs
40C	Correct answer
40D	Has worked out that 24 tiles are needed but multiplied by 3 instead of dividing to get the number of packs

Alternative Mathematics GCSE resit curriculum

Exemplar Paper 2

Time 1½ hours

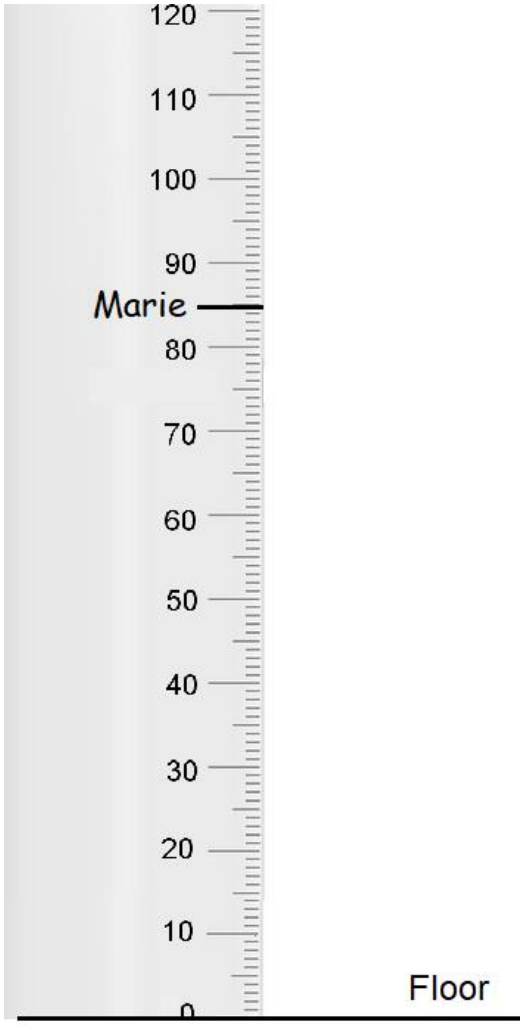
No calculator allowed

80 marks

Answer all questions

Note: This exemplar paper has been prepared by MEI as part of an exploration of an alternative to GCSE resit. It is not part of any recognised qualification.

1 Part of a vertical measuring scale is shown on the right.
The scale is in centimetres and starts at floor level.
It is used to measure children's heights.



(a) The mark on the scale shows Marie's height.

How tall is Marie?

_____ **[2]**

(b) Sadia is 1.05 m tall.

Put a mark on the scale to show Sadia's height. **[1]**

(c) How much taller is Sadia than Marie?

_____ cm **[2]**

2 Aisha is planning a party at a hotel.

(a) It costs £200 for the room and an extra £15 per person.

How much will it cost for 30 people?

£_____ **[2]**

(b) Aisha has made a spreadsheet to work out the cost at a different hotel.

	A	B
1	**No. of people**	**Cost (£)**
2	30	=40*A2
3		

How much will this hotel cost for 30 people?

£_____ **[1]**

3 Bottles of water cost 61p each.
Wesley has £5.

(a) What is the maximum number of bottles of water Wesley can buy?

_____ **[2]**

(b) How much change does Wesley get?

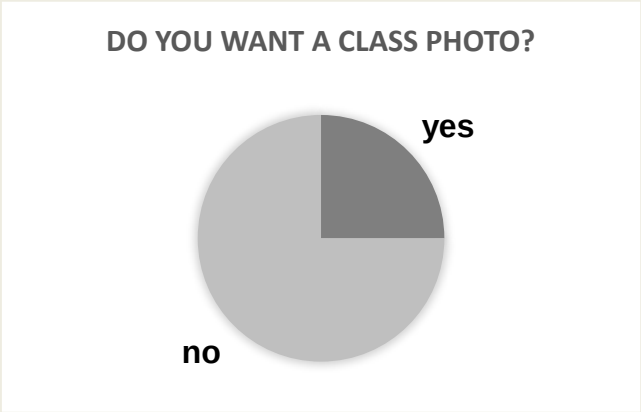
_____ **[2]**

4 A coat normally costs £70.
It is reduced by 20% in a sale.

What is the sale price of the coat?

£_____ **[2]**

5 28 students are asked whether they want to buy a class photo.
The results are shown in the pie chart below.



(a) How many said yes?

_____ **[2]**

(b) How many said no?

_____ **[2]**

6 Three athletes jump the following distances in a long jump competition.

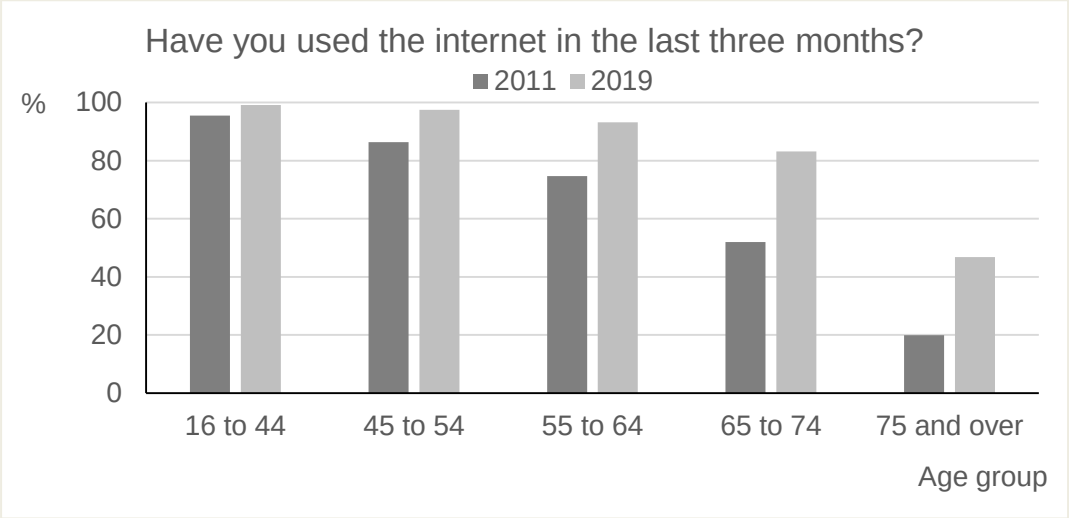
Name	**Distance (m)**
Bob	5.41
Carl	6.7
Juan	6.42

Write the names of the athletes who came first, second and third in the table below.

Place	**Name**
First	
Second	
Third	

[2]

7 The chart below shows answers to a survey question about internet use in two different years.



(a) What percentage of people aged 75 and over used the internet in the last three months in 2011?

_____% **[1]**

(b) Which age group had the smallest change in internet use from 2011 to 2019?

_____ **[1]**

(c) Lara says that most people who were in the 55 to 64 age group in 2011 will be in the 65 to 74 age group in 2019. Is Lara right? Explain your reasoning.

.....

.....

.....

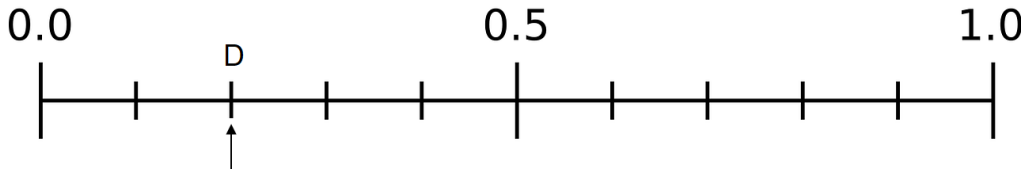
.....**[2]**

8 Look at the following information.

A The probability of rain tomorrow is 80%	**B** One in three babies born today will live to be 100
C Volcano has 50/50 chance of erupting	**D** One in five chance of winning a prize

Put the probabilities from the statements on the probability scale below.

Statement D has been done for you.



[3]

9 A carton of fruit juice contains 1.2 litres.

(a) How many 200 ml glasses will it fill?

_____ **[2]**

The carton is a cuboid made of thin card.
David suggests a new design for the carton.
David's design is a cuboid with square base 5 cm long, 5 cm wide.

(b) Work out the height of David's design.

_____ cm **[3]**

(c) Give one reason why a cuboid with smaller height would be a better design.

……

……

……

……**[1]**

(d) Write down the measurements of a cuboid with square base that will contain 1 litre.
Your cuboid must have a smaller height than David's design.

Length _____ cm

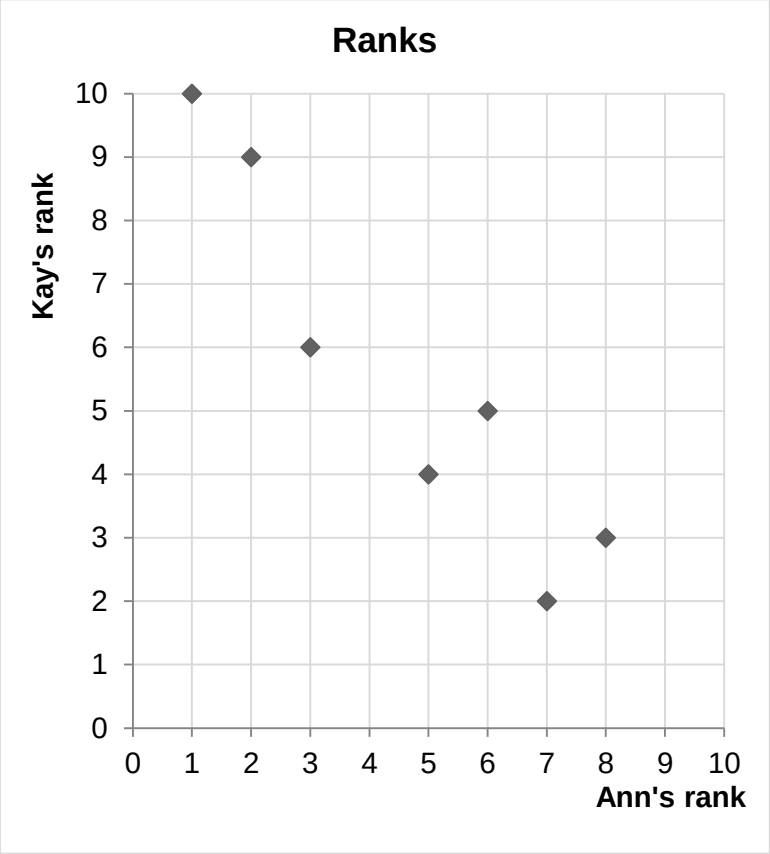
Width _____ cm

Height _____ cm **[3]**

10 Ann and Kay rank the 10 films showing at the cinema.
Each of them gives rank 1 to the film she would most like to see, 2 to the next one and so on.

Ann's rank	2	7	3	5	1	8	6	4	9	10
Kay's rank	9	2	6	4	10	3	5	8	1	7

The first seven pairs of ranks are plotted in the scatter diagram below.



(a) Plot the remaining points in the scatter diagram. **[2]**

(b) Describe the correlation in the scatter diagram.

……

……**[1]**

(c) Comment on whether Ann and Kay agree about the films.

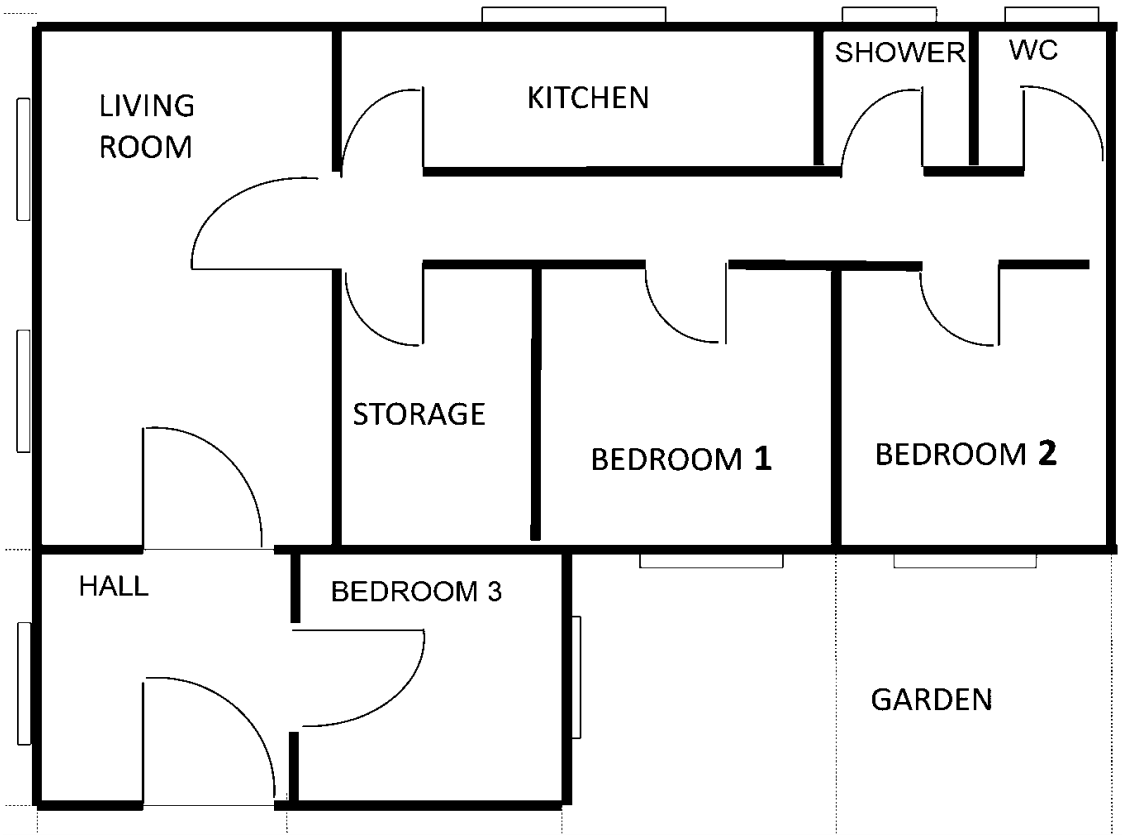
……

……

……

……**[1]**

11 The scale drawing below shows a house.



The living room measures 16 feet by 28 feet.

Find the length and width of the kitchen.

Length _____ feet

Width _____ feet **[6]**

12 The graph below shows the number of meals served each week by a café.

(a) Estimate the number of meals sold in week 10.

_____ **[1]**

(b) The café had a buy one get one free offer on meals one week.
Which week do you think it was?

Explain your reasoning.

……

……

……

……**[2]**

(c) A customer complained on Facebook that the café had given her food poisoning. This turned out not to be true.
When do you think the customer complained?

Explain your reasoning.

……

……

……

……**[2]**

13 A survey of 1000 people asks how much exercise they do.
The results are shown in the following table.

	Men	Women	Total
Get enough exercise	355		
Do not get enough exercise	145		348
Total			1000

(a) Fill in the missing numbers in the table

[4]

(b) What percentage of people do **not** get enough exercise?

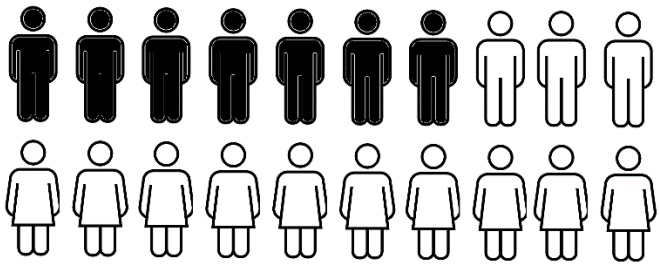
_____ **% [2]**

(c) What percentage of men get enough exercise?

_____ **% [2]**

To the nearest 10%, 40% of women do **not** get enough exercise.

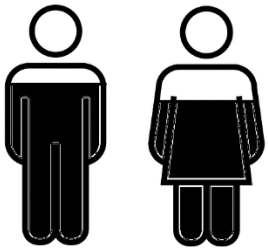
(d) Aisha is making the following graphic to show the results of the survey.
She has rounded percentages to the nearest 10%.
The graphic is not finished.
Aisha has shaded the correct number of men.



How many of the women should be shaded?

_____ **[2]**

(e) Barry suggests using the following graphic instead of Aisha's graphic.



Give one advantage of Aisha's graphic.

……

……

……

……**[1]**

14 Hayley's grandma give her this recipe for shortbread.
An ounce is an unit of weight.
The recipe makes 20 pieces.

4 ounces butter 2 ounces sugar 6 ounces flour

(a) Hayley has 500 g of butter.
She uses all the butter to make shortbread.

How much of each of the other ingredients will she need?

……

……

……

……**[3]**

(b) 16 ounces = 454 g.

How many pieces of shortbread will Hayley make?

_____ **[3]**

(c) 1 kg of flour costs £1.10.
1.5 kg of flour costs £1.60.

Which pack is better value? You must show your working.

_____ **[3]**

15 A runner runs 200 m in 28.87 seconds.

(a) Estimate her average speed in km/h.

_____ km/h **[4]**

(b) Estimate her average speed in miles per hour.

_____ mph **[3]**

(c) Is your estimate in part **(a)** greater or smaller than her actual speed?
Justify your answer.

……

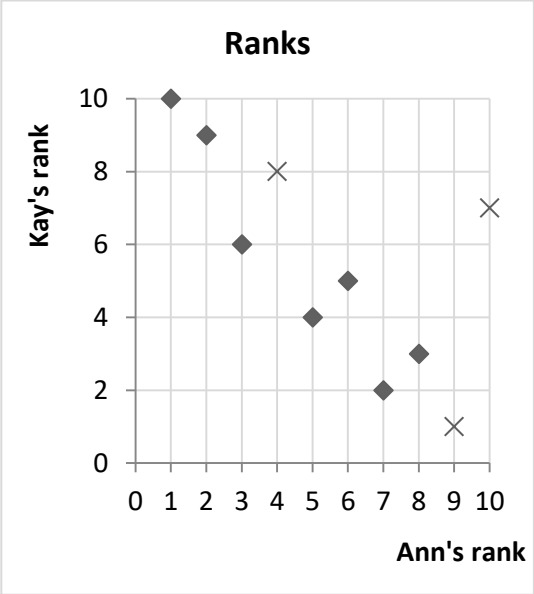
……

……

……**[2]**

Exemplar Paper 2 mark scheme

Question	Answer	Marks	Notes
1(a)	85 cm or 0.85 m	B2 [2]	B1 if units missing
1(b)	Mark at 105 cm	B1	need not be labelled, ±1 cm
1(c)	105 − 85 20	M1 A1 [2]	both measurements in same units
2(a)	30 × 15 650	M1 A1 [2]	
2(b)	1200	B1	
3(a)	500 ÷ 61 OR 500 ÷ 60 8	M1 A1 [2]	OR at least two terms from sequence 61, 122, 183, …
3(b)	500 − 488 12p or £0.12	M1 A1 [2]	OE FT *their* (a) must include units
4	[£]14 56	M1 A1 [2]	
5(a)	¼ OR 90° 7	M1 A1 [2]	
5(b)	¾ OR 270° 21	M1 A1 [2]	OR 28 – *their* (a)
6	Carl, Juan, Bob	B2	B1 for reverse order
7(a)	20	B1	
7(b)	16 to 44	B1	
7(c)	Adding 8 on to any age in the 55 to 64 range Convincing completion e.g. most ages go into the next group (so most people do)	M1 A1 [2]	
8	A at 0.8 B between 0.2 and 0.4 C at 0.5	B1 B1 B1 [3]	

Question	Answer	Marks	Notes
9(a)	1200 [ml] 6	M1 A1 [2]	OR 0.2 [l]
9(b)	1200 5 × 5 48	B1 M1 A1 [3]	
9(c)	Suitable reason e.g. • would use less card • less likely to fall over • would fit in fridge better	B1	
9(d)	length = width height < *their* (b) Measurements multiply to give 1200	B1 B1 B1 [3]	Allow two other measurements equal
10(a)		B2	B2 if all three points correct and no extras B1 if one point correct
10(b)	Negative correlation	B1	
10(c)	They disagree	B1	
11	Width of living room = 4 cm Scale is 1 cm to 4 feet Kitchen length 6.5 cm Kitchen width 2 cm 26 feet 8 feet	M1 M1 A1 A1 A1 A1 [6]	OR length = 7 cm +/− 2 mm +/− 2 mm For either length or width FT their measurements

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Notes</th></tr>
<tr><td>12(a)</td><td>1250</td><td>B1</td><td>Value in range 1200 to 1300</td></tr>
<tr><td>12(b)</td><td>Week 21
There were more meals sold that week</td><td>B1
B1
[2]</td><td></td></tr>
<tr><td>12(c)</td><td>Any time between week 2 and week 3
Sales dropped</td><td>B1
B1
[2]</td><td>inclusive (or 21 to 22)</td></tr>
<tr><td>13(a)</td><td><table><tr><th></th><th>Men</th><th>Women</th><th>Total</th></tr><tr><td>Get enough exercise</td><td>355</td><td>297</td><td>652</td></tr><tr><td>Do not get enough exercise</td><td>145</td><td>203</td><td>348</td></tr><tr><td>Total</td><td>500</td><td>500</td><td>1000</td></tr></table></td><td>B4

[4]</td><td>all correct B4
4 correct B3
3 correct B2
1 or 2 correct B1</td></tr>
<tr><td>13(b)</td><td>$\frac{348}{1000}$
34.8</td><td>M1

A1
[2]</td><td>

or 35</td></tr>
<tr><td>13(c)</td><td>$\frac{355}{500}$
71</td><td>M1

A1
[2]</td><td></td></tr>
<tr><td>13(d)</td><td>6</td><td>B2
[2]</td><td>B1 for 4</td></tr>
<tr><td>13(e)</td><td>advantage given
e.g. easier to read/draw</td><td>B1</td><td></td></tr>
<tr><td>14 (a)</td><td>500 ÷ 2
250 g sugar
750 g flour</td><td>M1
A1
B1
[3]</td><td></td></tr>
<tr><td>14(b)</td><td>454 g makes 4 × 20 = 80
45 g makes 8
88 pieces</td><td>M1
M1
A1
[3]</td><td></td></tr>
<tr><td>14(c)</td><td>For 1 kg pack 0.5 kg costs 55p

For 1.5 kg pack 0.5 kg costs 53.3p
1.5 kg pack is better</td><td>M1

M1
A1
[3]</td><td>Finding cost of different amount to those given for one pack
cost for two identical amounts
correct completion</td></tr>
</table>

Question	Answer	Marks	Notes
15(a)	28.87 secs ≈ 30 secs	M1	Or 0.5 minutes
	400 m in 1 minute	M1	
	24 000 m in 1 hour	M1	Ft going from minutes to hours
	24 km/h	A1	Correct conversion of *their* speed in m/h to km/h [or distance from m to km]
		[4]	
15(b)	8 km ≈ 5 miles	B1	Could be implied by some correct working
	3 × 5	M1	
	15	A1	FT *their* (a)
		[3]	
15(c)	She will run more than 200 m in 30 s	M1	If answer to (a) incorrect allow reason based on speed being unrealistic and resulting conclusion
	So actual speed will be greater	A1	Or estimate will be smaller
		[2]	

Alternative Mathematics GCSE resit curriculum

Exemplar Paper 3

Time 1½ hours

Calculator allowed

80 marks

Answer all questions

Note: This exemplar paper has been prepared by MEI as part of an exploration of an alternative to GCSE resit. It is not part of any recognised qualification.

1 Alice has a part time job selling ice cream.
She is paid 20p for each ice cream she sells.
One day Alice sells 46 ice creams.

How much does she earn?

£_____ **[2]**

2 Manny wants to pay exactly 59p using 5p and 2p coins.
He wants to use the smallest possible number of coins.

How many of each coin does Manny need?

5p coins_____

2p coins_____ **[4]**

3 Alia is planning a party for 30 children.
There are two possible venues for the party.

Play Park
£8 for each child

Fun Hotel
£150 plus £4 for each child

Which venue will be cheaper? You must show your working.

……

……

……

……

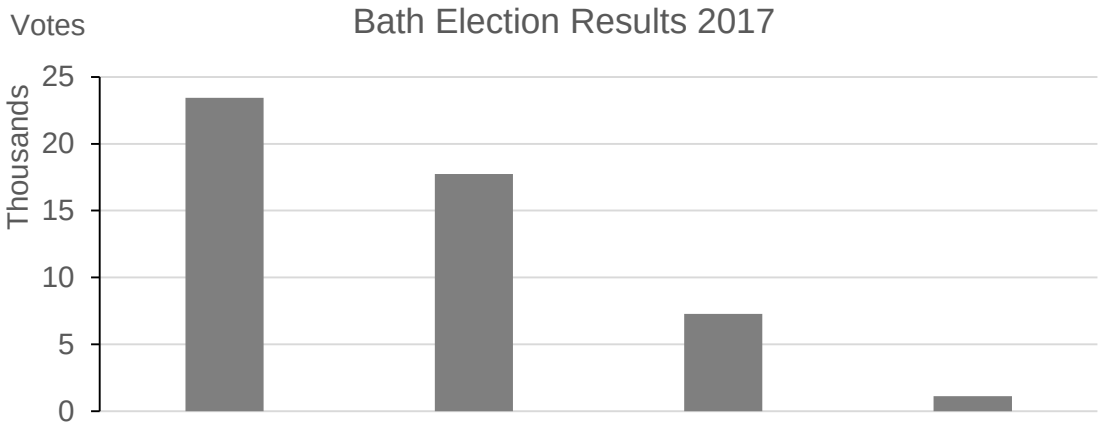
……

……

……**[4]**

4 The election results for Bath in 2017 are shown in the table and bar chart below.

Party	Votes
Cons	17 742
Green	1126
Labour	7279
Lib Dem	23 436



(a) Write the name of the correct party under each bar of the bar chart. **[2]**

(b) Which party won?

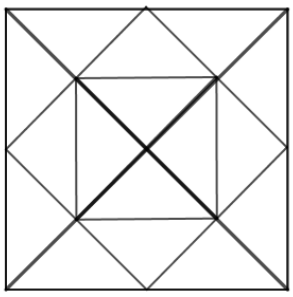
_____ **[1]**

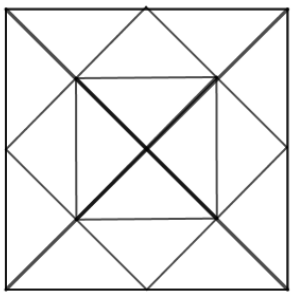
(c) What was the total number of votes?

_____ **[1]**

5 Shade some of the triangles in the design below so that half the area is shaded. Your shaded design should have at least one line of symmetry.

Spare



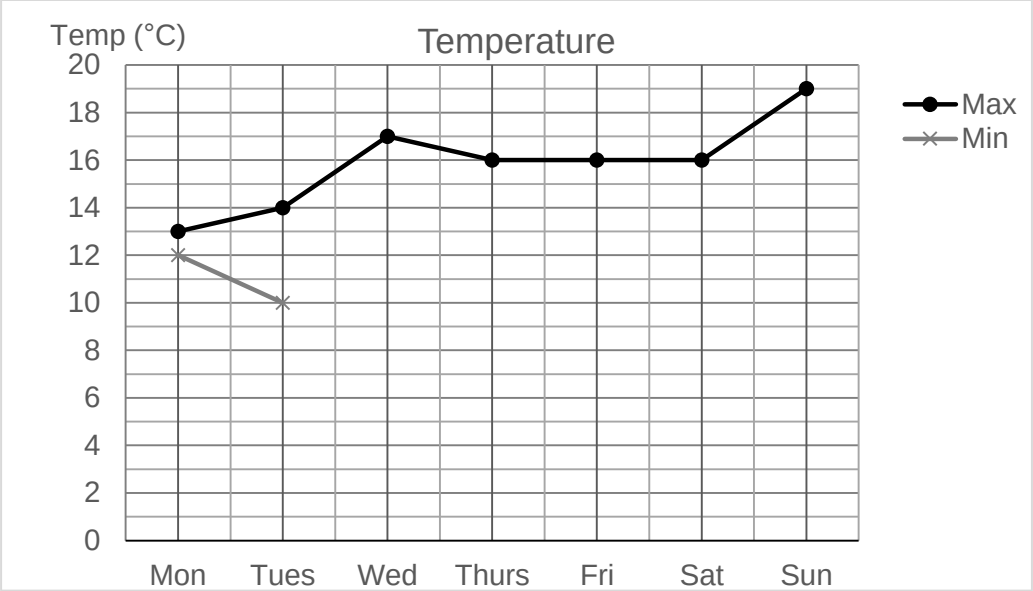


[2]

6 The table shows the maximum and minimum temperatures each day in a town.

°C	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Max	13	14	17	16	16	16	19
Min	12	10	10	10	11	12	11

(a) The maximum temperatures have been plotted on the graph below.



Finish the graph for minimum temperatures. **[3]**

(b) Which day had the biggest range of temperatures?

_____ **[2]**

(c) What was the range on that day?

_____°C **[1]**

7 **Undiscounted** tickets for a particular train journey are priced as follows.

- £36 for an adult
- £18 for a child

A family and friends railcard gives the following discounts off train tickets.

- ⅓ off for adults
- 60% off for children

One child and one adult buy tickets using a railcard. What is the **total** cost?

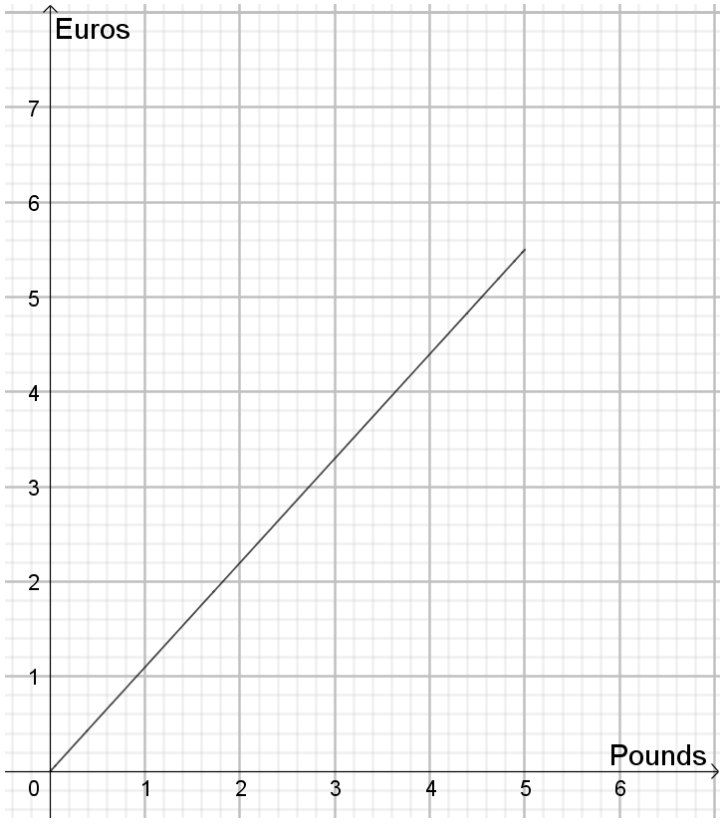
£_____ **[4]**

8 **(a)** A road sign shows the distance to Paris in **kilometres**.

How far is it to Paris in miles?

_____ miles **[2]**

(b) A conversion graph between pounds (£) and euros (€) is shown below.



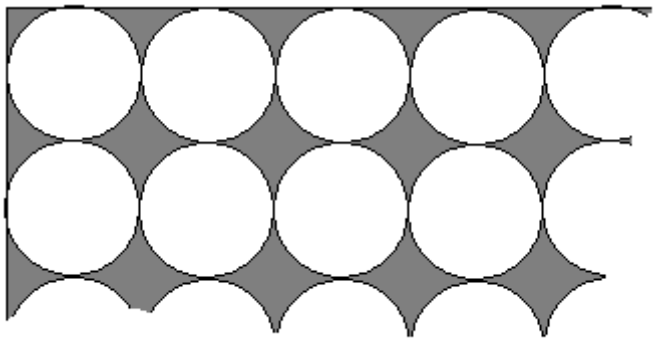
(i) What is 4 euros in pounds?

£ _____ **[2]**

(ii) What is 10 pounds in euros?

€ _____ **[3]**

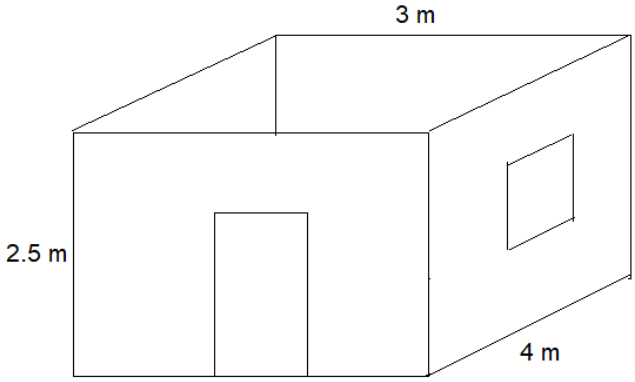
9 A rectangular sheet of metal is 50 cm long and 30 cm wide.
Circles of diameter 2 cm are cut from the sheet.
The circles are in rows and as close together as possible.
The **corner** of the sheet is shown in the diagram below.



How many circles can be cut from the sheet?

_____ **[4]**

10 A room is 2.5 m tall, 4 m long and 3 m wide.
It has one door and one window.
The window is a rectangle, 1 m wide and 0.5 m high.
The door is 75 cm wide and 2 m high.



(a) On the centimetre square grid below, draw a plan of the room.

Use a scale of 1 cm to 1 m.

[2]

(b) All 4 walls need to be painted with two coats of paint.
Paint is sold in 2.5 litre tins; each tin will cover 30 m^2.

How many tins of paint are needed?

_____ **[5]**

11 Jill buys a car costing £5000.

(a) Jill borrows money to buy the car.
She pays back £163 a month for 3 years.

How much does Jill pay back altogether?

£_____ **[3]**

(b) The table below shows what Jill spends each **month** on her house and car.

Description	Expenditure (£)
Rent	350
House bills	155
Car loan	163
Petrol	140
Car insurance, tax and servicing	80

Jill spends £1980 a **year** on food.
Jill's take home pay is £1200 a month.

How much does Jill have left every **month** after spending on house, car and food?

£_____ **[3]**

(c) Jill's car depreciates in value by 15% a year.
It is worth £5000 when she buys it.

How much will it be worth after 3 years?
Give your answer to the nearest pound

£_____ **[4]**

12 The salary for a particular job is £19,000 a year.
The worker does not pay any income tax on the first £12,500 of this.
Income tax is 20% of the rest of the salary.

(a) How much income tax does the worker pay each year?

£_____ **[3]**

(b) What percentage of the total salary does the worker pay in income tax?
Give your answer to one decimal place.

_____ % **[3]**

The spreadsheet below calculates the income tax for a group of workers.
Each worker does not pay any income tax on the first £12,500 of salary.
Income tax is 20% of the rest of the salary.

	A	B	C	D
1	**Salary**	**Taxable salary**	**Tax**	**Percentage of salary paid in tax**
2	£15,960	£3460	£692.00	4.3
3	£16,500	£4000	£800.00	4.8
4	£16,600	£4100	£820.00	4.9

(c) Write down a suitable spreadsheet formula which could be put in the following cells for copying down the column.

(i) Cell B2

……

……

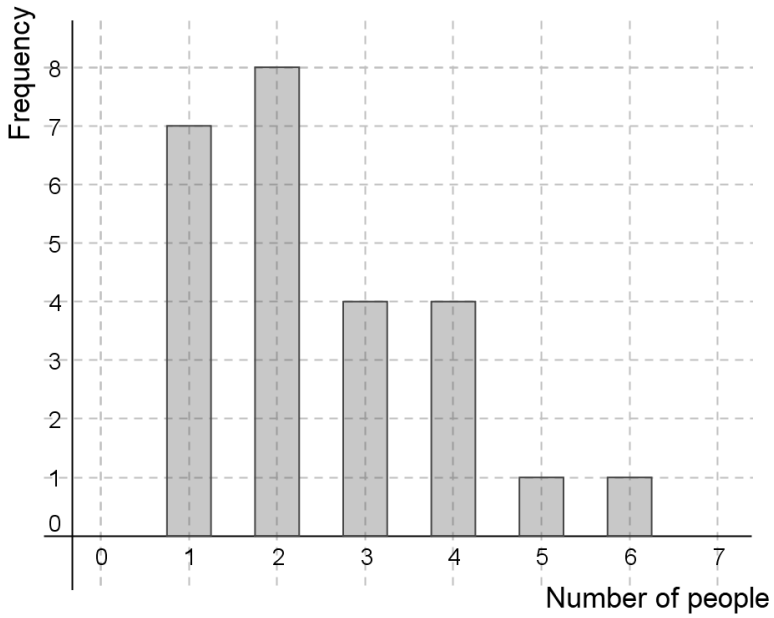
(ii) Cell D2

……

……

[3]

13 The bar chart below shows the number of people living in each house on a particular street.



(a) Find the total number of people living on the street.

_____ **[2]**

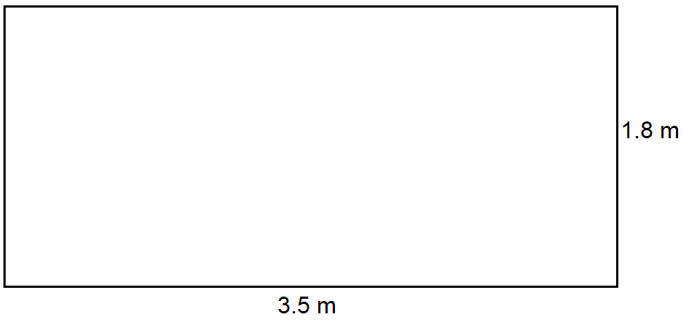
(b) 100 years ago the mean number of people in a house on the same street was 4.2.

Calculate the total number of people living on the street 100 years ago.

_____ **[3]**

14 Annie is marking out a rectangular garden plot.
The plot is 3.5 m long and 1.8 m wide.

Not to scale



Annie wants to be sure that she has marked out a rectangle and that it is the right size.
She will measure **lengths** on the plot to check.

What lengths should she measure and what should she get if the plot is correct?

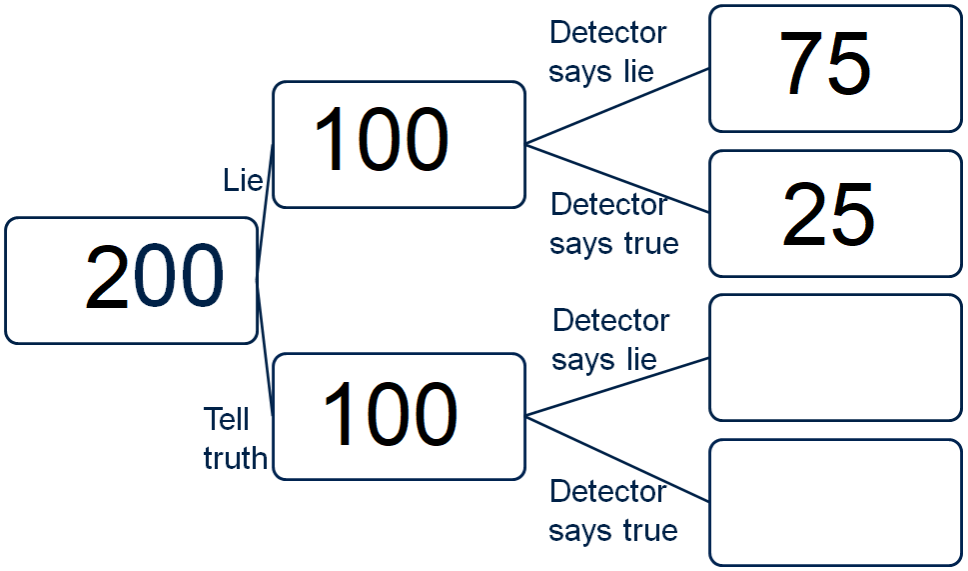
You might not need to use all the rows in the table.

She should measure	She should get

[3]

15 Lie detectors are not completely accurate.
A lie detector will either identify someone as a liar or as someone who is telling the truth.
It will correctly identify a liar 75% of the time.
It will correctly identify that someone is telling the truth 60% of the time.

(a) Fill in the missing numbers in the frequency tree for 200 people.
Assume that 50% of them tell the truth and 50% lie.



[2]

(b) The lie detector identifies someone as a liar.
What is the probability that they were telling the truth?

_____ **[2]**

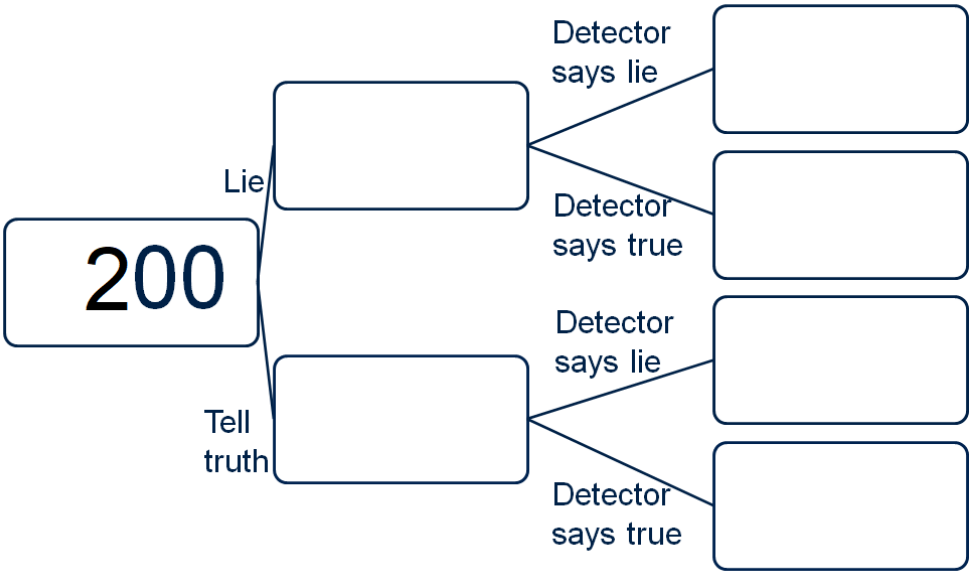
(c) Suppose the assumption in part **(a)** is not correct.
In fact, 80% of people tell the truth.

What effect would this have on the probability calculated in part **(b)**?

Tick one box.

☐	It would be bigger
☐	It would be the same
☐	It would be smaller

Show working to justify your answer.



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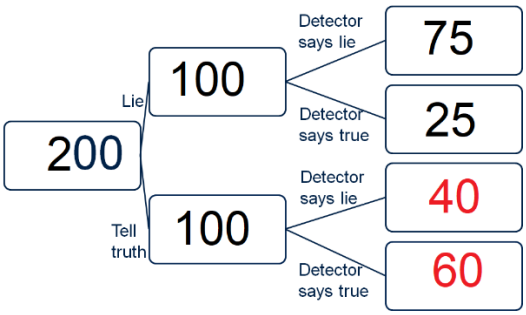
[5]

Exemplar Paper 3 mark scheme

Question	Answer	Marks	Notes
1	46 × 20 9.20	M1 A1 [2]	= 920
2	59 ÷ 5 Any combination of 5p and 2p coins that adds up to 59p 11 (5p) 2 (2p)	M1 B1 A1 A1 [4]	or any combination of 5p and 2p coins totalled
3	8 × 30 240 4 × 30 Play Park is cheaper	M1 A1 M1 A1 [4]	 270 and 240 seen
4(a)	Lib Dem, Cons, Labour, Green	B2	B1 for either reverse order or two correct
4(b)	Lib Dem	B1	
4(c)	49 583	B1	
5	half area shaded at least one line of symmetry	B1 B1 [2]	8 small triangles
6(a)	All 5 points plotted correctly *their* points joined dot to dot	B2 B1 [3]	B1 3 points correct
6(b)	One range calculated Sunday	M1 A1 [2]	Or vertical line on graph from max to min on one day
6(c)	8	[1]	FT *their* (b)
7	36 ÷ 3 0.6 × 18 OR 0.4 × 18 36 – 12 + 7.20 31.20	M1 M1 M1 A1 [4]	= 12 (implied by 24) = 10.80 = 7.20

Question	Answer	Marks	Notes
8(a)	5 miles ≈ 8 km 10	M1 A1 [2]	oe
8(b) (i)	Value between 3.6 and 3.7 stated to 2 dp e.g. 3.60	M1 A1 [2]	
8(b) (ii)	Value for a factor of £10 read from graph Multiplying by correct number to get to £10 11	M1 M1 A1 [3]	e.g. £1 = €1.10 e.g. 1.10 × 10 10.80 to 11.20 (2 d.p if not whole number)
9	50 ÷ 2 = 25 30 ÷ 2 = 15 25 × 15 375	M1 M1 M1 A1 [4]	
10(a)	Rectangle 3 cm by 4 cm	M1 A1 [2]	For plan
10(b)	3 × 2.5 4 × 2.5 4 walls area = 35 [m^2} 2 coats so 70 [m^2} 3 tins	M1 M1 M1 M1 A1 [5]	 OR 33 without window and door OR 66
11(a)	36 36 × 163 5868	M1 M1 A1 [3]	
11(b)	1980/12 888 147	M1 M1 A1 [3]	165 OR 1053
11(c)	4250 3612.5 3070.625 3071	M1 M1 M1 A1 [4]	OR 0.85 seen OR *their* 0.85 to the power 3 allow all values rounded or truncated for M marks

Question	Answer	Marks	Notes
12(a)	19 000 − 12 500 Working out 20% of either *their* taxable pay OR of £19 000 £1300	M1 M1 A1 [3]	
12(b)	*their* (a)/ 19 000 6.8	M1 A2 [3]	 FT *their* (a) A1 if more than 1 dp but rounds to correct ans
12(c) (i)	=A2−12500	B1	condone missing =
12(c) (ii)	=100*C2/A2	B2 [3]	condone missing = and use of × instead of * [B1 for C2/A2]
13(a)	correct method to find total 62	M1 A1 [2]	eg $7 + 2 \times 8 + 3 \times 4$..... may be done by entering data into calculator; full marks for correct answer
13(b)	25 *their* 25 × 4.2 105	M1 M1 A1 [3]	 e.g. 6 × 4.2
14	both short sides 1.8 m each both long sides 3.5 m each Both diagonals the same OR one diagonal 3.9(357…) m	B1 B1 B1 [3]	

Question	Answer	Marks	Notes
15(a)		B1 B1 [2]	60 40
15(b)	$\frac{40}{40+75}$ $\frac{40}{115}$ oe	M1 A1 [2]	numerator correct FT *their* tree 0.348
15(c)	bigger ticked 40 or 160 30 64 $\frac{64}{30+64} = 0.68...$	B1 M1 M1 M1 A1 [5]	

Appendix 4 Comparative judgement report from No More Marking

The full report on the comparative judgement study carried out by No More Marking[106] is published separately alongside this report as an external appendix.

[106] Barmby, P., & Wheadon, C. (2019). *A comparative judgement study of MEI GCSE exam items*. No More Marking Ltd.

Appendix 5 Further research that could be done

While working on this project, a number of areas of possible further research emerged:

- Reviewing whether the change to allow students with grade 2 in GCSE Mathematics at the end of KS4 to aim for level 2 Functional Skills Mathematics results in a greater proportion of these students achieving a level 2 pass by age 18.
- Comparing the approaches to problem solving, including regulatory approaches, in GCSE Mathematics and Functional Skills Mathematics to assess which is more effective.
- Comparing the judgements of difficulty of exam questions found by comparative judgement with how difficult students sitting examinations found them.
- A general review of the process of qualification development, with the aim of making recommendations to improve the process and so produce better qualifications, more able to meet the needs of students, employers and HEIs.
- Investigating the typical strengths and weaknesses of understanding which GCSE resit students have, in order to inform teaching.
- Tracking long-term outcomes for students who take the proposed qualification against outcomes for students who resit GCSE Mathematics.

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