Mathematical transitions:

a report on the mathematical and statistical needs of students undertaking undergraduate studies in various disciplines

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The Higher Education Academy

The Higher Education Academy STEM project series

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Foreword

This report provides a comprehensive and collective overview of the outcomes from a series of studies that the Higher Education Academy STEM team undertook to look at the requirements for mathematical and statistical skills in a range of discipline areas. The discipline areas explored were Business and Management, Chemistry, Computing, Economics, Geography, Sociology and Psychology. This work addressed a nationally recognised issue of the need to address these skills in undergraduate degree programmes. Individual reports were produced for these discipline areas. Although only a sample of discipline areas were investigated, the work fits in with the needs of many, if not all, discipline areas.

This report and the recommendations within are a demonstration of the importance that the Higher Education Academy places on the topic. The Higher Education Academy STEM team focused its efforts and work on this topic over two years to ensure maximum benefit to the sector. The studies encompassed the way in which mathematical and statistical skills form part of the discipline landscape, the signalling higher education provides about the need for these skills, sector requirements within the discipline (e.g. from accreditors and Quality Assurance Agency subject benchmark statements), the use of diagnostic testing and the support provided for students to improve and develop their mathematical and statistical skills.

The Higher Education Academy STEM team would like to thank the authors across all reports for their invaluable efforts and input in writing the reports. We were also very pleased to work with the sector and would like to thank higher education institutions, professional bodies and charities for their support for and input to the tackling transition events, the surveys and the reports. All of these fed valuable information into the overarching report.

The Higher Education Academy STEM team feels that the outcome and recommendations are valuable to the sector in providing a strong evidence base to further inform the dialogue between the secondary and tertiary sectors, to inform policy development and teaching practice to address the importance of mathematical and statistical skills.

Dr Janet De Wilde
Head of STEM, The Higher Education Academy

Foreword

Undergraduate student choice is now a fundamental driver of UK higher education - what to study and where to study it? This means that it is important to ensure that would-be university students are fully informed about all aspects of the courses they are considering applying for.

This is particularly important with regard to the mathematical aspects of university courses where the nature of the transition from school to university can be especially challenging. It is therefore a cause of considerable concern to note the key finding of this report that "too many students arrive in higher education without a realistic understanding either of the relevance of Mathematics and Statistics to their discipline or of the demands that will be put upon them".

We must address this problem and I very much welcome the contribution of this report to identifying key issues of concern and signalling possible ways forward.

Professor Sir Adrian Smith FRS
Vice-Chancellor, University of London

Executive summary

What the report is about

Mathematics and Statistics are essential to the university curricula of many disciplines. The purpose of the Higher Education Academy STEM project was to investigate the mathematical and statistical requirements in seven of these disciplines: Business and Management, Chemistry, Computing, Economics, Geography, Sociology and Psychology. Reports were commissioned from discipline experts to provide a strong evidence base to inform developments within the disciplines and dialogue between the higher education and pre-university sectors.

This report summarises the findings of these reports and of similar work in other disciplines. It introduces some high-level contextual evidence from the pre-university sector, in particular data about trends in public examinations, and highlights important policy developments in pre-university Mathematics education that are set to have a major influence on the issue of students' transitions into higher education.

Key findings

1. All the disciplines in the Higher Education Academy STEM project require Mathematics and/or Statistics to some extent.

2. Some 85,000 students are admitted into university each year to study the seven disciplines in the project. Other disciplines will have similar mathematical and statistical needs. The number of students affected is of the order of 200,000 pa.

3. The number of students entering the disciplines with an A or AS-level in Mathematics has increased in recent years but has probably reached a limit. Many students will only have a GCSE grade B or C and may have done little or no Mathematics for at least two years.

4. Important government policy changes in pre-university Mathematics teaching are underway. In particular the development of "Core Maths", a new pathway designed for students who achieve a grade C or better at GCSE, has the potential to bring about real improvements.

5. Many students arrive at university with unrealistic expectations of the mathematical and statistical demands of their subjects. Lack of confidence and anxiety about Mathematics/Statistics are problems for many students.

6. Many universities provide valuable support in Mathematics and Statistics at institutional level but too few students make use of it.

7. Diagnostic testing linked to purposeful interventions can be an effective tool but it is not used widely in the disciplines.

Recommendations

1. There should be clear signalling to the pre-university sector about the nature and extent of mathematical and statistical knowledge and skills needed in undergraduate degree programmes.

2. As part of this signalling university tutors should consider recommending the benefits of continuing with mathematical/statistical study beyond the age of 16. They should be aware of the full range of post-16 Mathematics qualifications, in particular the new "Core Maths" qualification.

3. Guidance documentation should be commissioned to provide university staff with a description of the range of knowledge and skills that students with GCSE Mathematics at different grades can be expected to demonstrate when they start their undergraduate studies.

4. Key stakeholders within the disciplines should actively engage with current and future developments of discipline A-levels as well as those in post-16 Mathematics qualifications, (e.g. "Core Maths").

5. University staff should consider the benefits of diagnostic testing of students' mathematical and statistical knowledge and skills at the start of degree programmes, and of using the results to inform feedback and other follow-up actions.

6. Teaching staff should be made aware of the additional support in Mathematics and Statistics that is available to students. Students should be actively encouraged to make use of these resources and opportunities.

A key factor in achieving progress with all of the above will be dialogue and the development of mutual understanding between the higher education and pre-university sectors.

1 Introduction

1.1 The Higher Education Academy STEM project

Mathematical and statistical concepts and techniques are embedded within the university curricula of a wide range of disciplines and are fundamental tools which many students need to apply throughout their undergraduate studies and beyond. Yet many university degree programmes do not explicitly provide information about their mathematical content (Advisory Committee on Mathematics Education, 2011). The presence of mathematical content is a challenge for many students. Some are surprised at the extent of the mathematical demands of their programmes and some struggle to cope with those demands.

The purpose of the Higher Education Academy STEM project was to investigate the requirements for mathematical and statistical knowledge and skills at undergraduate level across a selection of disciplines in higher education. The work was motivated by views expressed by some Discipline Leads within the Higher Education Academy who said that mathematical and statistical skills within the context of their disciplines were areas requiring attention. The project sought to provide a strong evidence base to inform dialogue and promote greater understanding, between the higher education and pre-university sectors within the UK, about the need for students to develop these skills. Throughout the work a particular emphasis was placed on the point of transition into higher education study.

This overview report synthesises key findings and recommendations from the series of discipline reports produced during the project (Shallcross and Yates, 2014; Souch *et al.*, 2014; Scott Jones and Goldring, 2014; Cottee *et al.*, 2014; Field 2014; Dawson, 2014).[1] It also presents some high-level contextual evidence from the pre-university sector, in particular data about trends in public examinations and observations on current policy developments in Mathematics education that will potentially have an important bearing on the issue of students' transitions into higher education.

1.2 Overview of the project

Recent reports have been published on the mathematical and statistical requirements of university degrees in Physics and Engineering (Institute of Physics, 2011) and the Biosciences (Koenig, 2011). To complement these studies, seven discipline areas were selected for the project, namely: Business and Management, Chemistry, Computing, Economics, Geography, Sociology and Psychology. Broadly speaking these are disciplines where there is a clear need for Mathematics and Statistics, but where an A-level in Mathematics is not, in general, a pre-requisite for acceptance at university (as it would be for Physics and Mathematics, say). In some of the disciplines involved in the project, the term 'quantitative methods' is most commonly used to refer to content of a mathematical and/or statistical nature. Throughout this report the terms Mathematics and Statistics will be used. The project was run by the Higher Education Academy STEM team. Discipline specialists were commissioned to write reports focusing on the mathematical and statistical skills needed for higher education study in their individual discipline areas. The discipline report authors were provided with a set of common areas for investigation in their work. These were:

1 At the time of writing, Spring 2014, work on a further report for the Computing discipline was still in progress.

- the mathematical and statistical skills requirements for higher education study within the context of the disciplines;
- the signalling from higher education about the need for these skills, incorporating the nature of entrance qualifications;
- the higher education sector requirements for these skills within individual disciplines;
- diagnostic testing of students' mathematical and statistical knowledge and skills;
- support for mathematical and statistical skills development;
- issues and areas for attention.

As the work progressed these general areas were developed and refined collectively by the project team with input from a project steering group which included Higher Education Academy and sector representation.

1.3 Scope of the analysis

Some 85,000 students (UCAS, 2013) are admitted into university each year to study the seven disciplines in the project. There are many other disciplines where the students will have similar mathematical and statistical needs and where the issues discussed in this report will be germane, for example Biological Sciences, Medicine and Dentistry, subjects allied to Medicine, Architecture, Building and Planning and various technology degrees. A recent report from the Advisory Committee on Mathematics Education estimates that the total number of students who would benefit from having continued experience of Mathematics beyond GCSE but do not currently do so is at least 200,000 pa, around 50% of the undergraduate student cohort. (Advisory Committee on Mathematics Education, 2011).

1.4 Evidence base for the discipline reports

Evidence for the discipline reports was obtained by means of literature reviews, discussion events, survey work and additional desk-based research. The surveys were based around a common core of questions used across all disciplines. Questions seeking specific opinions on areas the authors considered to be particularly important varied from discipline to discipline.

Three types of survey were developed. The **staff survey** was designed for those teaching the mathematical/statistical components of the degree programmes. The **heads survey** was designed for those making the management decisions about the allocation of staff to this teaching. These surveys differed only in the questions which specifically referred to how management decisions were made. The **student survey** was designed for students who had experience of mathematical/statistical work as part of their degree programmes. In the case of the Economics discipline it was decided not to undertake a survey, in the light of existing work in this area.

The discussion events were open to those involved in teaching the disciplines in the pre-university sector and higher education as well as other interested stakeholders including examination boards and professional bodies. The discussions focused on:

- higher education lecturers' expectations of the mathematical and statistical skills students have on arrival at university and FE lecturers' and school teachers' expectations of the mathematical and statistical skills students will need at university;
- how and where the mathematical and statistical work is taught in the two sectors;
- barriers and enablers within the context of mathematical/statistical skills development and what could be done in this area to ease the transition into higher education study in the discipline.

The survey response rates varied between the disciplines and were in some cases low. In all cases the respondents were self-selected, so there are limitations on the conclusions that can be drawn from the data. That said the results of the surveys for the most part support the findings of other published work in the area as revealed by the literature reviews. They are also consistent with the views expressed at the discipline events and elsewhere.

2 Findings

2.1 Demand for Mathematics and Statistics

All the disciplines demand some level of Mathematics and/or Statistics from their students. There were no departments mentioned in any of the discipline reports that were running degree programmes that are purely qualitative in nature. Within all the disciplines there is variation between degree programmes in the amount and level of Mathematics and Statistics required. For students studying Business and Management, Geography or Sociology this variation is wide, ranging from degree programmes that are highly mathematical to some that are mainly qualitative. For Chemistry, Economics and Psychology the range is narrower and in general the threshold is higher. In these three subjects Mathematics/Statistics will be an integral and important part of any degree programme students will take. There are differences between the disciplines in terms of the kinds of Mathematics and Statistics needed: Chemistry and Economics need calculus; Psychology mainly needs Statistics; some Computing degree programmes need discrete Mathematics, and so on. The discipline reports describe these needs in some detail. All the discipline reports identify a need for Statistics at some level.

2.2 Benchmarks and accreditation

The QAA subject benchmark statements set out expectations about the content and standards of degrees. They: "...define what can be expected of a graduate in terms of the abilities and skills needed to develop understanding or competence in the subject" (Quality Assurance Agency, 2014). In addition some degrees are accredited by professional bodies. Psychology degrees are accredited by the British Psychological Society and Chemistry degrees by the Royal Society of Chemistry. For the disciplines under consideration all the relevant QAA subject benchmark statements make reference to Mathematics and Statistics, or to quantitative methods, but for the most part the authors of the discipline reports have the view that they are general, and not explicit as to content. The statements say in effect that Mathematics and Statistics are important for the discipline but do not describe in any detail the type and level of mathematical and statistical knowledge and skills that are needed and the extent to which they should be included in undergraduate degree programmes.

The Sociology, Geography, Chemistry and Business and Management reports all recommend that the benchmark statements for their subjects should be strengthened and made more explicit about the requirements for mathematical and/or statistical content of degree programmes in the discipline.

2.3 Entry qualifications

2.3.1 What Mathematics entrance qualifications do the disciplines require?

Most, but not all, degree programmes set a minimum Mathematics entrance requirement. Business and Management, Geography and Sociology report that many of their degree programmes do not specify a minimum requirement beyond a general university entrance requirement (which will often include at least a C grade at GCSE Mathematics or equivalent). For those that do have a requirement the reports say that GCSE Grade C or equivalent is the minimum requirement for most degree programmes. Some degree programmes require a higher GCSE grade. An A-level Mathematics or equivalent requirement is unusual; Chemistry and Economics report this although even here it is restricted to a small number of universities. A minimum requirement of AS-Mathematics seems to be rare.

The actual attainment of students is a different matter and in practice the Mathematics entrance qualifications of the students are often higher than the minima and for some undergraduate programmes considerably so.

2.3.2 Pre-university Mathematics

Table 1 shows the proportions of UK students with A-level Mathematics in 2013 across the seven discipline areas together with Biology and how these have changed since 2006. In most disciplines there has been a steady increase, reflecting the increased numbers taking A-level Mathematics (see below). These students are unevenly distributed across university mission groups: 51% of Russell group entrants have A-level Mathematics compared to 43% for the 1994 group, 21% for the University Alliance, 16% for non-aligned universities (Rodiero and Sutch, 2013).

	JACS3 Subject Lines	2006	2009	2013
Biology	C1	26%	34%	38%
Business & Management	N1 & N2	10%	12%	15%
Chemistry	F1	53%	63%	71%
Computing related [2]	Group I	30%	32%	50%
Economics	L1	57%	65%	69%
Geography	F8 & L7	15%	18%	20%
Psychology	C8	9%	11%	13%
Sociology	L3	2%	4%	4%

Table 1: The proportion of UK students entering university with A-levels in 2006, 2009 and 2013 who have a full A2 A-level in Mathematics for each discipline. Source: UCAS data.[3]

2 Computing related includes all of Group I JACS3 Subject Lines in 2013. In 2006 and 2009, Computing related subjects were included within Group G, Mathematical and Computer Sciences. Hence, figures for Computing related subjects are calculated by aggregating JCAS3 Subject Lines, G4, G5, G6 and G7, and are not directly comparable to the 2013 figure. The Computer Science entrants with A-levels constitute about a third of total Computing related entrants, because many entrants to Computing related degree programmes have BTEC qualifications rather than A-levels.

3 We are grateful to UCAS for these data and note that UCAS cannot accept responsibility for any inferences or conclusions derived from the data by third parties. The data shown are proportions of the UK domiciled accepted candidates who have at least 1 GCE A-level qualification.

Table 2 shows the proportions of UK students with A-level and AS-level Mathematics in 2013.

	A-level	AS-level only	AS-level or above
Biology	38%	12%	50%
Business & Management	15%	6%	20%
Chemistry	71%	8%	79%
Computing related [4]	43%	5%	48%
Economics	69%	6%	76%
Geography	20%	8%	28%
Psychology	13%	5%	18%
Sociology	4%	2%	6%

Table 2: The proportion of UK students entering university with A-levels in 2013 who have A-level or AS-level in Mathematics for each discipline. Source: UCAS data. (The numbers in the final column are the sum of those in the previous two columns, subject to small rounding errors.)

Students with A-level Mathematics (or in Scotland with a Higher in Mathematics)[5] are relatively scarce. In 2010 approximately 13% of 16-18 year olds in the UK studied A-level Mathematics (Hodgen *et al.*, 2010)[6]. The Higher Mathematics participation rate in Scotland is greater than that for England, Northern Ireland and Wales at around 27% (Hodgen *et al.*, 2013).

A-level numbers in Mathematics have risen over the past decade in England, Northern Ireland and Wales. (See Figure 1 which shows participation trends in England between 1996 and 2013.) Following the introduction of the AS qualification in 2002 there was a fall but numbers have recovered and are now considerably above pre-2002 levels. In 2013, across the UK, 89,000 16-18 year olds gained the equivalent of A2 Mathematics and a further 39,000 took an equivalent of AS-Mathematics.

4 Computing related based on all JACS3 Group I Subject Lines.

5 The qualification system in Scotland is different. Students preparing for university entrance take a broader range of subjects, taking typically six Highers rather than A-levels. Highers are equivalent to AS-levels.

6 The UK's participation rate in advanced Mathematics at upper secondary is relatively low internationally, but comparable to that of Germany (Hodgen *et al.*, 2010).

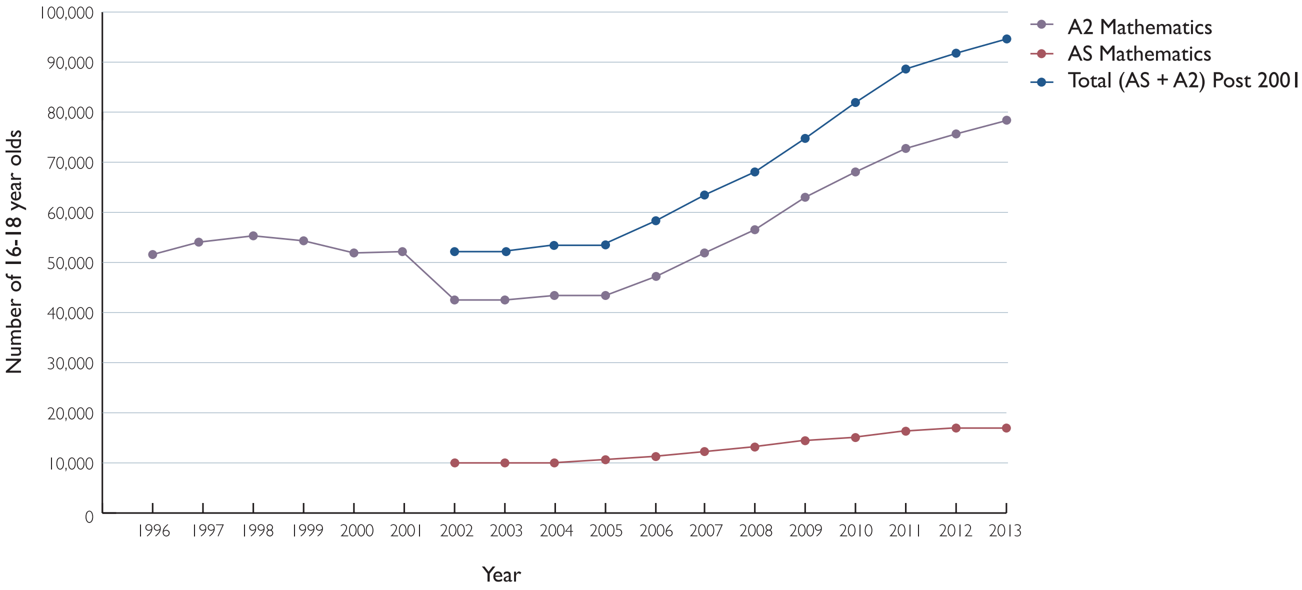

Figure 1: Numbers of 16-18 year olds taking A-level Mathematics in England between 1996 and 2013: A2, AS and total shown. Source: Joint Council for Qualifications (JCQ, 2014) data.[7]

It is unlikely that A-level participation will increase further. The increases in A-level participation have risen in parallel with GCSE grades, particularly the rising proportion of students achieving A and A* grades.[8] Given an increasing political concern with maintaining standards at GCSE, it seems unlikely that the proportion of A and A* grades at GCSE will increase any further. Consequently participation at A-level is likely to remain static at best, and may decrease.

A number of other reforms to A-level are currently being implemented. From 2015, AS will become a standalone qualification that is "de-coupled" from an A-level, so that students will not be able to use the results from their AS to contribute to a full A-level qualification, as they can at present. It is unclear how these changes will affect participation rates, although the experience of the introduction of AS-levels in 2001 as part of Curriculum 2000 suggests that Mathematics participation rates may be vulnerable. Intended to broaden the curriculum, as Figure 1 shows, it led to a large drop in numbers taking the full A-level and no overall increase in participation in Mathematics.

7 Although JCQ is used, the data overestimates participation numbers for AS Mathematics. The AS numbers are estimated using the same methodology as Advisory Committee on Mathematics Education (2011, p.8): "At AS and A-level, a new regulation took effect in 2009 allowing students to apply for certification on more than one occasion, with the result that some may be double-counted. … the changed regulation makes it impossible to know with any accuracy the number taking only AS Mathematics. The estimate is based on historical continuation rates of 83–85% from AS to A-level Mathematics." AS was first examined as part of Curriculum 2000 reforms in 2001. Although students first took AS examinations in 2001, most students "cashed in" the qualification in 2002 at age 18. Hence, AS numbers are first shown for 2002 not 2001.

8 Based on JCQ data, 2002-2011.

In England, Wales and Northern Ireland there is no widely available qualification that sits between GCSE and A/AS-level.[9] As a result, the rate of participation in study for post-16 mathematical qualifications is unusually low by international standards: around 20-26% of 16-18 year olds (Hodgen *et al.*, 2010, op cit). Almost all UK entrants to university have at least a C grade in GCSE Mathematics (or its Scottish equivalent). However Scottish students apart most will not have studied Mathematics since the age of 16 and close to 40% will have only a C grade at GCSE.[10]

The problem is further exacerbated by the limited mathematical content required in numerate subjects at A-level (SCORE, 2012; Nuffield, 2012). A recent consultation document from Ofqual proposes that ability to use mathematical skills at A-level appropriate for a number of GCE A-level subjects including the Sciences, Psychology, Geography, Business, Computer Science and Economics must be tested across the assessment objectives and proposes minimum weightings for each subject, so there is progress in prospect in this regard (Ofqual, 2013).

2.3.3 GCSE Mathematics

A detailed description of GCSE Mathematics is beyond the scope of this report. However given the role Statistics plays for all the disciplines considered here, it is important to note that the current Mathematics GCSE includes relatively little Statistics.

GCSE Mathematics is not a straightforward qualification. GCSE Mathematics can be taken at foundation or higher tier. Students who take a foundation tier course (for which the maximum they can achieve is a grade C) are likely to have followed a restricted curriculum. The distinction is important because by the time foundation tier students arrive at university they will have covered less ground than higher tier students with the same grade and are likely to find the going tougher. A revised GCSE Mathematics is being introduced that is intended to raise mathematical standards (DfE 2013a). The first cohort of students will be examined in 2017 and will enter higher education in 2019.

It would be helpful for higher education staff to have guidance indicating both the content of the existing and new GCSEs and the expectations for different grades in order to help them to judge the likely strengths and weaknesses of their students, and to inform the setting of entrance qualifications.

9 A further 13,500 studied some advanced Mathematics either as part of a Level 3 vocational qualification or by taking a Free-Standing Mathematics Qualification (FSMQ). The advanced Mathematics content in the FSMQ is relatively limited and equivalent to around a sixth of a full A-level.

10 In 2013, there were 433,600 UK domiciled students accepted to higher education in the UK (UCAS Analysis and Research, 2013). It is likely that these students' GCSE grades in Mathematics roughly follow the pattern across the UK and thus at least 40% have a Grade C in Mathematics (JCQ, n.d.).

2.4 Policy changes: new developments to improve the Mathematics and Statistics qualifications of university entrants

At the time of writing (Spring 2014) there are a number of policy initiatives underway that seek to address the mathematical preparation of UK university entrants. In particular, there is a political consensus behind the government's ambition that the overwhelming majority of young people in England should study Mathematics at least to age 18 by 2020. Key to this is the introduction of "Core Maths", a new level 3 Mathematics pathway suitable for the 40% of students, (over 200,000 each year) who achieve a grade C or better at GCSE but who do not progress into A-level or AS Mathematics.

"Core Maths courses will be distinct from A-level maths. The main purpose of the latter is to prepare students for higher level study with a significant mathematical focus, such as engineering, economics and the sciences. Core Maths will prepare students for further study and careers without such a mathematical focus but where mathematical knowledge and its application are nonetheless important, such as geography, business and the social sciences," (DfE, 2013b p5).

The detailed content and specification of these qualifications are under discussion, although they are likely to focus on Statistics, modelling and the application of largely GCSE level mathematical content and techniques. There is to be a focus on:

- "the application of mathematical knowledge to address problems and questions;
- representing situations mathematically; and
- use of mathematical and statistical knowledge to make logical and reasoned arguments in a variety of contexts"

(DfE, 2014).

"Core Maths" will be introduced for all schools and colleges from September 2015 and first assessments will take place in 2017. The Department for Education has encouraged higher education to participate in shaping this and other new qualifications (including the mathematical content of other A-levels). This provides an important strategic opportunity for higher education to influence the mathematical preparation of future students and the discipline reports unanimously recommend that key stakeholders should actively engage in these developments.

Scotland is also engaged in a broader-based reform, Curriculum for Excellence, which is at a later stage of implementation. Nevertheless, it provides some opportunity for higher education to influence the mathematical preparation of future students.

2.5 Teaching and learning

2.5.1 How is the teaching organised?

Mathematics and Statistics, or quantitative methods, are most often taught in compulsory standalone modules in year 1. More mathematically demanding degree programmes may also have compulsory standalone modules in year 2. Thereafter mathematical/statistical content is most often embedded within modules in the student's specialist subject area as needed. Several reports refer to the advantages of embedding this teaching, principally the motivation that comes when students see the point of the Mathematics/Statistics they are learning. It is however reliant on the availability of staff with the relevant expertise.

Additional resources and support are widely available to students to develop their mathematical and statistical skills. The reports refer to many online resources and support mechanisms including workshops, mentoring, revision and remedial classes. University Mathematics and Statistics support centres also provide opportunities for dedicated tutor support which reside outside the context of the student's home department.

The discipline reports refer to the importance of disseminating and sharing information about these resources and about good practice generally. Evidence in the reports identifies that students are generally made aware of resources that are available to them. However, survey responses also indicated that not all staff are aware of the full range of support opportunities that are available to students. In some cases it is left to the students to seek out and use these additional mechanisms and resources. There are comments throughout the reports to the effect that students do not take advantage of these resources as much as they should. Several of the reports recommend that more should be done to make sure that students are aware of them and are actively encouraged to use them. (See also comments on diagnostic testing in Section 2.5.3.)

For institutions setting up and running Mathematics/Statistics support centres, **sigma**, the network for excellence in Mathematics and Statistics support, provides guidance, resources and assistance. (**sigma**, 2013). The reports also mention the Q-step initiative (Nuffield, 2013) which, through major funding, is seeking to address strategically the quantitative skills training shortage in the Social Sciences.

2.5.2 Who does the teaching?

Survey respondents said that Mathematics and Statistics, or quantitative methods, are for the most part taught by department staff who feel confident in their subject knowledge. Very few have received training in the teaching of Mathematics/Statistics, although where postgraduate students are involved in providing supplementary teaching or support they do sometimes receive training. Sociology faces particular issues arising from the recent history of the discipline. (See the Sociology discipline report and references therein (Scott Jones and Goldring, 2014).)

2.5.3 Diagnostic testing

Diagnostic testing (LTSN, 2003) can be used to provide information about the actual level of students' mathematical/statistical knowledge and skills at a point in time and can be a powerful tool when used effectively. It is not usually part of summative assessment and students need clear reassurance when this is the case. Testing which is only used to provide information about the mathematical background of a student cohort has limited benefit. To maximise the potential benefits of diagnostic testing students need rapid feedback on their performance in a supportive way. It is essential that this is accompanied by structured follow-up activities that will enable students to develop their skills further and achieve higher levels of attainment. For students who struggle with mathematical and statistical work, (for the reasons given in the next section) a referral to a resource is unlikely to be sufficient, and more direct support is likely to be needed.

Several of the reports discuss the benefits that diagnostic testing can bring. In practice it appears to be used only rarely, but several of the reports include a recommendation that diagnostic testing should be used more often and more purposefully.

2.5.4 Issues and challenges

All the reports speak of students who enjoy the mathematical dimensions of their degree programmes and thrive on the challenges and insights that a mathematical approach brings. However, a consistent message from all the reports is that Mathematics and Statistics are for a variety of reasons a problem for many students and that this creates teaching challenges for university departments. Some students find it difficult to apply mathematical ideas, even apparently basic ones; some lack confidence in their abilities; some are surprised at the mathematical demands of their disciplines; and some shy away from quantitative approaches. The picture across the disciplines is not homogeneous, but some findings are reported consistently.

Anxiety and lack of confidence: Lack of confidence and anxiety about Mathematics/Statistics are problems for many students. This theme is identified as a key factor in nearly all the reports, cited by staff and students alike. It is clear that anxiety and lack of confidence are not merely symptomatic but that they influence and exacerbate the difficulties encountered by students in using Mathematics and Statistics.

Prior knowledge and attainment: A common theme throughout the reports is that not all students know, or can apply, mathematical ideas and methods that staff expect them to know, or to be able to apply. Issues that occur frequently are the following:

- Students struggle with apparently elementary ideas and methods that staff expect them to cope with easily, eg changing the subject of an equation, calculating percentages, interpreting graphs.
- Students struggle to apply techniques and methods to new problem areas, even when on the face of it the techniques and methods are ones they know well.
- Students appear to have been trained to answer certain kinds of problems, but become lost when the same knowledge is needed in unfamiliar contexts.

The discipline reports are not unusual in reporting these problems; similar findings are made in two recent reports about Biology (Koenig, 2011) and Physics (Institute of Physics, 2011).

Time elapsed since last used Mathematics: This problem is cited in most of the reports and is identified as important by students and staff alike. Students who have only studied to GCSE level have probably done no Mathematics or Statistics for two or more years. It seems highly likely that this lack of opportunity to use their mathematical and statistical skills is a major contributor to some students' lack of fluency and confidence. In Scotland students follow a broader curriculum and, as already noted, a higher proportion of students will have continued to study Mathematics in some form between the ages of 16 and 18. There is some illustrative evidence internationally that suggests that this may lead to fewer difficulties (Hodgen *et al.*, 2013 op cit).

Failure to see the relevance of Mathematics: A common problem, mentioned in all the reports, is a failure on the part of some students to appreciate the relevance of Mathematics to their subject. The picture here is complex, not least because there seem in some instances to be notable differences in the perceptions of students and staff.

Dealing with the range of prior attainment: All the reports are concerned, to a greater or lesser extent, with the problems that arise because of the range of attainment. This appears to be perceived by higher education staff as one of the biggest challenges facing the disciplines.

2.6 Staff and student expectations

In the surveys students were asked whether they expected to encounter Mathematics and Statistics as part of their degree programmes and staff were asked whether they thought the students' expectations were realistic.

In four of the disciplines (Business and Management, Chemistry, Geography, Psychology) there is a striking difference between the views of students and staff. Most students think they have realistic expectations and most staff think they do not. The data source for Economics is different but the picture is the same; students report that they are surprised by the amount of Mathematics/ Statistics.

2.7 Signalling

A clear conclusion from the evidence in the reports is that too many students arrive in higher education without a realistic understanding either of the relevance of Mathematics and Statistics to their discipline or of the demands that will be put on them.

There are multiple means by which information reaches students. These include, most obviously, departmental websites or prospectuses and the more general sources of information developed by subject groups, learned societies, professional bodies and others. A less direct but still powerful message will be given by the department's mathematical entrance requirements. This will be read alongside any further information (if any) on the department's website about the mathematical and statistical content of undergraduate degree programmes and the skills needed for those programmes.

A different form of signalling comes from subject A-levels. As discussed above, the reports observe that their subject A-levels do not adequately reflect the quantitative nature of their discipline. This is important both because students following those degree programmes will not have experience of using Mathematics in the context of their subjects and because it conveys a misleading message (SCORE, 2012; Nuffield, 2012. Op cit).

The unanimous recommendation in the reports is that universities should provide clear signalling to the pre-university sector about the mathematical and statistical (or quantitative methods) content of undergraduate degree programmes and the skills needed for those programmes. To support this, the reports call for better dialogue and understanding between the pre-university sector and higher education, a sentiment that reflects recent government policy statements (see for example DfE, 2013c). The outreach work that many universities do with schools and colleges has a potentially important role here. At the heart of this recommendation, as indeed of this report, is a desire that pre-university students should have a better understanding of what is expected of them and that higher education should have a better understanding of what their new undergraduates can do.

3 Recommendations

This overview report has drawn on the extensive evidence assembled in the discipline reports and has presented broader contextual information about the current state of pre-university education in Mathematics. Each of the discipline reports contains recommendations aimed at the specific needs of the individual disciplines. The following recommendations draw on these discipline-specific recommendations and findings and also on the wider educational and policy developments discussed in sections 2.3 and 2.4.

Recommendation 1

Key stakeholders in academic disciplines within higher education should provide clear signalling to the pre-university sector about the nature and extent of mathematical and statistical knowledge and skills needed in undergraduate degree programmes.

Recommendation 2

A key factor contributing to the problems some students experience with mathematical/statistical work at university is that they have undertaken no mathematical study since completing their Mathematics GCSE (or equivalent).

Higher education admissions tutors and those responsible for managing degree programmes should be aware of this fact and as part of their signalling to the pre-university sector should consider recommending the benefits of continuing with mathematical/statistical study beyond the age of 16. In so doing they should be aware that post-16 Mathematics encompasses a growing portfolio of qualifications, including the new "Core Maths" qualification currently under development, as well as the more established AS and A-level qualifications.

Recommendation 3

Key stakeholders in academic disciplines within higher education need a clear understanding of the knowledge and skills that students with GCSE Mathematics (or equivalent) at different grades can be expected to demonstrate when they start their undergraduate studies.

Suitable guidance documentation should be commissioned from qualified experts to provide this information. This should be written for a higher education audience and include a description of the typical range of knowledge, understandings and skills that could be expected of students at any given grade, and the mathematical and statistical difficulties that students might be expected to encounter in the transition to higher education.

Recommendation 4

Ongoing policy developments at both higher education and pre-university levels mean that there is currently a unique opportunity for higher education to influence the mathematical preparation of university entrants across the disciplines.

Key stakeholders in disciplines with mathematical/statistical content should actively engage with current and future developments of discipline A-levels (and their Scottish equivalents) as well as developments in post-16 Mathematics qualifications, (e.g. "Core Maths").

Recommendation 5

University staff with responsibility for managing degree programmes in disciplines with mathematical/statistical content should, (where they do not already do so), consider the potential benefits of diagnostic testing of students' mathematical and statistical knowledge and skills at the start of degree programmes, and of using the results to inform feedback and other follow-up actions.

Recommendation 6

Many universities already provide additional support in Mathematics and Statistics at institutional level, which resides outside the structure of any individual degree programme.

Members of staff responsible for managing degree programmes should ensure that all staff are aware of this support. Teaching staff and tutors should actively encourage students to make use of these resources and opportunities for assistance with their mathematical/statistical work.

Observation

In addition to these high level recommendations it is notable that several of the discipline reports have recommended that the QAA subject benchmark statements for their disciplines should be revised to include more specific information on the type and level of Mathematics and Statistics that should be present in undergraduate degree programmes. Other disciplines in higher education may wish to consider their subject benchmark statements from a similar perspective.

4 Conclusion

This report and the associated discipline reports on which it draws have described a series of issues and challenges that affect a very large number of students in UK universities. For all their prevalence these issues and challenges are not intractable and the reports contain many examples that show how universities are making constructive interventions to enable students to overcome their difficulties. Current policy developments in higher education and the pre-university sector mean that there is opportunity for substantial progress to be made over the next few years, so that students will arrive at university better prepared and better able to cope with the mathematical and statistical demands of their undergraduate studies. A key factor in achieving this progress will be dialogue and the development of mutual understanding between the sectors. By drawing on evidence from both domains the hope is that this report has exemplified and contributed to that process.

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