Lesson Plan: Independent and Mutually Exclusive Events

Aim

For students to understand ideas of independence and mutual exclusivity

Background

Students should have encountered conditional probability and may be the multiplication law and addition laws of probability

Rationale

The main activity, the worksheet, is very structured and hopefully leads students to the results desired without the teacher explaining to the whole class In classrooms. Where group work is the norm this might be an occasion when you ask students to work on something by themselves before any discussion takes place. However, going through this worksheet in pairs (just one copy between two) would encourage collaborative learning'.

Thought needs to be given on how the discussion during the last part of the lesson is managed eg: in fours- one person selected after discussion to feedback , teacher led whole class etc ?

By the end of the lesson

Students should understand that

- Independent events are ones which have no effect on the probability of each other happening.
- Mutually exclusive events cannot both happen at the same time

They should also be more confident in working through a situation with minimal guidance and spotting results for themselves.

The next lesson

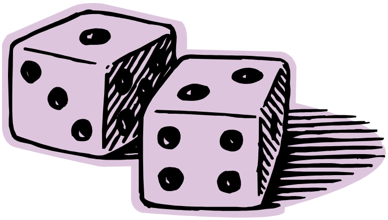
Students will work on various probability problems pulling all the probability topics together and even tackle exam questions.

Resources: display pdf (below), Mini white boards, pens, wipes, calculators, worksheets (p3 - 5 in this pdf doc)

<table>
<tr><th></th><th>CONTENT</th><th>ACTIVITY/ QUESTIONS</th><th>COMMENTARY</th></tr>
<tr><td>INTRODUCTION (5 mins)</td><td>Tossing two dice (recap of idea of conditional probability)</td><td>Imagine I roll two dice, one black and one white. I can see the outcome but you can’t. As far as you’re concerned, the probability that I rolled a 6 on the black die is obviously $\frac{1}{6}$.
I now give you some additional information: the score on the black die is greater than the score on the white.
What difference does this make?
.see diagram (in this doc below)
Taking this new information into account you can now say that the probability that I rolled a 6 on the black die is $\frac{1}{3}$.
ie: $P\left(B=6 \mid B>W\right)=\frac{1}{3}$.
This is what we mean by conditional probability</td><td><table>
<tr><td></td><td></td><td colspan="6">Black die</td></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td rowspan="6">White die</td><td>1</td><td></td><td>×</td><td>×</td><td>×</td><td>×</td><td>×</td></tr>
<tr><td>2</td><td></td><td></td><td>×</td><td>×</td><td>×</td><td>×</td></tr>
<tr><td>3</td><td></td><td></td><td></td><td>×</td><td>×</td><td>×</td></tr>
<tr><td>4</td><td></td><td></td><td></td><td></td><td>×</td><td>×</td></tr>
<tr><td>5</td><td></td><td></td><td></td><td></td><td></td><td>×</td></tr>
<tr><td>6</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table></td></tr>
<tr><td>ACTIVITY (30 + mins)</td><td>Worksheet</td><td>Worksheet involves:
conditional probability,
dependent events,
independent events and
mutually exclusive events.</td><td>Students to work through this worksheet, in pairs ideally and their outcomes/discoveries are used to inform the discussion in the next stage of this lesson.</td></tr>
<tr><td rowspan="2">PLENARY(mins)</td><td>Class Discussion</td><td>Discussion point 1
Two events are either dependent or independent. Is it possible for two events to be:
both dependent and mutually exclusive?
both independent and mutually exclusive?</td><td>These discussion points are copied below for display

Discussion in groups first (pairs of pairs?)followed by whole class may be appropriate here</td></tr>
<tr><td></td><td>Discussion point 2
A bag contains 5 green cubes and 4 yellow cubes. Two cubes are removed.
Together or separately ?</td><td>see tree diagram</td></tr>
<tr><td></td><td></td><td>Discussion point 3
Find context and $P(A)=\frac{4}{12}$ find :
$P(B),\quad P(A\cap B),\quad P(A|B),\quad P(B|A)$</td><td>see venn diagram</td></tr>
</table>

Thinking About Probability



Imagine I roll two dice, one blue and one red and you can't see what the outcome is.

The 36 outcomes can be shown using a possibility space:

		Blue die					
		1	2	3	4	5	6
Red die	1	(1,1) Sum=2	(1,2) Sum=3	(1,3) Sum=4	(1,4) Sum=5	(1,5) Sum=6	(1,6) Sum=7
	2	(2,1) Sum=3	(2,2) Sum=4	(2,3) Sum=5	(2,4) Sum=6	(2,5) Sum=7	(2,6) Sum=8
	3	(3,1) Sum=4	(3,2) Sum=5	(3,3) Sum=6	(3,4) Sum=7	(3,5) Sum=8	(3,6) Sum=9
	4	(4,1) Sum=5	(4,2) Sum=6	(4,3) Sum=7	(4,4) Sum=8	(4,5) Sum=9	(4,6) Sum=10
	5	(5,1) Sum=6	(5,2) Sum=7	(5,3) Sum=8	(5,4) Sum=9	(5,5) Sum=10	(5,6) Sum=11
	6	(6,1) Sum=7	(6,2) Sum=8	(6,3) Sum=9	(6,4) Sum=10	(6,5) Sum=11	(6,6) Sum=12

Obviously the score on the red die is **independent** of the score on the blue die.

Now you are asked to think about events which might be dependent or they might be independent but it won't be quite so obvious which!

Imagine you are interested in the following events:

Event A: You roll a double.

Event B: The sum of the two scores is even.

Event C: The score on the blue die is greater than the score on the red die.

Event D: You get a 6 on the red die.

It might help to shade the events on the table above with different colours

1. The probabilities of these four events are:

P(A)= P(B)= P(C)= P(D)=

Now think about two of these events **both** happening in one roll of the two dice. The probability that events A and D both occur is $\frac{1}{36}$ because only a double 6 satisfies the requirements.

2. Now work out and fill in the other five probabilities:

$P(X \cap Y)$	D	C	B
A	$\frac{1}{36}$		
B			
C			

3. Circle each pair of events which are mutually exclusive:

A & B A & C A & D B & C B & D C & D

Explain how you decided. You should also explain what you understand by the term 'mutually exclusive':

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

4. Circle the pairs of events for which $\mathrm{P}(X \cap Y) = \mathrm{P}(X) \times \mathrm{P}(Y)$:

A & B A & C A & D B & C B & D C & D

Now imagine that I've rolled the two dice and I tell you that the score on the red die is 6; in other words event D has occurred. You should be able to see that you are now concerned with only 6 of the initial 36 outcomes.

5. Given this information, what can you say about the probabilities that the other events have also occurred in this one throw? Fill in the conditional probabilities:

$\mathrm{P}(A|D)=\frac{1}{6}$ because……………………………………………………………….

……………………………………………………………………………………………

$\mathrm{P}(B|D)=$…………………………………………………………………………..

……………………………………………………………………………………………

$\mathrm{P}(C|D)=$…………………………………………………………………………..

……………………………………………………………………………………………

6. Using your answers to question 5 and with further calculations circle the correct statements:

$\mathrm{P}(A|D)=\mathrm{P}(A)$ $\quad$ $\mathrm{P}(B|D)=\mathrm{P}(B)$ $\quad$ $\mathrm{P}(C|D)=\mathrm{P}(C)$

$\mathrm{P}(A|C)=\mathrm{P}(A)$ $\quad$ $\mathrm{P}(B|C)=\mathrm{P}(B)$ $\quad$ $\mathrm{P}(A|B)=\mathrm{P}(A)$

7. What is the link between your answers to questions 4 and 6? Explain.

……………………...……………………………….

……………………...……………………………….

ANSWERS (Also see discussion points below)

1. P(A)= $\frac{1}{6}$ P(B)= $\frac{1}{2}$ P(C)= $\frac{15}{36}$ P(D) = $\frac{1}{6}$

2.

$P(X \cap Y)$	D	C	B
A	$\frac{1}{36}$	0	$\frac{1}{6}$
B	$\frac{3}{36}$	$\frac{1}{6}$	
C	0		

3. The mutually exclusive events are A & C and C & D

4. The events for which $\mathrm{P}(X \cap Y) = \mathrm{P}(X) \times \mathrm{P}(Y)$ are A & D and B & D

5. $\mathrm{P}(A|D) = \frac{1}{6}$ $\mathrm{P}(B|D) = \frac{1}{2}$ $\mathrm{P}(C|D) = 0$

6. The correct statements are $\mathrm{P}(A|D) = \mathrm{P}(A)$ and $\mathrm{P}(B|D) = \mathrm{P}(B)$

7. $\mathrm{P}(X \cap Y) = \mathrm{P}(X) \times \mathrm{P}(Y)$ and $\mathrm{P}(X|Y) = \mathrm{P}(X)$ are equivalent ways of saying events X and Y are independent

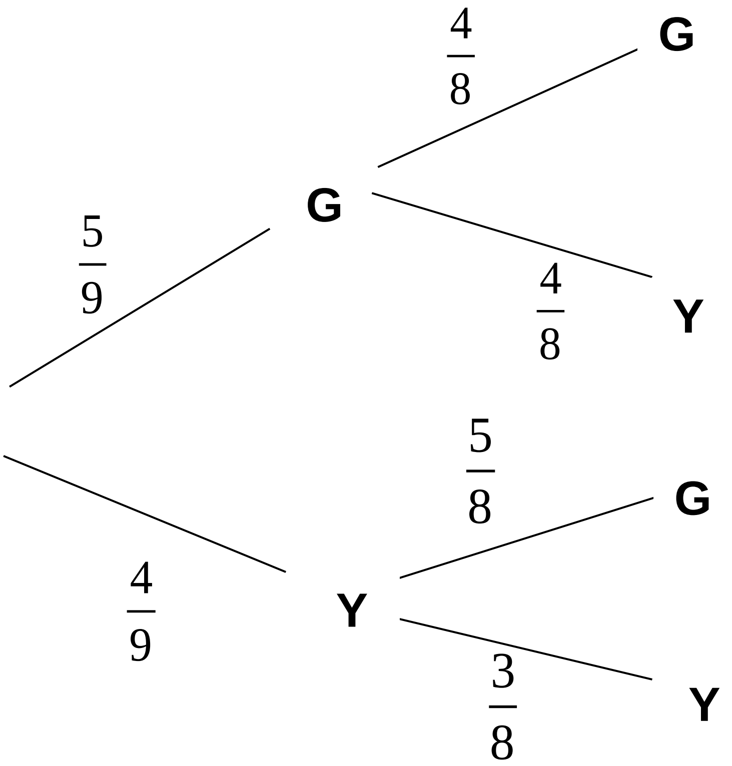
Discussion point 1

Two events are either dependent or independent.
Is it possible for two events to be:

- both dependent and mutually exclusive?
- both independent and mutually exclusive?

Discussion point 2

A bag contains 5 green cubes and 4 yellow cubes. Two cubes are removed.

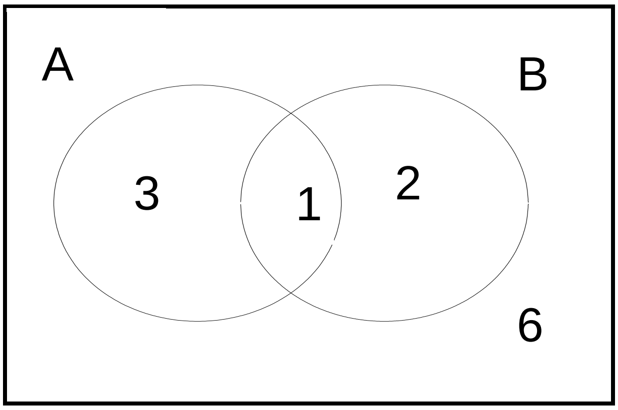


Were the two cubes withdrawn together or was one removed first and then the other?

I remove two cubes. Given that they are the same colour what is the probability that they are both yellow?

Discussion point 3

Give a context to this Venn diagram and answer the questions:



$P(A)=\dfrac{4}{12}$.

Find: $P(B)$, $P(A\cap B)$, $P(A|B)$, $P(B|A)$