Lesson Plan for Introducing the Binomial Distribution

Aim

For students to understand that we use the binomial probability distribution to model the numbers of successes, when n independent events occur,

Background

Students are likely to have encountered ideas of probability and independence. They would have used tree diagrams to illustrate consecutive events and understand the concept of conditional probability.
They may or may not have encountered the binomial expansion in core maths and know about ${}^nC_r$; this will make a difference to how this topic is approached, and it would be helpful to find out about prior knowledge before planning a set of lessons on the binomial distribution.

Rationale

Studying Statistics without ever tossing dice in class is unthinkable, and this topic is an ideal one in which to do so. Calculating relative frequencies reminds students what probabilities are all about, and producing results themselves makes it very clear what one means by '5 fours' say.

The fact that the relative frequencies in this activity do usually turn out to be close to those predicted by the mathematical model is as convincing to some students as a mathematical argument.

Note: neither 'sixes' nor 'six dice' are used in this activity because having too many sixes can cause some students confusion; P(X = score on a dice) = 1/6 as well! Historically, the interest of gamblers when tossing dice was always in how many sixes occur when a certain number of dice are tossed, but the study of such problems can wait until later if there are any doubts about the capacity of the class to cope easily with this introductory lesson.

By the end of the lesson

Students should understand that the binomial distribution is a good model for a scenario with has:

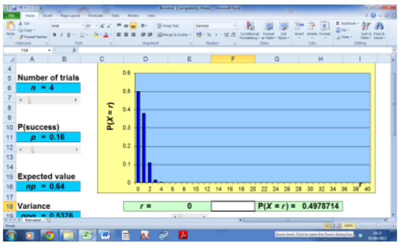
- a success or fail situation
- a series of independent events
- a constant probability of success

Students should also have a picture of what a binomial distribution looks like.

The next lesson

Students will be able to recognise what situations are appropriate to model using the binomial distribution, and calculate simple probabilities of the form P(X=a) where X is a discrete random variable which is binomially distributed.

Resources: display pdf (below) or ppt , 4 dice per pair, record sheet per student (below), mini white boards, pens, wipes, calculators, Autograph open

	CONTENT	ACTIVITY/ QUESTIONS	COMMENTARY
INTRODUCTION (5 mins)	**Setting the scene**	Display 'The experiment' sheet p3 Possible Questions: i) What do you think will be the most common number of fives? Vote – record result. ii) Can you work out the probability of being very lucky and getting 4 fives?	
MAIN ACTIVITY (30+ mins)	**The experiment and the theory** **p4 , p5, p6**	Students work in pairs , both i) record the experimental results ii) work on theoretical probabilities iii) Plot their relative frequency columns next to the ones on the printed Autograph page	The theoretical results page should be structured enough for most students to derive the probabilities working in pairs – however, some classes may benefit from more guidance via a whole class discussion after the results have been collected. There is no need to explore the ^nC_r formula at this stage, beyond being short hand for the lists they produce (number of ways of choosing r objects from n)
PLENARY(20+ mins)	**Exploring the binomial distribution together.**	a) Using the spread sheet and with starting with n= 4 and p = 0.166 i) check the students' theoretical calculations (table on right) ii) explore what happens to the graph and the heights of the columns if: a) n changes (had more dice) b) p changes iii) ask for explications of what P(X=r) means in words iv) show ^nC_r button on calculator b) Introduce concepts of a discrete probability distribution parameters Binomial PLENARY QUESTIONS If you had 5 dice write down expressions for calculating P(X = r) r is some of 0,1,2,3,4,5	

BINOMIAL DISTRIBUTION

The Experiment

Toss 4 dice 40 times and record how many fives you get each time.

Number of fives																				

Make a tally and work out the relative frequency of getting no fives, 1 five, 2 fives, 3 fives or 4 fives.

Number of fives	*tally*	*frequency*	$Relative\ frequency = \frac{frequency}{40}$
0			
1			
2			
3			
4			

Total = 40 ✓

The Theory

Number of fives	*workings*	***probability***
0		
1		
2		
3		
4		

Total = 1 ✓

List of ways of getting 0 fives on 4 dice	List of ways of getting 1 five on 4 dice	List of ways of getting 2 fives on 4 dice	List of ways of getting 3 fives on 4 dice	List of ways of getting 4 fives on 4 dice
	5 X X X **X 5 X X** **X X 5 X** **X X X 5**			
${}^4C_0 =$	${}^4C_1 = 4$	${}^4C_2 =$	${}^4C_3 =$	${}^4C_4 =$

This graph shows the theoretical probabilities when there are 4 independent events each with a probability of success of 1/6.
Plot the results for your experimental relative frequencies alongside these columns.