Mathematical Problem Solving
AS/A Level example

Example 1

The class have been studying straight line coordinate geometry and have covered the gradient and equation of a straight line as well as the gradient conditions for two straight lines to be either parallel or perpendicular.

Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; gradient conditions for two straight lines to be parallel or perpendicular.

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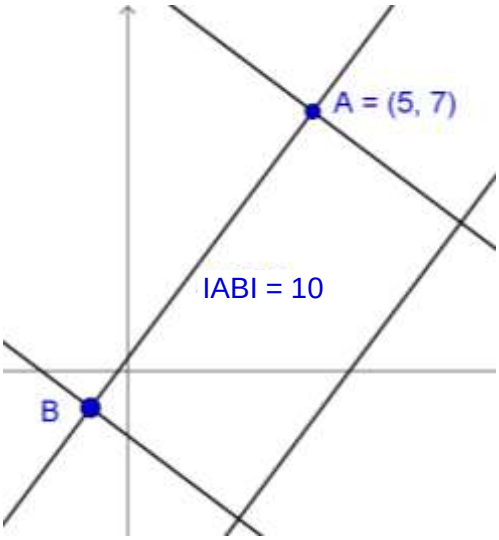
In order to check the students' understanding of the methods used, the teacher presents the class with this problem on the board.



Find the equations of the four straight lines in this diagram.

At this point it is important that the teacher makes it clear that the task is also to develop the students' problem-solving skills. The students need to be aware that there is some vital information missing and that they will have to ask questions to find that information. The teacher should point out that some obvious questions such as "what are the coordinates of the other points?" will not be answered.

The students should be given two minutes time to consider the problem without writing anything down. They should then spend two more minutes writing down as many questions as they can that would give key information for solving the problem. Students should then spend two more minutes comparing their questions with those of one or two other students. It is important at this stage for the students to say why they think their question will reveal some important information.

The students should hopefully come up with some questions like these:

- Does the line through AB pass through $(0,0)$ – is the diagram just badly drawn?
- Are those lines perpendicular? Are those lines parallel?
- Do we know the coordinates of any of the other points?
- Do we know the lengths of any of the other line segments?
- Are the scales on the x axis and the y axis the same?
- Are the x and y coordinates of each point integer values?
- Is there only one solution?

These are all questions that the teacher should be prepared to answer. By answering these questions, the teacher should be able to give the students enough information to be able to solve the problem.

The teacher should encourage and model the use of accurate mathematical language wherever possible.

Working through the questions in order:

Question	Answer
Does the line through AB pass through $(0,0)$?	No, the line AB passes above $(0,0)$.
Are those lines perpendicular?	Adjacent lines are perpendicular.
Do we know the coordinates of any of the other points?	No, that would make it too easy to solve!
Do we know the lengths of any of the other line segments?	Yes, given that adjacent lines are perpendicular, we now know that the line segment opposite AB must also be of length 10 units.
Are the scales on the x axis and the y axis the same?	Yes, but this is now redundant information. It will not give any additional help in solving the problem. **Note:** the teacher may ask why it is now redundant information

Question	Answer
Are the x and y coordinates of each point integer values?	Yes, they are. This is a key question. Knowing this should 'unlock' the problem. **Note:** the teacher may have to prompt to get this question.
Is there only one solution?	That's part of the problem – some problems do have more than one solution. We'll have to find that out.

There may well be other sensible questions that the students ask and the teacher should be prepared to answer these.

The students should consider the answers to each question and how that may help them to solve the problem. At each stage the teacher can give prompts to help the students think, in particular encouraging them to combine pieces of information e.g. "The length of AB is 10 units long and point B is an integer. How does that help you?"