

Mathematical Problem Solving

GCSE example

Example 10

A class has been studying indices. The teacher uses the following problem, which includes indices, to assess their problem solving skills.

- **calculate with roots, and with integer and fractional indices**

DfE: Mathematics GCSE subject content and assessment objectives 2013

Five numbers are arranged in order from least to greatest:

$$x, x^3, x^4, x^2, x^0$$

Where does $-x^{-1}$ belong in the list above?

Possible questions and prompts

The questions and prompts are examples in each case. A teacher helping a student would not necessarily ask any or all of these. Questions and prompts should be selected based on the ability of the student the teacher is helping.

Problem	Stimulus questions	Additional questions	Direct prompt
A student really doesn't know where to start.	Are all of these values unknown? Could you give a numerical (number) value for one of these? What indices facts do you know? Do any of them help here?	If that is 1 and the numbers are in order, what does that tell you about the other numbers? What does this tell you about x which is over here (point)?	One of the numbers in the list given doesn't depend on x , which one? What does this tell you about x ?

Problem	Stimulus questions	Additional questions	Direct prompt
The student has identified that the greatest value is 1 and that $x < 1$ but is trying to find out the value of x .	<p>What information have they given you in the problem? Is it enough to find x?</p> <p>Do you really need to find x?</p>	<p>Do you think it is possible to find the value of x?</p> <p>Would a number line help?</p>	Draw a number line and mark on the region where x could be.
The student has identified that the greatest value is 1 and that $x < 1$ but is unable to draw any more conclusions.	<p>Look at where x^2 is. What does this tell you about the value of x?</p> <p>When you square a number which is between 0 and 1 is the answer bigger or smaller than the number you started with?</p>	<p>Draw a rough number line with -1, 0 and 1 on it. Where would x^2 be if x was here? Or here? (pointing to a few positions on the line)</p> <p>Combined with the information you've been given what does this tell you about x?</p>	
The student has identified that $-1 < x < 1$ but is unable to draw any more conclusions.	<p>Look at x^3 and x^4. Which one is greater?</p> <p>Pick an easy fraction between 0 and 1. Square it, cube it and raise it to the power of 4. Which order do they go in?</p>	<p>How can you simply change your fraction to make x^4 greater than x^3? Remember that x is between -1 and 1.</p>	

Problem	Stimulus questions	Additional questions	Direct prompt
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The student has identified that $-1 < x < 0$ but cannot think how to position $-x^{-1}$

What does the power -1 mean?

If you find the reciprocal of a number between 1 and 0, what happens to the number?

What happens if the number is between -1 and 0? Where would it be on this number line? What happens to it when you change its sign?